



STATE RESEARCH CENTER OF RUSSIA  
INSTITUTE FOR HIGH ENERGY PHYSICS

IHEP 2000-36

A.A. Bogdanov<sup>1</sup>, S.B. Nurushev<sup>1,2</sup>, A. Penzo<sup>3</sup>, M.F. Runtzo<sup>1</sup>,  
O.V. Selyugin<sup>4</sup>, M.N. Strikhanov<sup>1</sup>, A.N. Vasiliev<sup>2</sup>

**NEW APPROACHES  
TO THE  $PP$  TOTAL CROSS SECTION MEASUREMENTS  
AT POLARIZED COLLIDERS**

Submitted to *Nuclear Physics*

---

<sup>1</sup> Moscow Engineering Physics Institute, Kashirskoe Ave. 31, 115409 Moscow, Russia

<sup>2</sup> Institute for High Energy Physics, 142284 Protvino, Moscow Region, Russia

<sup>3</sup> Universita di Trieste and sezione INFN, Triest, Italy

<sup>4</sup> BLTPH, JINR, Dubna, Russia

**Abstract**

Bogdanov A.A., Nurushev S.B., Penzo A. et al., New approaches to the  $pp$  total cross section measurements at polarized colliders: IHEP Preprint 2000-36. – Protvino, 2000. – p. 14, figs. 3, tables 5, refs.: 25.

It is proposed to extract the  $pp$  total cross section,  $\sigma_T(pp)$ , from the analyzing power measurement in elastic  $pp$  scattering in the Coulomb-Nuclear Interference region. Contributions to an accuracy of  $\sigma_T(pp)$  determination are estimated, which arise from different sources, including the single spin-flip interaction. Applicability of the factor of merit to the extraction of the  $\sigma_T(pp)$  from experimental data is briefly discussed. The conclusion is made that under some conditions the precisely measured analyzing power,  $A_N(t)$ , might be a good approach for  $\sigma_T(pp)$  determination.

**Аннотация**

Богданов А.А., Нурушев С.Б., Пенцо А. и др. Новые подходы к измерениям полных сечений на поляризованных коллайдерах: Препринт ИФВЭ 2000-36. – Протвино, 2000. – 14 с., 3 рис., 5 табл., библиогр.: 25.

Предлагается определять полное сечение  $pp$ -взаимодействия,  $\sigma_T(pp)$ , из измерения анализирующей способности упругого  $pp$ -рассеяния в области Кулон-ядерной интерференции. Оцениваются вклады в точность определения  $\sigma_T(pp)$ , возникающие от разных источников, включая взаимодействие с однократным переворотом спина. Кратко обсуждается применимость фактора качества для извлечения  $\sigma_T(pp)$  из экспериментальных данных. Делается заключение, что точно измеренная анализирующая способность,  $A_N(t)$ , при некоторых условиях может быть хорошим подходом для определения  $\sigma_T(pp)$ .

## Introduction

It has been recently suggested to apply a new approach to the measurements of the  $pp$  total cross section,  $\sigma_T(pp)$ , at present and future accelerators [1]. The approach relies on the relation between  $\sigma_T(pp)$  and the position of maximum of the so called factor of merit,  $M(t) = A_N^2(t)d\sigma/dt$ , introduced in polarimetry [2]. There is another observable, analyzing power,  $A_N(t)$ , which is also sensitive to the total cross section. These new concepts are very attractive for us, since the  $pp2pp$  (R7) experiment at RHIC [3] aims to measure independently three observables  $\sigma_T(pp)$ ,  $d\sigma(t)/dt$  and  $A_N(t)$  and able to check feasibility of the proposed new methods. This paper is devoted to a detailed study of new concepts and organized in the following way. In section 1 we discuss briefly standard techniques of measuring the total cross sections at unpolarized colliders in order to illustrate the reached accuracy in  $\sigma_T(p\bar{p})$  and  $\sigma_T(pp)$  measurements. In section 2 we outline possible new techniques which might be applicable to the total cross section determination at unpolarized and polarized colliders. Section 3 is devoted to the application of  $A_N(t)$  and  $M(t)$  to the E704 experimental results. In section 4 we discuss the simulated data of pp2pp experiment at RHIC and the expected corrections to  $\sigma_T$  from different sources. In section 5 we make an estimate of the single spin flip contribution. The last section summarizes our study of the new approaches to  $\sigma_T(pp)$  extraction at the polarized colliders.

### 1. Survey of Schemes of Total Section Measurements

The total cross section measurements at colliders (ISR,  $Spp\bar{S}$  and Tevatron) have been made by four methods. They are the following:

**1.1. The direct method.** This means the measurement of the total interaction rate,  $\dot{N}_T$

$$\dot{N}_T = \dot{N}^{el} + \dot{N}^{inel}, \quad (1)$$

where  $\dot{N}^{el}$  and  $\dot{N}^{inel}$  are the total elastic and inelastic rates measured respectively by Elastic Scattering Detector (ESD) and Inelastic Scattering Detector (ISD). The apparatus must cover a  $4\pi$  solid angle. In reality the vacuum pipe, the beam parameters, and backgrounds in colliders put a limit on a minimum acceptable angle  $\Theta_{min}$ . Therefore, the measurements are made up to this angle and then one makes an extrapolation to correct the  $\sigma_{obs}$  (observed cross section) for undetected events. For such an extrapolation one needs to use a phenomenological model for hadron amplitudes and this is a weak point of this approach.

Assuming that the machine luminosity  $L$  is known from an independent measurement, the total cross section can be determined through the relation

$$\sigma_T = N_T/L, \quad (2)$$

where  $N_T$  and  $L$  are summed up over a whole run. The examples of such a direct measurement of  $\sigma_T$  are presented in Table 1, column I, taken from experiments [4] and [5]. It is evident, that the best precision in  $\sigma_T$  of order 0,5 - 1% was reached at ISR by thorough application of the special techniques for improving the beam quality. The precision of  $\sigma_T$  measurement at Tevatron was limited to  $\pm 7.5\%$  by an accuracy of the luminosity determination.

