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# PECULIARITIES OF PARTICLE MOTION IN THE PLANE MONOCHROMATIC ELECTROMAGNETIC WAVE FIELD

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**Abstract**

Boldyrev E.M. Peculiarities of Particle Motion in the Plane Monochromatic Electromagnetic Wave Field: IHEP Preprint 2000-55. – Protvino, 2000. – p. 9, tables 10, refs.: 6.

With a spiral approach of a charged particle motion (into a beam) in the plane monochromatic elliptically polarized electromagnetic wave field some peculiarities of this motion were brought out. It is shown that the motion of a low energy particle is physically predicable. The motion doesn't depend on a particle movement along or against the wave spreading; the particle frequency is comparable with a wave frequency; the deflection of particle from the axis of initial motion is inversely proportional to its mass. It is shown that the motion of a high energy particle (an ultrarelativistic particle) has a number of peculiarities. The motion depends essentialy on the way the particle moves along or against the wave spreading. If the particle moves along a wave spreading, the particle frequency is a few cycles only or even less. The deflection of particle from the axis of initial motion may be proportional to the particle mass. If the particle moves against the wave spreading, the frequency is twice as much as the wave frequency and the motion of particle is independent of the particle mass. The peculiarities associated with the particle mass were first established.

**Аннотация**

Болдырев Е.М. Особенности движения частицы в электромагнитном поле плоской монохроматической волны: Препринт ИФВЭ 2000-55. – Протвино, 2000. – 9 с., 10 табл., библиогр.: 6.

При условии винтового приближения движения частицы (в пучке) в электромагнитном поле плоской монохроматической эллиптически поляризованной электрогнитной волны выявлены определенные особенности указанного движения. Показано, что если для частицы низкой энергии движение физически предсказуемо: движение практически не зависит от того, как движется частица — вдоль или против распространения волны; частота колебания частицы сравнима с частотой волны; отклонение частицы от оси начального движения обратно пропорционально ее массе, то для частицы высокой энергии (ультрарелятивистской частицы) движение обладает рядом особенностей: движение частицы существенно зависит от того, как движется частица — вдоль или против распространения волны. В первом случае частота колебания частицы меньше более чем на десять порядков длины волны и существует возможность того, что отклонение частицы от оси начального движения пропорционально ее массе. Во втором случае частота колебания частицы вдвое больше частоты волны и движение частицы не зависит от ее массы. Особенности, связанные с массой частицы, выявлены впервые.

## INTRODUCTION

In high-energy physics considerable attention has been recently focused on the experimental set-ups using the beam motion of charged particles in the electromagnetic laser wave field ([1],[2]). This invites further study of a charged particle motion in the plane monochromatic elliptically polarized electromagnetic wave field (TEM) as most fitting the description of a laser wave field.

Motion of charged particles (electron, proton) of both low ($10^3$ eV) and high energy ($10^{12}$ eV) for different parameters of TEM was calculated from the formulae in [3]. The calculational results (see APPENDIX A) have revealed some peculiarities of the motion of a charged high energy particle. The motion depends essentially on how the particle moves along or against the wave spreading. If the particle moves along the wave spreading, the particle frequency is a few cycles only or even less. The deflection of the particle from the axis of its initial motion may be proportional to the particle mass. If the particle moves against the wave spreading, the particle frequency is twice as much as the wave frequency and the motion of particle is independent of the particle mass.

This is in contrast to the case with a low energy particle. The motion does not depend on how a particle moves along or against the wave spreading; the particle frequency is comparable with a wave frequency; the deflection of particle from the axis of initial motion is inversely proportional to its mass. It is physically predictable.

In recent paper [4], to account for these facts an analysis of the motion of particle has been made for a special case: the zero initial conditions (with the exception for an initial longitudinal momentum) and with the phase of TEM being virtually zero.

In the present paper we pursue a related analysis for the initial conditions characteristic of a particle moving in the beam and with the phase of TEM being not virtually zero.

The stated generality of this analysis has made possible practical estimations of the particle motion in electromagnetic laser wave field.

## 1. PRELIMINARIES

The $[x, y, z, ct]$ is the laboratory system of coordinates and it has the signature $[-,-,-,+]$ ($c$ is the velocity of light). The $\vec{V} = (V_x, V_y, V_z)$ is the vector. $V_x, V_y, V_z$ are the coordinates of $\vec{V}$. $V = |\vec{V}| = \sqrt{(V_x^2 + V_y^2 + V_z^2)}$.

Infinitesimal quantities are considered in comparison with 1. In this case $I(\pm a) = 1 \pm a$ if $a \ll 1$.

We consider a particle with mass m and charge e (e $= g_e|e|$, $g_e = \pm 1$, $|e|$ is the magnitude of e).