Table 1. Summary of experimental data on  $\sigma_T$  measurements at unpolarized colliders

Facility, reaction, experiment.	$\sqrt{s}$ GeV	$\sigma_T, mb$				Ref.
		I Dir. Meth.	II Indir. Meth.	III L.I. Meth.	IV Coul. Scat.	
<u>Tevatron, <math>p\bar{p}</math></u>						
E710	1800			$71.71 \pm 2.02$		[11]
	1800	$78.3 \pm 5.9$				[5]
CDF	546			$61.26 \pm 0.93$		[12]
	1800			$80.03 \pm 2.24$		[12]
	1800		$72.0 \pm 3.6$			[6]
<u><math>Sp\bar{p}S</math></u>						
UA1	540		$67.9 \pm 5.9$			[7]
UA4	541		$63.0 \pm 1.5$			[8]
UA4/2	541		$63.0 \pm 2.1$			[9]
UA4	546			$61.9 \pm 1.5$		[10]
<u>ISR</u>						
$pp$	23.5	$38.80 \pm 0.25$	$39.01 \pm 0.27$	$39.22 \pm 0.55$		[4]
$pp$	30.6	$40.07 \pm 0.24$	$40.38 \pm 0.31$	$40.53 \pm 0.62$		
$pp$	44.7	$41.90 \pm 0.24$	$41.45 \pm 0.23$	$41.00 \pm 0.43$		
$pp$	52.8	$42.71 \pm 0.35$	$42.38 \pm 0.27$	$42.02 \pm 0.47$		
$pp$	62.7	$42.96 \pm 0.38$	$43.07 \pm 0.30$	$43.20 \pm 0.54$		
$pp$	23.0				$38.9 \pm 0.7$	[14]
$pp$	31.0				$40.2 \pm 0.8$	

**1.2. The indirect method.** This is based on the optical theorem relating the differential cross section of  $pp$  nuclear elastic scattering in forward direction  $(\frac{d\sigma^{el}}{dt})_{t=0}$  and the total cross section  $\sigma_T$  :

$$\left(\frac{d\sigma^{el}}{dt}\right)_{t=0} = \frac{\sigma_T^2(1 + \rho^2)}{16\pi(\hbar c)^2}, \quad (3)$$

where  $\rho$  is a ratio of the real to imaginary part of the nuclear spin nonflip amplitude. Such presentation assumes that any spin contribution is negligible. Usually the differential cross section  $\frac{d\sigma^{el}}{dt}$  is presented in the region of nuclear scattering as

$$\frac{d\sigma^{el}}{dt} = \left(\frac{d\sigma^{el}}{dt}\right)_{t=0} \cdot e^{bt}, \quad (4)$$

where  $b$  is a slope parameter. Therefore,  $(\frac{d\sigma^{el}}{dt})_{t=0}$  should be defined by extrapolation of elastic events (measured)  $\frac{dN^{el}(t)}{dt}$  to the  $t = 0$  point:

$$\frac{dN^{el}(t)}{dt} = L \cdot \frac{d\sigma^{el}}{dt} = L \left( \frac{d\sigma^{el}}{dt} \right)_{t=0} \cdot e^{bt} = \frac{dN^{el}(0)}{dt} \cdot e^{bt}, \quad (5)$$

where

$$\frac{dN^{el}(0)}{dt} = L \cdot \left( \frac{d\sigma^{el}}{dt} \right)_{t=0}. \quad (6)$$

From relations 3 and 6 one can extract

$$\sigma_T = \frac{1}{\sqrt{L}} \left[ \frac{16\pi(\hbar c)^2}{(1 + \rho^2)} \cdot \frac{dN^{el}(0)}{dt} \right]^{1/2}. \quad (7)$$

So, taking  $L$ , luminosity, and  $\rho$  value from independent measurements, one can determine  $\sigma_T$ . The fractional error in  $\sigma_T$  due to the uncertainty in the luminosity is half that in the first method. This is the simplest way to determine  $\sigma_T$ . The examples are presented in Table 1, column II ( [4], [6], [7], [8], [9]). The most limiting contribution comes from uncertainty in the luminosity  $L$  measurements at Tevatron (CDF) and  $Spp\bar{p}S$  (UA1). UA4 was able to reach the best precision of order  $\pm 2\%$  by a thorough measurement of beam parameters. As it seen the best accuracy in  $\sigma_T$  measurement was achieved at ISR.

**1.3. The Luminosity Independent Method.** In order to avoid the luminosity measurement, the following technique was invented [4], [10], [11], [12]. From (2) and (7) one gets an expression without luminosity

$$\sigma_T = \frac{16\pi(\hbar c)^2}{(1 + \rho^2)N_T} \frac{dN^{el}(0)}{dt}. \quad (8)$$

Two types of detectors, ESD and ISD, must be used in this case. Additionally it is assumed that  $\rho$  is known. The results of using such method are given in column III. It is seen that the best precision in  $\sigma_T$  measurement was reached at ISR. We can note an inconsistency of order 10% between E710 and CDF measurements at  $\sqrt{s}=1800$  GeV which may illustrate a hidden systematic error.

The above three approaches to the  $\sigma_T$  measurement need to extrapolate the measured quantities to the zero degree. The inelastic scattering data are extrapolated by gaussian function in the production angle or by exponential or other functions. The final result does not seem very sensitive to the exact shape of the curve. But the elastic scattering data were constrained by several conditions like:

- spin contributions are neglected;
- the imaginary part of the nuclear amplitude has an exponential form in the momentum transfer in the small  $t$  region;
- the real and imaginary parts of the nuclear amplitude have the same  $t$ - dependence, thus  $\rho$  is independent of momentum transfer.

The last two requirements were questioned in paper [13].

**1.4 The Coulomb Scattering.** This method is based on the theoretically very reliable differential cross-section and allows one to avoid the luminosity measurement.

$$\frac{d\sigma_c(t)}{dt} = \frac{4\pi\alpha^2(\hbar c)^2}{t^2} F^4(t), \quad (9)$$

where  $\alpha$  is the fine structure constant and  $F(t)$  is the proton electromagnetic form factor. Its application requires the reaching a net Coulomb scattering region, which sits at  $|t| < |t_0|$ , where  $|t_0| = 1.6 \cdot 10^{-3}(\text{GeV}/c)^2$  (for  $\sigma_T = 40\text{mb}$ ) is a point where the Coulomb cross section is equal to the nuclear one. At varying initial momentum  $p_{in}$  the Coulomb scattering angle should vary as

$$\Theta_0(\text{mrad}) = \frac{\sqrt{|t_0|}}{p_{in}} \approx \frac{42}{p_{in}}. \quad (10)$$

For ISR momentum  $p_{in} = 15 \text{ GeV}$   $\Theta_0 \approx \frac{42 \cdot 10^{-3}}{15} \approx 3\text{mrad}$ . This corresponds to the displacement of the scattered proton from the beam axis of about 3 cm at the end of the ISR 10 m long straight section. The experiment [14] was able to use this technique at two energies  $\sqrt{s} = 23\text{GeV}$  and  $\sqrt{s} = 31\text{GeV}$  on the base of the 10 m space (as it was impossible to operate at an angle smaller than 2 mrad, the Coulomb measurements could not be performed for beam momenta above 15.4 GeV/c [14]). The results of this experiment are presented in Table 1 (column IV). The 2% precision in  $\sigma_T$  measurement is a good achievement. But according to relation (10) this method cannot be directly used at higher energies. The modification of this technique using the accelerator lattice structure and a high  $\beta^*$  insertion was successfully applied at  $Spp\bar{p}S$  [10] and the Tevatron [15] for measuring the  $\rho$ -parameter. But in these experiments the absolute normalization comes from direct luminosity measurements, not from the Coulomb cross section. No publication exists about using this method for  $\sigma_T$  measurement at  $Spp\bar{p}S$  and Tevatron. Presumably, it is a difficult technique for application.