**Particle parameters**

$\vec{r}_0 = (x_0, y_0, z_0)$, $\vec{P}_0 = (P_{0x}, P_{0y}, P_{0z})$, and $\mathcal{E}_0$ are the respective radius-vector of the initial position of particle, initial momentum of particle, and initial energy of particle at the initial time instant $t_0$. In this case, $P_{0x} = P_0 cos\varphi_0 sin\theta_0$, $P_{0y} = P_0 sin\varphi_0 sin\theta_0$, $P_{0z} = P_0 cos\theta_0$, $P_{0xy} = max(|P_{0x}|, |P_{0y})$, $\vec{\pi}_0 = \frac{\vec{P}_0}{mc}$.

**Wave parameters**

$\vec{E}$ and $\vec{H}$ are the respective electric and magnetic fields. $E_1$ and $E_2$ are the respective amplitudes of $E_x$ and $E_y$, $E_{max} = max(|E_1|, |E_2|)$. $\omega$ and $\varphi$ are the respective frequency and phase of TEM. $g = \pm 1$ is the degree of polarization. $\xi = t - \frac{z}{c}$.

$$\vec{E} = (E_1 \cos(\omega\xi - \varphi), gE_2 \sin(\omega\xi - \varphi, 0).$$
$$\vec{H} = (-gE_2 \sin(\omega\xi - \varphi), E_1 \cos(\omega\xi - \varphi), 0),$$

$\xi_0 = t_0 - \frac{z_0}{c}$, $\phi_0 = \omega\xi_0 - \varphi$, $\phi = \omega\xi - \varphi$, $C(\phi) = cos\phi - cos\phi_0$, $S(\phi) = sin\phi - sin\phi_0$.

The TEM propagation in the z direction.

$T = t - t_0$.

$\vec{r} = (x, y, z) = (x(t), y(t), z(t))$, $\vec{P} = (P_x, P_y, P_z) = (P_x(t), P_y(t), P_z(t))$, $\vec{v} = (v_x, v_y, v_z) = (v_x(t), v_y(t), v_z(t))$, $\vec{a} = (a_x, a_y, a_z) = (a_x(t), a_y(t), a_z(t))$, and $\mathcal{E} = \mathcal{E}(t)$ are the respective radius-vector of particle, momentum of particle,velocity of particle, acceleration of particle, and energy of particle at $t$.

## 2. ANALYSIS OF MOTION

With our conventions the classical equations of motion are

$$\vec{P} = \gamma m\vec{v}, \quad \mathcal{E} = \gamma mc^2,$$
$$\frac{d\vec{P}}{dt} = e\vec{E} + \frac{e}{c}[\vec{v}, \vec{H}], \quad \frac{d\vec{r}}{dt} = \vec{v}, \qquad (1)$$
$$\vec{P}(t_0) = \vec{P}_0, \quad \vec{r}(t_0) = \vec{r}_0,$$

where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, $\quad \beta = \frac{v}{c}$.

It follows that the motion of particle in TEM is well determined if $\vec{r}(t)$, $\vec{P}(t)$, $\mathcal{E}(t)$, $\vec{v}(t)$, and $\vec{a}(t)$ of particle are determined in the laboratory system of coordinates ($t \in [t_0, \infty)$).

Here the motion of particles of different mass are determined at the same initial conditions. On this assumption, $\vec{P}_0$, and $\vec{r}_0$ are independent of the particle mass.

The solution of Eq. (1) can be considered as a spiral approximation. Hereafter we shall use the term “a quasi-spiral motion” for the spiral approximation.

The quasi-spiral motion is defined as the motion wherein $z(t)$ is linear in t, $v_z(t)$ and $P_z(t)$ are constant, and $|a_z(t)| \ll |a_x(t)|, |a_y(t)|$ with an accuracy of infinitesimals.

That approximation is due to the following fact: For a spiral motion, the quasi-spiral motion is the simplest and it spans a wide enough class of problems, in particular, so important for the practical implementation the class of problems such as the motion of high energy particles. As an illustration, the data given in APPENDIX A showed that only rows 7, 8 of Tables 1, 2 did not fit this approximation. At the same time the spiral motion of a particle is possible in TEM of a circular polarization only under those initial data conditions that they cannot be realized for the beam of particles. In this the case of a linear polarization is ruled out [3].

The quasi-spiral motion is determined by the requirement so that $\Gamma \ll 1$, where

$$\Gamma = 4\frac{|e|}{\omega}E_{max}(\frac{|e|}{\omega}E_{max} + P_{0xy})G \tag{2}$$

and G is determined below.

The area of particle and wave parameters, where the above requirement (2) is obeyed, will be called the area of quasi-spiral motion.

The motion of the particle in the beam is determined by the requirement so that $P_{0x} \ll P_{0z}$ and $P_{0y} \ll P_{0z}$. For this it will suffice to require that $\theta_0 = \theta \quad (n_0 = 1)$ and $\theta_0 = \pi - \theta \quad (n_0 = -1)$ where $\theta \ll 1$ and $\pi = 3.14....$

Then $P_{0x} = P_0 cos\varphi_0\theta$, $P_{0y} = P_0 sin\varphi_0\theta$, and $P_{0z} = n_0 P_0 I(-\theta)$.