## 2. The new approaches to the $\sigma_T(pp)$ measurements

In principle any relation between the total cross section,  $\sigma_T(pp)$ , and the measurable observable can be used for the extraction of  $\sigma_T(pp)$ . We have discussed four such relations in section 1. We are planning to discuss here some new approaches.

**2.1 Collisions of unpolarized protons.** Assume we can independently measure with high accuracy two differential cross sections for  $pp$  elastic scattering. The Coulomb one is

$$\frac{d\sigma_c}{dt} = \pi |f_c|^2, \quad (11)$$

where

$$f_c = \pm \frac{2\alpha F^2(t)}{t} \cdot (\hbar c) \cdot \exp(\mp i\alpha\varphi). \quad (12)$$

Here the upper and lower signs refer to  $p\bar{p}$  and  $pp$  respectively.  $F(t) = (1 + \frac{|t|}{\Lambda^2})^{-2}$  is the dipole form factor of the proton,  $\Lambda^2 = 0.71\text{GeV}^2$ .  $\varphi$  is the Coulomb phase

$$\varphi = \ln \frac{2}{|t|(b + \frac{8}{\Lambda^2})} - 0.5772. \quad (13)$$

The nuclear elastic scattering cross section is

$$\frac{d\sigma_n}{dt} = \pi |f_n|^2, \quad (14)$$

where

$$f_n = \frac{\sigma_T}{4\pi}(\rho + i)e^{-\frac{1}{2}b|t|}. \quad (15)$$

The quantity  $\rho$  is the ratio of the real to imaginary part of the nuclear amplitude at  $|t| = 0$ ,  $b$  is a slope parameter. We can extrapolate both cross sections (4) and (9) to the point,  $t_0$ , where they are equal to each other

$$\frac{d\sigma_c(t_0)}{dt} = \frac{d\sigma_n(t_0)}{dt}. \quad (16)$$

By using the previous relation one can find

$$\frac{|t_0|e^{-\frac{1}{2}b|t_0|}}{F^2(t_0)} = \frac{8\pi\alpha}{\sigma_T}. \quad (17)$$

Therefore, for small  $|t_0|$

$$\sigma_{tot} \simeq \frac{8\pi\alpha}{|t_0|}. \quad (18)$$

We do not know any use of this relation in high energy experimental physics. The mere explanation of this situation might present the experimental difficulty in measuring the Coulomb cross section at the smaller than  $|t_0|$  values of the invariant transverse momenta. One can hope that  $pp2pp$  experiment may reach the smallest  $t$ -values (around  $2 \times 10^{-4}(GeV/c)^2$ ) and make a precise measurement of  $\frac{d\sigma_c}{dt}$  (out of the CNI region). Then this technique can be applied at RHIC.

We try now to look for the additional specific features of the pp differential cross section, which can be used for extraction of the total cross section. It is well known that the elastic pp-differential cross section at the CNI region can be presented as

$$\frac{d\sigma^{el}(t)}{dt} = \frac{d\sigma_c}{dt} + Int + \frac{d\sigma_n}{dt}, \quad (19)$$

where the first and third terms are given above by relations (9) and (14), respectively. "Int" means the interference cross section, which can be presented in the following way:

$$Int = \mp 2Re(f_c^* f_n) = \mp \frac{\alpha F^2(t)\sigma_T}{t} e^{bt/2} [\rho \cos(\alpha\varphi) \pm \sin(\alpha\varphi)].$$

Here the upper and lower signs refer to the  $pp$  and  $\bar{p}p$  scattering, respectively. The last two relations furnish two specific points in  $t$  axis. The first point  $t_{in}$  is defined through the equality of nuclear and interfering cross sections

$$Int = \frac{d\sigma_n}{dt}.$$

Therefore, for the pp case

$$t_{in} = -\frac{16\pi\alpha\rho(\hbar c)^2}{(1+\rho^2)\sigma_T} e^{bt_{in}/2} [\rho \cos(\alpha\varphi) + \sin(\alpha\varphi)].$$

Assuming  $bt/2 \ll 1$  and neglecting also  $\alpha\varphi$  one can find

$$t_{in} = \frac{16\pi\alpha\rho}{(1+\rho^2)\sigma_T}(\hbar c)^2 = \frac{2\rho}{1+\rho^2}t_0.$$

At  $\sqrt{s} = 541$  GeV  $\sigma_T = 63$  mb,  $\rho = 0.135$  so  $t_{in} = 3.1 \times 10^{-4}$  (GeV/c)<sup>2</sup>. For  $\sqrt{s} = 19.4$  GeV  $\sigma_T = 38$  mb,  $\rho = -0.034$  and  $t_{in} = 1.3 \times 10^{-4}$  (GeV/c)<sup>2</sup>. A very interesting situation appears in the elastic pp-scattering. Since the interfering term has a negative sign at  $t = t_{in}$  this term completely compensates the nuclear scattering. Therefore, at this point there is only the Coulomb scattering which can be used for normalization of the counting rates. The only problem is that the pp2pp experiment plans to reach the  $t_{min} = 7 \times 10^{-4}$  (GeV/c)<sup>2</sup> which is higher than one needs. There is another peculiar point,  $t_{ic}$ , where

$$Int = \frac{d\sigma_c}{dt}.$$

This is

$$t_{ic} = \frac{4\pi\alpha F^2(t)(\hbar c)^2}{\sigma_T\rho} e^{bt_{ic}/2}.$$

For small t this expression simplifies

$$t_{ic} = -\frac{4\pi\alpha(\hbar c)^2}{\sigma_T\rho} = \frac{1}{2\rho} \cdot t_0. \quad (20)$$

At  $\sqrt{s}=541$  GeV  $-t_{ic} = 4.2 \times 10^{-3}$  (GeV/c)<sup>2</sup>. This is an accessible point in the pp2pp experiment. Since at this point we have the pure nuclear scattering we can use the measurement at this point for checking some hypotheses such as the change of the slope parameter or oscillation of the differential cross section [13]. From the above consideration one can hope that after a time by improving the beam quality and the apparatus resolution we will be able to realize all the discussed above approaches, i.e. the three possibilities for the extraction of the  $\sigma_T$ .