In that approximation solution (1) is presented in APPENDIX B.

**A low energy particle.** $\mathcal{E}_0 \approx mc^2$ i.e. $\pi_0 \ll 1$.

In this case we have: $R = \frac{1}{m}$, $\quad Z = n_0 P_0 \frac{1}{m}$, $\quad P = n_0$, $\quad A_1 = I(-n_0\pi_0)$, $\quad A_2 = \frac{1}{m^2c^2}$, $\quad G = \frac{1}{mc}\frac{1}{P_0}I(n_0\pi_0 + \theta^2)$.

$$\mathcal{E} = mc^2 I(n_0\pi_0\Gamma^2 + \pi_0^2), \quad v_z = \frac{1}{m}n_0 P_0 I(-n_0\pi_0\Gamma^2 - \pi_0^2).$$

It immediately shows that the motion of the particle is inversely related to the particle mass. The particle frequency is nearly equal to the wave frequency. The area of quasi-spiral motion is essentially bounded as $P_0 \ll 1$. Moreover, it can be enhanced at the cost of an increase of the wave frequency. In other words, $P_0$ and the wave frequency balance out each other, which narrows down this area.

**A high energy particle.** $\mathcal{E}_0 \gg mc^2$ and thus $\pi_0^{-1} \ll 1$. In this case we have $R = \frac{c}{P_0}$, $\quad A_2 = \frac{1}{P_0^2}$, and

$$\mathcal{E} = cP_0 I(\Gamma^2 + \pi_0^{-2}).$$

Furthermore, a distinction needs to be made between the case $n_0 = 1$ and the case $n_0 = -1$.

**Case $n_0 = 1$.** In this case: $Z = cI(\Gamma^2 - \theta^2)$, $\quad P = I(\Gamma^2 - \theta^2)$, and $v_z = cI(-\theta^2 - \pi_0^{-2})$.

Here a possibility exists that $\phi \ll 1$.

In the case of $n_0 = 1$, a distinction needs to be made between the case $\pi_0^{-2} < \theta^2$ and the case $\theta^2 < \pi_0^{-2}$.

In the case of $\pi_0^{-2} < \theta^2$, we have $A_1 = \theta^2$, $\quad G = \frac{1}{P_0^2}\theta^2$.

It immediately follows that in this case TEM becomes less and less discriminate to the particle mass as $\theta$ increases (see Tables 4,5). In this case the area of the quasi-spiral motion is sufficiently large, because in contrast to the foregoing case, the particle parameters and the wave parameters complement each other from the viewpoint of expanding this area and enclose the values of all the parameters used in APPENDIX A. In rows 7, 8 of Table 4 $\Gamma \approx 0.01$.

In the case of $\theta^2 < \pi_0^{-2}$, we have $A_1 = \frac{1}{2}\frac{m^2c^2}{P_0^2}$, $\quad G = \frac{1}{2}\frac{1}{m^2c^2}I(\theta^2)$.

If $\phi \ll 1$, the motion of particle varies in direct proportion to the particle mass (see Tables 3,9). The area of the quasi-spiral motion is narrowed as compared with to preceding case. For example, in rows 7, 9 of Table 3 $\Gamma \approx 0.1$.

In both cases, the particle frequency $\omega_p = A_1\omega$ falls far short of the wave frequency (see Tables 5,6,9,10), in this with decreasing $\theta$ the particle frequency diminishes especially for electron (cf. Tables 3,4).

**Case $n_0 = -1$.**
$A_1 = 2, \quad Z = -cI(\Gamma^2 - \theta^2), \quad P = -I(\Gamma^2 - \theta^2), \quad G = \frac{1}{2}\frac{1}{P_0^2}$, and

$$v_z = -cI(-\theta^2 - \pi_0^{-2}).$$

That is to say, in this case the particle frequency is twice as much as the wave frequency. The motion of particle is independent of its mass (see Table 6). At least, the quality is still retained in the circular cone with the axis along Oz and with apex angle of cone $(\pi - 10^{-3})$ rad (see Table 5). In this case the area of the quasi-spiral motion is the greatest, since the particle parameters and the wave parameters complement each other from the viewpoint of expanding this area. For example, $\Gamma \approx 10^{-13}$ in rows 7, 8 of Table 5.

## Conclusion

In summary let us take up the results of APPENDIX A.

Comparing rows 5, 7 of Tables 1, 2 and rows 7, 8 of Tables 7, 6 we see that the motion of particle is practically the same in these cases. It allows one to conclude that the motion of a low energy particle in high-powered TEM is independent of the polar angle in the range from 0 to $10^{-3}$ rad and, thus, the particles, entering TEM with an angled initial momentum in this range move the same. Consequently, TEM may be used both as a focuser or as a means for deflecting the beam. In the latter case, $\rho$ remains invariant within the indicated limits of angles and it is possible that these limits may be belled up to $10^{-2}$ rad as the estimations above show.

Further note that $\Delta\mathcal{E}$ has a great value in rows 7, 8 of Tables 1, 2 (7, 8) and in the last rows of Tables 4, 5. This suggest the use of TEM as an accelerator of charged particles. But here, while the problem of radiative friction is not as challenging as the motion of a charged particle in the stationary uniform magnetic field, this problem requires a more sophisticated treatment with the use of TEM as an accelerater.

There is a need to note that the particle frequencies are various for the different $P_0$, in particular, for electrons with $\Theta_0 = 0$ (cf. Table 1 and Table 3). Here the difference of frequencies are by thirteen orders! And the particle frequency is twice that of the wave frequency when both the particle and TEN move to meet each other (see Table 6).

Finally, it should be noted that the particle parameters and the wave parameters given in APPENDIX A are the limiting values for $m, P_0, \Theta_0, P$, and $\lambda$ for the bulk of beams and TEM ([5],[6]). Since these parameters in $\vec{r}, \vec{P}_0$, and $\vec{a}$ change monotoneously [3], so the results of APPENDIX A provide a useful approximate estimate on the motion of an arbitrary particle in an arbitrary TEM.

**Table 1.**

**Particle is electron.**

$|\vec{P}_0| = 10^3 \frac{eV}{c} \quad \Theta = 0rad.$

| N | P | $\lambda$ | Sp | $\rho$ | $\Delta\mathcal{E}$ | $a_{xy}$ | $a_z$ | $\omega$ |
|---|---|---|---|---|---|---|---|---|
| 1 | $10^{-3}$ | 0.5 | $0_{(x)}$ | $0.7*10^{-9}$ | $0.5*10^{-15}$ | $0.2*10^{16}$ | $0.7*10^{5}$ | $0.6*10^{15}$ |
| 2 | - | - | $0_{(y)}$ | $0.4*10^{-15}$ | $0.1*10^{-15}$ | - | - | - |
| 3 | - | $10^3$ | $0_{(x)}$ | $0.1*10^{-5}$ | $0.2*10^{-8}$ | - | $0.1*10^{9}$ | $0.3*10^{12}$ |
| 4 | - | - | $0_{(y)}$ | $0.1*10^{-8}$ | $0.5*10^{-9}$ | - | - | - |
| 5 | $10^{10}$ | 0.5 | $0_{(x)}$ | $0.2*10^{-2}$ | $0.5*10^{-2}$ | $0.8*10^{22}$ | $0.5*10^{18}$ | $0.6*10^{15}$ |
| 6 | - | - | $0_{(y)}$ | $0.1*10^{-8}$ | $0.1*10^{-2}$ | - | - | - |
| 7 | - | $10^3$ | $0_{(x)}$ | $0.4*10^{1}$ | $0.2*10^{5}$ | - | $0.1*10^{22}$ | $0.3*10^{12}$ |
| 8 | - | - | $0_{(y)}$ | $0.5*10^{-3}$ | $0.4*10^{4}$ | - | - | - |

**Table 2.**

**Particle is electron.**

$|\vec{P}_0| = 10^3 \frac{eV}{c} \quad \Theta = 10^{-3}rad.$

| N | P | $\lambda$ | Sp | $\rho$ | $\Delta\mathcal{E}$ | $a_{xy}$ | $a_z$ | $\omega$ |
|---|---|---|---|---|---|---|---|---|
| 1 | $10^{-3}$ | 0.5 | $0_{(x)}$ | $0.6*10^{-4}$ | $0.5*10^{-15}$ | $0.2*10^{16}$ | $0.6*10^{5}$ | $0.6*10^{15}$ |
| 2 | - | - | $0_{(y)}$ | - | $0.2*10^{-10}$ | - | $0.5*10^{10}$ | - |
| 3 | - | $10^3$ | $0_{(x)}$ | - | $0.2*10^{-8}$ | - | $0.1*10^{9}$ | $0.3*10^{12}$ |
| 4 | - | - | $0_{(y)}$ | - | $0.2*10^{-7}$ | - | $0.5*10^{10}$ | - |
| 5 | $10^{10}$ | 0.5 | $0_{(x)}$ | $0.2*10^{-2}$ | $0.5*10^{-2}$ | $0.8*10^{22}$ | $0.6*10^{18}$ | $0.6*10^{15}$ |
| 6 | - | - | $0_{(y)}$ | $0.6*10^{-4}$ | $0.1*10^{-2}$ | - | - | - |
| 7 | - | $10^3$ | $0_{(x)}$ | $0.4*10^{1}$ | $0.2*10^{5}$ | - | $0.1*10^{22}$ | $0.3*10^{12}$ |
| 8 | - | - | $0_{(y)}$ | $0.4*10^{-2}$ | $0.4*10^{4}$ | - | - | - |