The additional point of interest arises from the function (also a measurable one)

$$f2(t) = t^2 \cdot \frac{d\sigma}{dt}. \quad (21)$$

This is

$$t2 = \frac{8\pi\alpha\rho}{\sigma_T(1+\rho^2)} = \frac{\rho}{1+\rho^2} \cdot t_0, \quad (22)$$

where  $-t_0$  is given in (18). For RHIC's top energy  $-t2 = 1.5 \times 10^{-4}$  (GeV/c)<sup>2</sup> and it is the smallest one among the four peculiar points. Experimentally it is difficult to reach  $t2$ , which is smaller than  $t_0$  by an order of magnitude.

The summary of this section is presented in the following Table 2.

**Table 2.** The specific points -t in the elastic pp differential cross section. The general formulae for -t as well as their magnitudes for the top RHIC energy are presented

No	labels	expression	value at $\sqrt{s} = 500$ GeV	Comments
1	$-t_0 =$	$\frac{8\pi\alpha(\hbar c)^2}{\sigma_T}$	$1.1 \cdot 10^{-3}(\text{GeV}/c)^2$	Coulomb=Nucl.
2	$-t_{ic} =$	$\frac{4\pi\alpha(\hbar c)^2}{\sigma_T\rho}$	$4.2 \cdot 10^{-3}(\text{GeV}/c)^2$	Coulomb=Int.
3	$-t_{in} =$	$\frac{16\pi\alpha\rho(\hbar c)^2}{\sigma_T(1+\rho^2)}$	$3 \cdot 10^{-4}(\text{GeV}/c)^2$	Int.=Nucl.
4	$-t2 =$	$\frac{8\pi\alpha\rho(\hbar c)^2}{\sigma_T(1+\rho^2)}$	$1.5 \cdot 10^{-4}(\text{GeV}/c)^2$	Extremum in $t^2 \frac{d\sigma}{dt}$



The practical way of finding  $-t_{in}$  and  $-t_{ic}$  might be the following. Taking the known apparatus at the fixed experimental conditions, we measure the differential cross section,  $d\sigma^{meas}/dt$ . Then we calculate by the Monte Carlo technique the Coulomb differential cross section,  $d\sigma_c/dt$ . Their difference versus  $-t$  should reach a zero at the point, which should be  $-t_{in}$ . In a similar way one may get  $-t_{ic}$ .

**2.2 Collisions of polarized protons.** In the following we discuss the possibility of extracting of  $\sigma_T(pp)$  from the measurement of the analyzing power,  $A_N(t)$ , and also a factor of merit,  $M(t)$ , as it has been proposed in [1]. First of all we need the analytical formulae for those parameters. In this section we present such formulae in an explicit form. Second we need to prepare the set of experimental data for  $M(t)$ . We do such job in the the following sections.

The simplest expression for  $A_N(t)$  in the CNI region was first given in [16]. A more complete formula including the  $\rho$  parameter, single and double spin flip interactions was recently published [17]. We borrow the formulae from this paper, but as is usually accepted in the standard  $\sigma_T$  measurements, we omit all the spin dependent terms. We shall later attempt to make an estimate of the single spin flip contribution to our procedure. At the moment there is no chance to make a similar estimate for the double spin flip interaction due to the lack of experimental information on double spin asymmetries. The expression for  $A_N(t)$  in our approach looks like

$$A_N(t) = C_0 \frac{\sigma_T(1 - \rho\alpha\varphi)(-t)^{3/2}}{1 + C_1(\rho + \alpha\varphi)\sigma_T|t| + C_2(1 + \rho^2)\sigma_T^2|t|^2}. \quad (23)$$

Here  $C_0 = \frac{\mu_p - 1}{8\pi\alpha m_p (\hbar c)^2} = 26.7735 \text{ GeV}^{-3} \text{ mb}^{-1}$ ,  $C_1 = -\frac{1}{4\pi\alpha (\hbar c)^2} = -27.9972 \text{ GeV}^{-2} \text{ mb}^{-1}$ , and  $C_2 = \frac{1}{[8\pi\alpha (\hbar c)^2]^2} = 195.9609 \text{ GeV}^{-4} \text{ mb}^{-2}$ . The  $\mu_p$  and  $m_p$  are the proton magnetic moment and mass, respectively. Assuming that the  $\rho$  parameter is known from  $\frac{d\sigma^{el}}{dt}$ , we can define the total cross section,  $\sigma_T$ , by one parameter fit to the experimental data.

The analyzing power has a maximum at the point  $-t_A$  [17]

$$-t_A = -t_0[\sqrt{3} - (\rho + \alpha\varphi) + \frac{8}{(\mu_p - 1)}(\rho I_5 - R_5)]. \quad (24)$$

The last term corresponds to the single spin flip contribution to the total cross section. Later we will use this formula in order to get a hint into such contribution to the position of the  $A_N(t)$  and to the magnitude of  $\sigma_T$ . From the same paper [17] we extracted the formula for the factor of merit

$$M(t) = \bar{C}_0 \frac{\sigma^2(1 - \rho\alpha\varphi)^2 e^{bt}|t|}{1 + C_1\sigma_T(\rho + \alpha\varphi)|t| + C_2\sigma_T^2(1 + \rho^2)t^2}. \quad (25)$$

Here  $\bar{C}_0 = \frac{(\mu_p - 1)^2}{16\pi m_p^2 (\hbar c)^2} = 0.1867 \text{ GeV}^{-4} \text{ mb}^{-1}$ . Other parameters were defined earlier. For small  $t$   $bt \ll 1$ ,  $\alpha\varphi \approx \alpha\varphi\rho \approx 0$ .  $M$  has a maximum at the point  $t_M$ , which is

$$-t_M = \frac{8\pi\alpha(\hbar c)^2}{\sigma_T\sqrt{1 + \rho^2}}. \quad (26)$$

Therefore the alternative to the fitting procedure is the extraction of  $\sigma_T$  from this relation. It is seen that  $t_M$  is not sensitive to the magnitude of  $\rho$  in the energy range of interest.