**Table 3.**

**Particle is electron.**

$|\vec{P}_0| = 10^{12} \frac{eV}{c} \quad \Theta = 0rad.$

| N | P | $\lambda$ | Sp | $\rho$ | $\Delta\mathcal{E}$ | $a_{xy}$ | $a_z$ | $\omega$ |
|---|---|---|---|---|---|---|---|---|
| 1 | $10^{-3}$ | 0.5 | $0_{(x)}$ | $0.1*10^{-28}$ | $0.1*10^{-21}$ | $0.8*10^{-10}$ | $0.8*10^{-27}$ | $0.8*10^{2}$ |
| 2 | - | - | $0_{(y)}$ | $0.8*10^{-12}$ | - | $0.2*10^{-3}$ | - | - |
| 3 | - | $10^3$ | $0_{(x)}$ | $0.7*10^{-32}$ | - | $0.4*10^{-13}$ | $0.9*10^{-27}$ | $0.4*10^{-1}$ |
| 4 | - | - | $0_{(y)}$ | $0.8*10^{-22}$ | - | $0.2*10^{-3}$ | - | - |
| 5 | $10^{10}$ | 0.5 | $0_{(x)}$ | $0.4*10^{-22}$ | 0.0 | $0.3*10^{-3}$ | $0.8*10^{-14}$ | $0.8*10^{2}$ |
| 6 | - | - | $0_{(y)}$ | $0.3*10^{-15}$ | $0.1*10^{-8}$ | $0.5*10^{3}$ | - | - |
| 7 | - | $10^3$ | $0_{(x)}$ | $0.2*10^{-25}$ | 0.0 | $0.1*10^{-6}$ | - | $0.4*10^{-1}$ |
| 8 | - | - | $0_{(y)}$ | $0.3*10^{-15}$ | $0.1*10^{-8}$ | $0.5*10^{3}$ | - | - |

| Table 4. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Particle is electron.** | | | | | | | | |
| $\|\vec{P}_0\| = 10^{12}\frac{eV}{c}$ $\Theta = 10^{-3}rad.$ | | | | | | | | |
| N | P | $\lambda$ | Sp | $\rho$ | $\Delta\mathcal{E}$ | $a_{xy}$ | $a_z$ | $\omega$ |
| 1 | $10^{-3}$ | 0.5 | $0_{(x)}$ | $0.3*10^{-1}$ | $0.2*10^{-15}$ | $0.6*10^{3}$ | $0.9*10^{-14}$ | $0.3*10^{9}$ |
| 2 | - | - | $0_{(y)}$ | - | $-0.2*10^{-1}$ | $0.2*10^{3}$ | 0.2 | - |
| 3 | - | $10^{3}$ | $0_{(x)}$ | - | 0.0 | 0.6 | $0.1*10^{-13}$ | $0.2*10^{6}$ |
| 4 | - | - | $0_{(y)}$ | - | $-0.4*10^{-1}$ | $0.6*10^{3}$ | 0.6 | - |
| 5 | $10^{10}$ | 0.5 | $0_{(x)}$ | - | $0.2*10^{-2}$ | $0.2*10^{10}$ | $0.9*10^{-1}$ | $0.3*10^{9}$ |
| 6 | - | - | $0_{(y)}$ | - | $0.1*10^{6}$ | $0.6*10^{9}$ | $0.1*10^{7}$ | - |
| 7 | - | $10^{3}$ | $0_{(x)}$ | - | $0.1*10^{-8}$ | $0.2*10^{7}$ | 0.1 | $0.2*10^{6}$ |
| 8 | - | - | $0_{(y)}$ | - | $0.1*10^{6}$ | $0.2*10^{10}$ | $0.2*10^{7}$ | - |

| Table 5. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Particle is electron (proton).** | | | | | | | | |
| $\|\vec{P}_0\| = 10^{12}\frac{eV}{c}$ $\Theta = (\pi - 10^{-3})rad.$ | | | | | | | | |
| N | P | $\lambda$ | Sp | $\rho$ | $\Delta\mathcal{E}$ | $a_{xy}$ | $a_z$ | $\omega$ |
| 1 | $10^{-3}$ | 0.5 | $0_{(x)}$ | $0.3*10^{-1}$ | $-0.1*10^{-21}$ | $0.2*10^{10}$ | $0.4*10^{-7}$ | $0.1*10^{16}$ |
| 2 | - | - | $0_{(y)}$ | - | $0.5*10^{-8}$ | - | $0.2*10^{7}$ | - |
| 3 | - | $10^{3}$ | $0_{(x)}$ | - | $0.5*10^{-15}$ | $0.2*10^{10}$ | $0.7*10^{-4}$ | $0.6*10^{12}$ |
| 4 | - | - | $0_{(y)}$ | - | $0.1*10^{-4}$ | - | $0.2*10^{7}$ | - |
| 5 | $10^{10}$ | 0.5 | $0_{(x)}$ | - | $0.1*10^{-8}$ | $0.8*10^{16}$ | $0.4*10^{6}$ | $0.1*10^{16}$ |
| 6 | - | - | $0_{(y)}$ | - | $0.1*10^{-1}$ | - | $0.8*10^{13}$ | - |
| 7 | - | $10^{3}$ | $0_{(x)}$ | - | $0.4*10^{-2}$ | $0.8*10^{16}$ | $0.7*10^{9}$ | $0.6*10^{12}$ |
| 8 | - | - | $0_{(y)}$ | - | $0.3*10^{2}$ | - | $0.8*10^{13}$ | - |

| Table 6. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Particle is electron (proton).** | | | | | | | | |
| $\|\vec{P}_0\| = 10^{12}\frac{eV}{c}$ $\Theta = \pi rad.$ | | | | | | | | |
| N | P | $\lambda$ | Sp | $\rho$ | $\Delta\mathcal{E}$ | $a_{xy}$ | $a_z$ | $\omega$ |
| 1 | $10^{-3}$ | 0.5 | $0_{(x)}$ | $0.4*10^{-15}$ | $-0.2*10^{-21}$ | $0.2*10^{10}$ | $0.4*10^{-7}$ | $0.1*10^{16}$ |
| 2 | - | - | $0_{(y)}$ | $0.9*10^{-22}$ | - | - | - | - |
| 3 | - | $10^{3}$ | $0_{(x)}$ | $0.6*10^{-12}$ | $0.5*10^{-15}$ | - | $0.7*10^{-4}$ | $0.6*10^{12}$ |
| 4 | - | - | $0_{(y)}$ | $0.4*10^{-15}$ | $0.1*10^{-15}$ | - | - | - |
| 5 | $10^{10}$ | 0.5 | $0_{(x)}$ | $0.1*10^{-8}$ | $0.1*10^{-8}$ | $0.8*10^{16}$ | $0.4*10^{6}$ | $0.1*10^{16}$ |
| 6 | - | - | $0_{(y)}$ | $0.3*10^{-15}$ | $0.3*10^{-9}$ | - | - | - |
| 7 | - | $10^{3}$ | $0_{(x)}$ | $0.2*10^{-5}$ | $0.5*10^{-2}$ | - | $0.7*10^{9}$ | $0.6*10^{12}$ |
| 8 | - | - | $0_{(y)}$ | $0.1*10^{-8}$ | $0.1*10^{-2}$ | - | - | - |

| Table 7. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Particle is proton.** | | | | | | | | |
| $\lvert\vec{P}_0\rvert = 10^3 \frac{eV}{c}$ $\Theta = 0 rad.$ | | | | | | | | |
| N | P | $\lambda$ | Sp | $\rho$ | $\Delta\mathcal{E}$ | $a_{xy}$ | $a_z$ | $\omega$ |
| 1 | $10^{-3}$ | 0.5 | $0_{(x)}$ | $0.4 * 10^{-12}$ | $0.3 * 10^{-18}$ | $0.1 * 10^{13}$ | $0.2 * 10^{-1}$ | $0.6 * 10^{15}$ |
| 2 | - | - | $0_{(y)}$ | $0.2 * 10^{-18}$ | $0.7 * 10^{-19}$ | - | - | - |
| 3 | - | $10^3$ | $0_{(x)}$ | $0.7 * 10^{-9}$ | $0.1 * 10^{-11}$ | - | $0.4 * 10^2$ | $0.3 * 10^{12}$ |
| 4 | - | - | $0_{(y)}$ | $0.7 * 10^{-12}$ | $0.2 * 10^{-12}$ | - | - | - |
| 5 | $10^{10}$ | 0.5 | $0_{(x)}$ | $0.1 * 10^{-5}$ | $0.3 * 10^{-5}$ | $0.4 * 10^{19}$ | $0.2 * 10^{12}$ | $0.6 * 10^{15}$ |
| 6 | - | - | $0_{(y)}$ | $0.6 * 10^{-12}$ | $0.5 * 10^{-6}$ | - | - | - |
| 7 | - | $10^3$ | $0_{(x)}$ | $0.2 * 10^{-2}$ | $0.1 * 10^2$ | - | $0.4 * 10^{15}$ | $0.3 * 10^{12}$ |
| 8 | - | - | $0_{(y)}$ | $0.3 * 10^{-5}$ | $0.2 * 10^1$ | - | - | - |