### 3. Extraction of $\sigma_T(pp)$ from E704 data

The E704 collaboration measured the analyzing power,  $A_N(t)$ , in reaction

$$p_{\uparrow} + p \rightarrow p + p, \quad (27)$$

at the initial momentum of the polarized beam,  $p_{in} = 200$  GeV/c in the region  $2 \cdot 10^{-3} \leq |t|[(\text{GeV}/c)^2] \leq 4 \cdot 75 \cdot 10^{-2}$  [18]. The experimental data are shown in Table 3.

Table 3.  $A_N(t)$ ,  $d\sigma^{el}/dt$ , and  $M(t)$  at  $\sqrt{s} = 19.4$  GeV

No	$-t, (\text{GeV}/c)^2$	$A_N \pm \Delta A_N$	$d\sigma^{el}/dt, \text{mb}/(\text{GeV}/c)^2$	$M \pm \Delta M$
1	0.00288	$0.0446 \pm 0.0316$	$106.4 \pm 2.3$	$0.212 \pm 0.295$
2	0.0083	$0.0311 \pm 0.0109$	$77.1 \pm 1.7$	$0.0745 \pm 0.0522$
3	0.0175	$0.0262 \pm 0.0101$	$64.5 \pm 1.2$	$0.0443 \pm 0.0342$
4	0.0273	$0.0317 \pm 0.0107$	$55.6 \pm 1.1$	$0.0559 \pm 0.0377$
5	0.0368	$0.0217 \pm 0.0139$	$52. \pm 1.$	$0.0245 \pm 0.0314$
6	0.0475	$0.0027 \pm 0.0277$	$49.9 \pm 0.8$	$0.0003 \pm 0.0064$

We should fix the parameter  $\rho$  at  $\sqrt{s} = 19.4$  GeV. This parameter was measured at FNAL at  $p_{lab} = 199$  GeV/c [19] and equaled  $\rho = -0.034 \pm 0.014$ . After inserting this number into formula (23) and fitting to the 6 experimental points on  $A_N(t)$ , we got

$$\sigma_T(pp) = (37.8 \pm 8.1) \text{mb}, \quad (28)$$

with  $\chi^2 = 1.49$  for  $\text{ndf}=5$  (see Fig.1a).

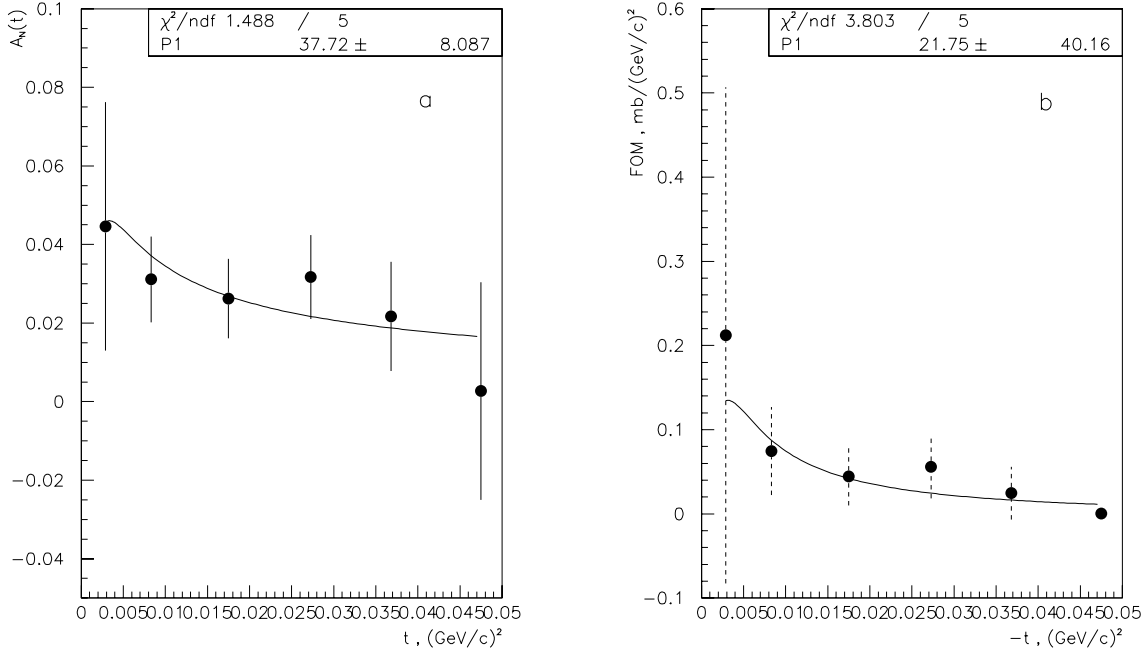


Fig. 1. a) The analyzing power,  $A_N(t)$ , measured by the E704 collaboration at  $\sqrt{s}=19.4$  GeV. A solid line is the result of one parametric fit. b) The similar fit to the factor of merit,  $M(t)$ , at the same energy.

This number is to be compared to the experimental value  $\sigma_T(pp) = 38.9 \pm 0.7$  measured at  $\sqrt{s} = 23.0$  GeV at ISR[14]. The compatibility of these two values proves the correctness of the new method, though the statistics of E704 data does not allow one to reach the precision of the standard technique: For that one needs to improve the E704 statistics by two orders of magnitude. The second limitation comes from the beam polarization,  $P_B$ , accuracy. In E704 the  $\Delta P_B/P_B$  was  $\pm 6.8\%$  [20] and this is an additional source of ambiguity in  $\Delta\sigma_T(pp)$ . The third source contributing to the  $\Delta\sigma_T(pp)$  is the detector resolution in  $\Delta t$ . The E704 setup had the resolution (geometrical)  $\Delta t/|t_{min}| = 0.10$ , smeared out additionally due to a multiple scattering,  $(\Delta t/|t_{min}|)_{ms} \simeq 0.07$ , due to a momentum resolution  $(\Delta t/|t_{min}|) = 0.03$ . All these sources add an additional systematic error of order  $\pm 14\%$  (summed in quadrature) to  $\Delta\sigma_T$  in (28) and put the E704 data out of the competition with the standard techniques.

The drastic improvements of E704 data can be made at RHIC by using a polarized jet target [21]. First of all the target polarization,  $P_T$ , can be measured with a better accuracy than the beam polarization ( $\Delta P_T/P_T \simeq 2\%$  seems feasible) [22]. Second, due to a large luminosity the  $A_N$  statistics may be increased by two orders of magnitude. Third, since the recoil Si detector has a good energy resolution  $\Delta T \simeq 50$  KeV [23], then at kinetic energy  $T_{rec} \simeq 1$  MeV (corresponding to  $|t_{max}| \simeq 2 \cdot 10^{-3}$  (GeV/c)<sup>2</sup>), one can reach an accuracy  $\Delta t/|t_{max}| \leq 5\%$  or better. So, we can overcome the main difficulties listed above. Therefore, the  $A_N(t)$  measurement in the CNI region at the fixed target mode (FTM) at RHIC is very desirable.

We turn now to the factor of merit,  $M(t)$ . This is not the direct observable as  $A_N(t)$  is. Therefore, we should make a product

$$M(t) = A_N^2(t) \cdot \frac{d\sigma^{el}}{dt}, \quad (29)$$

and calculate the error bar  $\Delta M(t)$ . We took the experimental data for  $d\sigma^{el}/dt$  from FNAL experiment [19] (see Table 3, column 4). Comparing the relative error bars in  $A_N(t)$  and in  $d\sigma^{el}/dt$ , one can conclude that

$$\frac{\Delta M(t)}{M(t)} \approx 2 \cdot \frac{\Delta A_N(t)}{A_N(t)}. \quad (30)$$

This relation already makes us pessimistic about the expected precision in the  $\sigma_T$  determination. Nevertheless we prepared the experimental data for the factor of merit and put them in the last column of Table 3.

Putting  $b = 12$  GeV<sup>-2</sup> and taking  $\rho$  the same as before, we made a fit (see Fig.1b) and got

$$\sigma_T = (22 \pm 40) mb, \quad (31)$$

at  $\chi^2 = 3.8$  for  $ndf=5$ . Therefore, we are not able to make any conclusion at such precision of  $\sigma_T$  extraction. As concerns the  $A_N(t)$ , we can conclude that the E704 data can be used only for a qualitative illustration of the correctness of a new approach to the total cross section extraction from the analyzing power measurements. But they can be essentially improved by repeating the E704 measurement at RHIC using the jet target.

#### 4. Extraction of the $\sigma_T(pp)$ from “simulated pp2pp data” at RHIC

One of the important tasks of  $pp2pp$  experiment at RHIC is to measure the analyzing power,  $A_N(t)$ , for elastic  $pp$ -scattering [3] in the CNI region. To study this, the performance of apparatus

to reconstruct an input  $A_N(t)$  was considered. The collision energy was taken as  $\sqrt{s} = 500$  GeV, the beam polarization was set to be 70%, the running luminosity was assumed to be  $2 \times 10^{29} \text{ cm}^{-2} \times \text{s}^{-1}$ . In order to optimize the left-right difference with vertically polarized beam, the events produced in the azimuthal region of  $|\cos \phi| > 1/\sqrt{2}$  were accepted. In such conditions the running time of about 3.7 hours will be required to collect  $2.5 \times 10^6$  events. For simulation of the “left-right” analyzing power a simple form of  $A_N(t)$ , given in equation (23) with condition  $\rho = 0$ , was applied. The asymmetry  $A_N(t)$  reconstructed with account for the difference in the number of events going left and right in the detector (the far pot installed at 143 m) is shown in Fig.2a and included in Table 4.

**Table 4.**  $A_N(t)$ (simulated),  $d\sigma^{el}/dt$ , and  $M(t)$  at  $\sqrt{s} = 500\text{GeV}$

No	$-t, (\text{GeV}/c)^2$	$A_N \pm \Delta A_N$	$d\sigma^{el}/dt, \text{mb}/(\text{GeV}/c)^2$	$M \pm \Delta M$
1	0.0015	0.034±0.0025	275±3.8	0.3179±0.0467
2	0.0025	0.033±0.003	216±3.1	0.2352±0.0429
3	0.0035	0.041±0.003	199±3.	0.3345±0.0492
4	0.0045	0.031±0.003	192±3.	0.1845±0.0358
5	0.0055	0.027±0.003	187±3.	0.1363±0.0304
6	0.0065	0.030±0.003	184±3.	0.1656±0.0332
7	0.0075	0.029±0.003	181±3.	0.1522±0.0316
8	0.0085	0.022±0.003	178±3.	0.0862±0.0235
9	0.0095	0.027±0.003	175±3.	0.1276±0.0284
10	0.0105	0.016±0.003	172±1.3	0.044±0.0165
11	0.0115	0.025±0.003	170±1.3	0.1063±0.0255
12	0.0125	0.021±0.003	167±1.3	0.0736±0.0210
13	0.0135	0.022±0.004	165±1.2	0.0799±0.029
14	0.0145	0.014±0.003	163±1.2	0.0319±0.0137
15	0.0155	0.018±0.003	160±1.2	0.0518±0.0173
16	0.0165	0.015±0.004	158±1.3	0.0356±0.019
17	0.0175	0.018±0.004	155±1.3	0.0502±0.0223
18	0.0185	0.01±0.004	153±1.3	0.0153±0.0122
19	0.0195	0.02±0.004	151±1.3	0.0604±0.0242

We took these simulated data as the “experimental measurement” and made a fit to the function given in equation 23 with  $\rho = 0$ . This fit is presented in Fig.2a by a solid line. At  $\chi^2/ndf = 26.64/18$ , we got

$$\sigma_T(pp) = (58.64 \pm 1.97) \text{ mb.} \quad (32)$$

This value has to be compared to the experimental data on  $\sigma_T(\bar{p}p)$  obtained at  $S\bar{p}pS$  at  $\sqrt{s} = 541: 63.0 \pm 1.5 \text{ mb}$  (UA4) [8] and at Tevatron: at  $\sqrt{s} = 546 \text{ GeV}$   $61.26 \pm 0.93 \text{ mb}$  (CDF) [12]. Consistency is very good proving that a new approach is workable and becoming competitive with standard technique.

Now we are going to apply the factor of merit for the extraction of  $\sigma_T$  from the “simulated” pp data. For that we should prepare the experimental data for  $M(t)$ . The pp elastic differential cross section at  $\sqrt{s} = 541\text{GeV}$  can be extracted from the UA4/UA2 results [24] assuming that the nuclear and Coulomb parts of the cross sections are equal, while the interfering term is different and has a different sign. The restored by such a way  $d\sigma^{el}/dt$  is included in Table 4. Finally the “experimental” data on  $M(t)$  are presented in the last column of Table 4. Applying formula (25) to these data, one gets

$$\sigma_T = 42.1 \pm 4.9,$$

with  $\chi^2/\text{ndf}=26/18=1.42$  (see Fig.2b). Though this value is much smaller than the expected one and the error bar is twice bigger than that in the  $A_N$  approach, nevertheless,  $M(t)$  also works in the right direction. One needs only more precise experimental data.