| Table 8. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Particle is proton.** | | | | | | | | |
| $\lvert\vec{P}_0\rvert = 10^3 \frac{eV}{c}$ $\Theta = 10^{-3} rad.$ | | | | | | | | |
| N | P | $\lambda$ | Sp | $\rho$ | $\Delta\mathcal{E}$ | $a_{xy}$ | $a_z$ | $\omega$ |
| 1 | $10^{-3}$ | 0.5 | $0_{(x)}$ | $0.3 * 10^{-7}$ | $0.3 * 10^{-18}$ | $0.1 * 10^{13}$ | $0.2 * 10^{-1}$ | $0.6 * 10^{15}$ |
| 2 | - | - | $0_{(y)}$ | - | $0.2 * 10^{-13}$ | - | - | - |
| 3 | - | $10^3$ | $0_{(x)}$ | - | $0.1 * 10^{-11}$ | - | $0.4 * 10^2$ | $0.3 * 10^{12}$ |
| 4 | - | - | $0_{(y)}$ | - | $0.2 * 10^{-10}$ | - | $0.1 * 10^4$ - | - |
| 5 | $10^{10}$ | 0.5 | $0_{(x)}$ | $0.2 * 10^{-5}$ | $0.3 * 10^{-5}$ | $0.4 * 10^{19}$ | $0.2 * 10^{12}$ | $0.6 * 10^{15}$ |
| 6 | - | - | $0_{(y)}$ | $0.4 * 10^{-7}$ | $0.5 * 10^{-6}$ | - | - | - |
| 7 | - | $10^3$ | $0_{(x)}$ | $0.2 * 10^{-2}$ | $0.1 * 10^2$ | - | $0.4 * 10^{15}$ | $0.3 * 10^{12}$ |
| 8 | - | - | $0_{(y)}$ | $0.3 * 10^{-5}$ | $0.2 * 10^1$ | - | - | - |

| Table 9. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Particle is proton.** | | | | | | | | |
| $\lvert\vec{P}_0\rvert = 10^{12} \frac{eV}{c}$ $\Theta = 0 rad.$ | | | | | | | | |
| N | P | $\lambda$ | Sp | $\rho$ | $\Delta\mathcal{E}$ | $a_{xy}$ | $a_z$ | $\omega$ |
| 1 | $10^{-3}$ | 0.5 | $0_{(x)}$ | $0.1 * 10^{-15}$ | $0.2 * 10^{-15}$ | $0.5 * 10^3$ | $0.7 * 10^{-14}$ | $0.3 * 10^9$ |
| 2 | - | - | $0_{(y)}$ | $0.2 * 10^{-15}$ | $0.1 * 10^{-15}$ | - | - | - |
| 3 | - | $10^3$ | $0_{(x)}$ | $0.8 * 10^{-19}$ | 0.0 | 0.5 | $0.1 * 10^{-13}$ | $0.1 * 10^6$ |
| 4 | - | - | $0_{(y)}$ | $0.3 * 10^{-15}$ | $0.4 * 10^{-15}$ | $0.5 * 10^3$ | - | - |
| 5 | $10^{10}$ | 0.5 | $0_{(x)}$ | $0.4 * 10^{-9}$ | $0.2 * 10^{-2}$ | $0.2 * 10^{10}$ | $0.7 * 10^{-1}$ | $0.3 * 10^9$ |
| 6 | - | - | $0_{(y)}$ | $0.7 * 10^{-9}$ | $0.1 * 10^{-2}$ | - | - | - |
| 7 | - | $10^3$ | $0_{(x)}$ | $0.3 * 10^{-12}$ | $0.7 * 10^{-9}$ | $0.1 * 10^7$ | 0.1 | $0.1 * 10^6$ |
| 8 | - | - | $0_{(y)}$ | $0.8 * 10^{-12}$ | $0.4 * 10^{-2}$ | $0.2 * 10^{10}$ | - | - |

| Table 10. | | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| **Particle is proton.** | | | | | | | | |
| $\vert\vec{P}_0\vert = 10^{12}\frac{eV}{c}$ $\Theta = 10^{-3}rad.$ | | | | | | | | |
| N | P | $\lambda$ | Sp | $\rho$ | $\Delta\mathcal{E}$ | $a_{xy}$ | $a_z$ | $\omega$ |
| 1 | $10^{-3}$ | 0.5 | $0_{(x)}$ | $0.3*10^{-1}$ | $0.3*10^{-15}$ | $0.1*10^4$ | $0.2*10^{-13}$ | $0.6*10^9$ |
| 2 | - | - | $0_{(y)}$ | - | $0.2*10^{-1}$ | $0.7*10^2$ | $0.1*10^1$ | - |
| 3 | - | $10^3$ | $0_{(x)}$ | - | $0.7*10^{-21}$ | $0.2*10^1$ | $0.5*10^{-13}$ | $0.3*10^6$ |
| 4 | - | - | $0_{(y)}$ | - | $0.4*10^{-1}$ | $0.8*10^2$ | $0.1*10^1$ | - |
| 5 | $10^{10}$ | 0.5 | $0_{(x)}$ | - | $0.3*10^{-2}$ | $0.4*10^{10}$ | 0.2 | $0.6*10^9$ |
| 6 | - | - | $0_{(y)}$ | - | $0.4*10^5$ | $0.2*10^9$ | $0.4*10^7$ | - |
| 7 | - | $10^3$ | $0_{(x)}$ | - | $0.7*10^{-8}$ | $0.7*10^7$ | 0.5 | $0.3*10^6$ |
| 8 | - | - | $0_{(y)}$ | - | $0.1*10^6$ | $0.2*10^9$ | $0.4*10^7$ | $0.3*10^6$ |

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## APPENDIX A

The particle (electron or proton) with the value of $P_0 = 10^3, 10^{12}$ $(\frac{\mathrm{eV}}{\mathrm{c}})$ in the initial point of four-dimension space [0,0,0,0] with $\Theta = 0, 10^{-3}, \pi, \pi - 10^{-3}$ (rad) and with $\phi_0 = 0$ (rad) enter TEM (a laser wave) with a wave power $P = 10^{-3}, 10^{10}$ (Watt) with a wave length $\lambda = 0.5, 10^3$ ($\mu$m), and with $\phi = 0$ (rad). TEM is a linear polarization with $Sp = 0_{(x)}$. This is true that TEM is polarized in the coordinate plane Oyz and with $Sp = 0_{(y)}$ this is true that TEM is polarized in the coordinate plane Oxz.