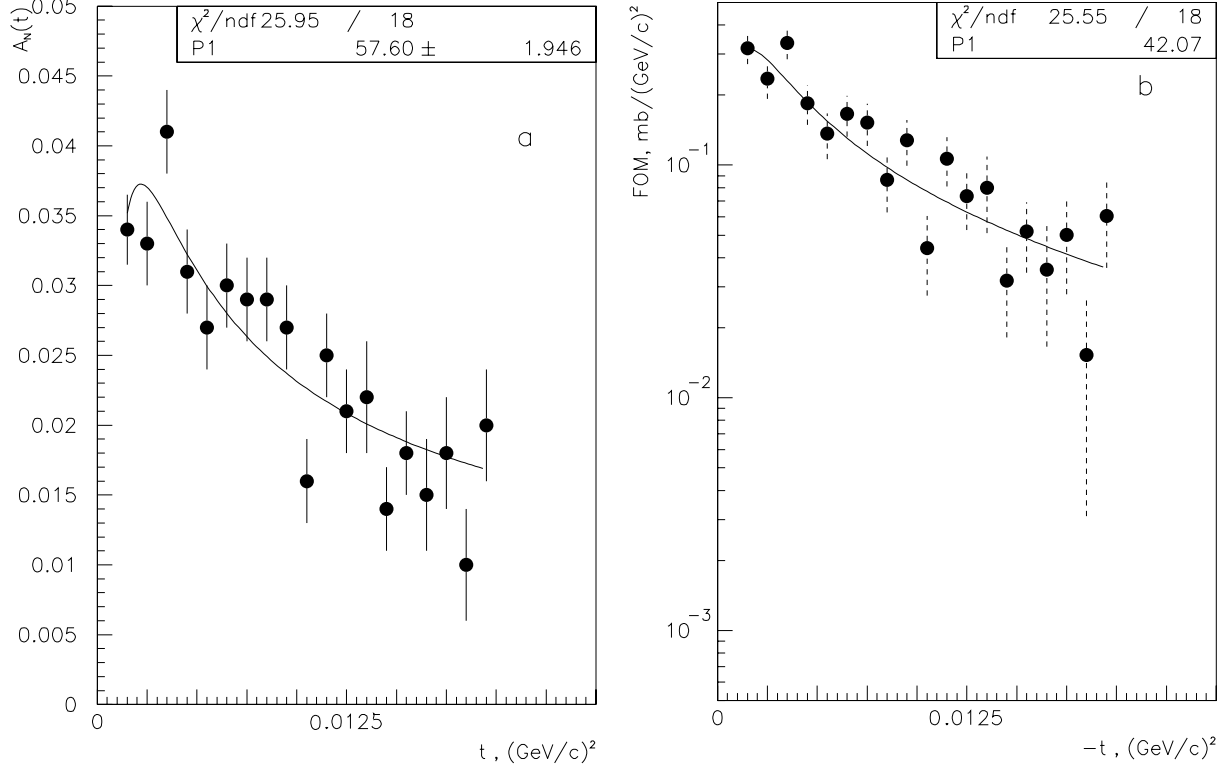


Fig. 2. a) The simulated  $A_N(t)$  data for  $pp2pp$  experiment at RHIC top energy,  $\sqrt{s}=500$  GeV. A solid line is the result of one parametric fit. b) The similar fit to the factor of merit,  $M(t)$ , at the same energy.

Now we turn to the discussion of difficulties which can be encountered in the application of a new scheme to the experiment. Mostly they are systematic errors.

First of all, the beam polarization,  $P_B$ , must be measured with a precision better, than an expected magnitude of the  $\Delta\sigma_T/\sigma_T$ . This is because  $P_B$  and  $A_N$  are related through relation

$$A_N = \frac{1}{P_B} \cdot \epsilon, \quad (33)$$

where  $\epsilon$  is a measurable (“raw”) asymmetry. At RHIC the main goal is to reach  $\Delta P_B/P_B = \pm 5\%$  and this error contributes directly to  $\Delta A_N$ . This puts a stringent limit on a precision of  $\sigma_T(pp)$  measurement. For the goal of the  $\sigma_T$  measurement at RHIC by a new technique one needs a measurement of beam polarization with a precision better than 1%.

The error in  $\Delta t$  is determined by the experimental conditions. They are the following ones: a) the error in the angle between the two beam axes is  $6 \mu$  rad. This leads to  $\Delta t/t \simeq 7\%$ , b) momentum resolution in measuring the scattered particle  $\Delta p/p \simeq 1.5\%$ . This leads to  $\Delta t/t = 3\%$ .

Adding all the listed error bars to quadrature one can expect their contribution to the  $\frac{\Delta\sigma_T}{\sigma_T}$  of order 9%. Obviously these contributions must be decreased in the experiment.

## 5. Single spin flip contribution to the $\sigma_T(pp)$

Now we attempt to estimate the single spin flip contribution to the extracted value of  $\sigma_T(pp)$ . For that we use the E-704 results for  $A_N(t)$ , the FNAL data for  $\frac{d\sigma^{el}}{dt}$  and reconstruct the measurable function

$$\psi(t) = \frac{m_p \sqrt{-t}}{\sigma_T} A_N(t) \frac{d\sigma^{el}}{dt}. \quad (34)$$

The experimental data for this function are presented in Table 5.

Table 5. Function  $\psi(t)$  at  $\sqrt{s} = 19.4$  GeV

No	$-t, (GeV/c)^2$	$\psi(t)$
1	0.00288	$0.0062863 \pm 0.004454$
2	0.0083	$0.0053902 \pm 0.0018892$
3	0.0175	$0.0055208 \pm 0.0021282$
4	0.0273	$0.0071884 \pm 0.0024264$
5	0.0368	$0.0053433 \pm 0.0034227$
6	0.0475	$0.007248 \pm 0.061125$

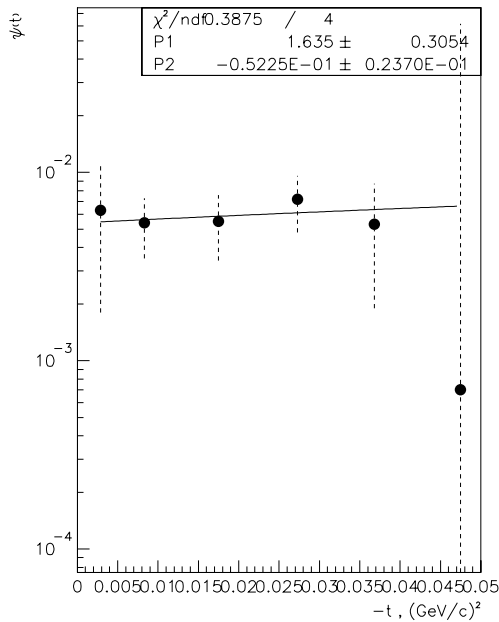


Fig. 3. The single spin dimensionless function,  $\psi(t)$ , versus  $-t$  at  $\sqrt{s}=19.4$  GeV. A solid line is the result of two-parametric fit.