The terminal time is $t = 10^{-9}$ (s).

From formulae [3] we calculate the following values corresponding to the motion of particle in TEM: $\rho = \sqrt{x^2(t) + y^2(t)}(sm)$, $\Delta\mathcal{E} = \mathcal{E}(t) - \mathcal{E}_0(eV)$, $a_{xy} = \sqrt{a_x^2 + a_y^2}(\frac{s}{s^2})$, $a_z(\frac{sm}{s^2})$, and $\omega(\frac{1}{sec})$ is the particle frequency.

## APPENDIX B

$$x = x_0 + R[(P_{0x} - \frac{1}{\omega}eE_1 sin\phi_0)T - A_1^{-1}\frac{1}{\omega^2}eE_1C(\phi)]$$
$$y = y_0 + R[(P_{0y} + g\frac{1}{\omega}eE_2 cos\phi_0)T - gA_1^{-1}\frac{1}{\omega^2}eE_2S(\phi)]$$
$$z = z_0 + ZT.$$

$$P_x = P_{0x} + \frac{1}{\omega}eE_1S(\phi),$$

$$P_и = P_{0y} - g\frac{1}{\omega}eE_2C(\phi),$$

$$P_z = PP_0.$$

$$a_x = eR[A_1 E_1 cos(\phi) + A_2 A_x],$$

$$a_y = eR[gA_1 E_2 sin(\phi) + A_2 A_y],$$

$$a_z = ecA_1 A_2 A_z.$$

where

$$A_x = \frac{e^2}{\omega^2} E_1 S(\phi)[-E_1^2 S(\phi)cos(\phi) + E_2^2 C(\phi)sin(\phi)] - \frac{e}{\omega}[gE_1 E_2 P_{0y} S(\phi)sin(\phi) + 2E_1^2 P_{0x} S(\phi)cos(\phi) - E_2^2 P_{0x} C(\phi)sin(\phi)] - P_{0x}(E_1 P_{0x} cos(\phi) + gE_2 P_{0y} sin(\phi),$$

$$A_y = g\frac{e^2}{\omega^2} E_2 C(\phi)[E_1^2 S(\phi)cos(\phi) - E_2^2 C(\phi)sin(\phi)] - \frac{e}{\omega}[gE_1 E_2 P_{0x} C(\phi)cos(\phi) + 2E_2^2 P_{0y} C(\phi)sin(\phi) - E_1^2 P_{0y} S(\phi)cos(\phi)] - P_{0y}(E_1 P_{0x} cos(\phi) + gE_2 P_{0y} sin(\phi),$$

$$A_z = \frac{e}{\omega}[E_1^2 S(\phi)cos(\phi) - E_2^2 C(\phi)sin(\phi)] + E_1 P_{0x} cos(\phi) + gE_2 P_{0y} sin(\phi).$$

## Case $n_0 = 1$ and $\phi \ll 1$

$$x = x_0 + R[(P_{0x} T - eE_1 A_1(-\frac{1}{2}T^2 cos\phi_0 + \frac{1}{6}A_1 \omega T^3 sin\phi_0)]$$
$$y = y_0 + R[(P_{0y} T + eE_2 A_1(\frac{1}{2}T^2 sin\phi_0 + \frac{1}{6}A_1 \omega T^3 cos\phi_0)]$$
$$z = z_0 + ZT.$$

$$P_x = P_{0x} - A_1 eE_1(-Tcos\phi + \frac{1}{2}A_1 T^2 \omega sin\phi_0),$$

$$P_y = P_{0y} + gA_1 eE_2(Tsin\phi + \frac{1}{2}A_1 T^2 \omega cos\phi_0),$$

$$P_z = PP_0.$$

$$a_x = R[A_1 eE_1(cos\phi_0 - A_1 \omega T sin\phi_0) + \omega A_2 A_x],$$

$$a_x = R[gA_1 eE_2(sin\phi_0 + A_1 \omega T cos\phi_0) + \omega A_2 A_y],$$

$$a_z = ecA_1 A_2 A_z.$$

Here $A_x, A_y, A_z$ are the preceding $A_x, A_y, A_z$ where the trigonometrical functions are expanded in $\phi \ll 1$.

In all cases

$$v_x = RP_x, \qquad v_y = RP_y.$$

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