The explicit form of function  $\psi(t)$  is taken from [17]

$$\psi(t) = -\alpha \left( \frac{\mu_p - 1}{2} - I_5 \right) + \frac{\sigma_T}{4\pi(\hbar c)^2} (\rho I_5 - R_5) t. \quad (35)$$

Putting the numerical values  $\sigma_T = 38$  mb,  $\rho = -0.034$ ,  $\mu_p = 2.793$ ,  $\alpha = 1/137$ , we got the final formula for the fit to the  $\psi(t)$  at  $\sqrt{s} = 19.4$  GeV

$$\psi(t) = -0.00654 + 0.0073 I_5 + 7.7(0.034 I_5 + R_5) |t|. \quad (36)$$

Two-parametric fit to the  $\psi(t)$  given in Table 5 lead to the following results:  $I_5 = 1.63 \pm 0.31$  and  $R_5 = -0.05 \pm 0.02$  (see Fig.3).

The value of  $I_5$  differs from the one given in [25], but for our estimate it is not so important. The position of the  $A_N$  maximum will be changed by the factor

$$\Delta t/t = \Delta_5 = \frac{8(\rho I_5 - R_5)}{(\mu_p - 1)(\sqrt{3} - (\rho + \alpha\phi))}, \quad (37)$$

so numerically the magnitude of  $\sigma_T(pp)$  will be changed by the same factor or  $\approx 4\%$ . Therefore, for the precise determination of the  $\sigma_T(pp)$ , let, say, of order 1%, one needs to make a better measurement of the parameters  $I_5$  and  $R_5$ .

## Conclusions

The suggestion to extract the  $\sigma_T(pp)$  from the measurement of the analyzing power,  $A_N(t)$ , in the elastic  $pp$ -scattering at the Coulomb-Nuclear Interference region, works, in principle, but encounters serious quantitative problems. The severe restrictions at present come from the following factors: 1) precision of beam polarization measurement (contribution to  $\Delta\sigma_T/\sigma_T \simeq 5\%$ ), 2)  $t$ -resolution of apparatus ( $\simeq 8\%$ ), 3) ambiguity in single spin flip term ( $\simeq 4\%$ ). In order for a new approach to  $\sigma_T(pp)$  extraction to be applicable at RHIC, we must find a way of improving precisions for items 1-3 listed above. Otherwise, this new approach will be incompetent with the standard technique of measuring the  $\sigma_T(pp)$ .

## Acknowledgements

We would like to thank V. Ejela, P. Gauron, W. Guryn, Yu. Kharlov and B. Nicolescu for useful discussions and help.

## References

- [1] Gauron P., Nicolescu B., and Selyugin O.V., *Phys. Lett.* **B390** (1997)405.
- [2] Kuroda K., In: AIP Proc., 1983, 95, p.618.
- [3] Experiment to measure total and elastic  $pp$  cross sections at RHIC. BNL Proposal, updated version, September 1995.
- [4] Amaldi U. et al., *Nucl. Phys.* **B145** (1978) 367.
- [5] Amos N.A. et al., *Phys. Rev. Lett.* **63** (1989) 2784.
- [6] White S. Measurements of the  $pp$  total cross sections at  $\sqrt{s} = 1800$  GeV. Preprint Fermilab-Conf-91/268-E, October 1991.
- [7] Armson G. et al., CERN-EP/83-70, May 27th, 1983.
- [8] Augier C. et al., *Phys. Lett.* **B316** (1993) 448.
- [9] Augier C. et al., *Phys. Lett.* **B344** (1995) 451.
- [10] Bozzo M. et al., *Phys. Lett.* **B147** (1984) 392.
- [11] Avila C. et al., *Phys. Lett.* **B446** (1999) 179.
- [12] Abe F. et al., *Phys. Rev.* **D50** (1994) 5550.
- [13] Gauron P., Nicolescu B., and Selyugin O.V. *Phys. Lett.* **B397** (1997)305.
- [14] Amaldi U. et al., *Phys. Lett.* **B43** (1973) 231.
- [15] Amos N.A. et al., *Phys. Rev. Lett.* **68** (1992)2433.
- [16] Vanja A.P., Lapidus I.I, and Tarasov A.V., *Yad. Fiz.* **16** (1972) 1023 (*Sov. J. Nucl. Phys.* **16**, (1972) 1023).

- [17] Buttimore N.H. et al., *Phys. Rev.* **D59** (1999) 114010-1.
- [18] Akchurin N. et al., *Phys. Rev.* **D48** (1993) 3026.
- [19] Gross D. et al., *Phys. Rev. Lett.* **41** (1978) 217.
- [20] Grosnick D.P. et al., *Nucl. Instr. and Meth.* **A290** (1990) 269.
- [21] Penzo A., In: Proc. of the Workshop *Polarized protons at High Energies - Accelerator Challenges and Physics Opportunities*, DESY, 17-20 May, 1999, p.489.
- [22] Baumgarten C., In: Proc. of the 13th Intern. Symposium on High Energy Spin Physics, September 8-12, 1998, Protvino, Russia, p.433.
- [23] K. Seth et al., *Phys. Lett.* **B385** (1996) 479.
- [24] C. Augier et al., *Phys. Lett.* **B316** (1993) 448.
- [25] Akchurin et al., *Phys. Rev.* **D51** (1995) 3944.

*Received August 8, 2000*



А.А.Богданов, С.Б.Нурушев, А.Пенцо и др.  
Новые подходы к измерениям полных сечений на поляризованных коллайдерах.

Оригинал-макет подготовлен с помощью системы  $\text{\LaTeX}$ .  
Редактор Е.Н.Горина. Технический редактор Н.В.Орлова.

---

Подписано к печати 10.08.2000. Формат  $60 \times 84/8$ . Офсетная печать.  
Печ.л. 1,75. Уч.-изд.л. 1,4. Тираж 160. Заказ 178. Индекс 3649.  
ЛР №020498 17.04.97.

---

ГНЦ РФ Институт физики высоких энергий  
142284, Протвино Московской обл.

