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V.V.Braguta and A.E.Chalov

PAIR PRODUCTION OF DOUBLY HEAVY DIQUARKS

Moscow Institute of Physics and Technology,
Dolgoprudny, Moscow region, 147000 Russia

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Abstract

Braguta V.V. and Chalov A.E. Pair production of doubly heavy diquarks: IHEP Preprint 2000-56. – Protvino, 2000. – p. 12, figs. 4, refs.: 6.

We analytically calculate the total and differential cross sections for the pair production of doubly heavy diquarks in the framework of diquark model. The processes of electron-positron and quark-antiquark annihilations are considered. The fractions of doubly heavy diquarks in the yields of heavy quarks are evaluated numerically.

Аннотация

Брагута В.В., Чалов А.Е. Парное рождение дважды тяжелых дикварков: Препринт ИФВЭ 2000-56. – Протвино, 2000. – 12 с., 4 рис., библиогр.: 6.

Проведен аналитический расчет дифференциальных и полных сечений для процессов парного рождения дважды тяжёлых дикварков в рамках дикварковой модели. Рассмотрены случаи электрон-позитронной и кварк-антикварковой аннигиляции. Даны численные оценки отношения полных сечений к соответствующим сечениям в два тяжёлых кварка.

1. Introduction

High luminosities of B-factories and hadron colliders open a real experimental possibility to observe doubly heavy baryons $\Xi_{QQ'}$ and $\Omega_{QQ'}$. This prospect stimulates a theoretical interest to study the physics of such baryons: the spectroscopy in the framework of both potential models [1] and QCD sum rules [2], the lifetimes and inclusive decay modes [3], the mechanism of production in various collisions and the rate of yield at accelerators [4].

The pair production can be essential at the energies close to the threshold we considered here. The physical approximation for the calculations is caused by the apparent diquark structure of doubly heavy baryons, since the diquark size is essentially less than the radius of confinement determining the motion of light quark inside the $QQ'q$ system. We suppose that, at first, the calculations of heavy diquark production should be made, and further, the models of diquark fragmentation into the baryon [5] can be applied.

In [6] the differential and total cross sections for exclusive production of meson pairs in e^+e^- annihilation were calculated in the framework of constituent quark model. The results were obtained for the case of final particles in pseudoscalar and vector states close to the threshold.

Following the same procedure, we examine the processes of $e^+e^- \to d\bar{d}$ and $q\bar{q} \to d\bar{d}$, where d, $\bar{d}$ are diquark and antidiquark, respectively (in our calculations we neglect masses of annihilating particles). Differential and total cross sections for the exclusive production of heavy diquark pairs in axial-axial, axial-scalar and scalar-scalar states are determined for the diquark composed of different heavy quarks. We also consider the case of axial diquark composed of equivalent quarks. We do not care for the annihilation into pseudoscalar and vector diquarks, since their production does not contribute to the leading order of $1/m$ expansion. If one supposes the fragmentation of diquarks into the doubly heavy baryons, the formulae derived may be useful in the calculation of cross sections for the pair production of baryons.

In section 2, we describe basic points of the constituent quark model. Sections 3 and 4 contain matrix elements, differential and total cross sections for the processes of e^+e^- and $q\bar{q}$ annihilation into the pairs of diquarks. In section 5 the numerical results are given. In Conclusion we summarize the consideration.

2. Basic points of the model

In this paper we use the constituent quark model [6], which considers the quark masses and leptonic constants as the only input parameters.

According to the model the diquark $d = (Q_1 Q_2)$ mass is equal to

$$M = m_1 + m_2.$$

Four-momenta of the quarks entering the diquark are given by

$$k_{Q_1} = \frac{m_1}{M} P + q, \qquad k_{Q_2} = \frac{m_2}{M} P - q,$$

where P is the diquark momentum.

To represent the Fock state of diquark with two different quarks in the model, we use the principle of superposition for the wave packages with distributions Ψ. Therefore, we find the following:

in the case of scalar diquark

$$|S_d^i\rangle = \frac{\epsilon_{ijk}}{\sqrt{2}} \int \frac{d^3q}{(2\pi)^3} \Psi_s(q) \sum_{\lambda_1 \lambda_2} \frac{(\Psi^\dagger_{\lambda_1} \hat{C} \gamma_5 \Psi_{\lambda_2})^*}{\sqrt{2}} \hat{a}^{j+}_{\lambda_1} \hat{b}^{k+}_{\lambda_2} |0\rangle, \tag{1}$$

and for the axial diquark

$$|A_d^i\rangle = \frac{\epsilon_{ijk}}{\sqrt{2}} \int \frac{d^3q}{(2\pi)^3} \Psi_a(q) \sum_{\lambda_1 \lambda_2} \frac{(\Psi^\dagger_{\lambda_1} \hat{C} \gamma_\mu \Psi_{\lambda_2})^*}{\sqrt{2}} e_\mu \hat{a}^{j+}_{\lambda_1} \hat{b}^{k+}_{\lambda_2} |0\rangle, \tag{2}$$

where $\hat{C}$ is a matrix of charge conjugation, i and j are the colour indices, $\hat{e} = e_\mu \gamma_\mu$, e_μ is a polarization vector of axial diquark, $\hat{a}$ and $\hat{b}$ denote the operators of quark creation, so that the axial and scalar diquarks in the Fock space are normilized in the following way:

$$\langle S_d^i(P) | S_d^j(P') \rangle = (2\pi)^3 \delta_{ij} \delta(\vec{P} - \vec{P}'), \tag{3}$$

$$\langle A_d^i(P, \lambda) | A_d^j(P', \lambda') \rangle = (2\pi)^3 \delta_{ij} \delta_{\lambda\lambda'} \delta(\vec{P} - \vec{P}'), \tag{4}$$

λ and λ' are diquark polarization indices.

If the heavy quarks are identical, then the Pauli principle must be taken into account. So, the above formulae have to be divided by $\sqrt{2}$ and antisymmetrized over the permutations of $\hat{a}$ and $\hat{b}$ operators.

The heavy quark propagator has the form

$$S(k) = (k_\mu \gamma_\mu + m) D(k),$$

where

$$D^{-1}(k) = k^2 - m^2.$$

3. Amplitudes and cross sections for the e^+e^- annihilation

In this section we consider the e^+e^- annihilation into the diquark and antidiquark pairs. The diagrams, which contribute into the processes in the leading order, are shown in Fig. 1. The colour factor for the processes involved is equal to

$$\text{Colour}_{ij} = -\frac{2}{3} \delta_{ij} \ ,$$

where i, j are the colour indices of diquark and antidiquark, correspondingly.

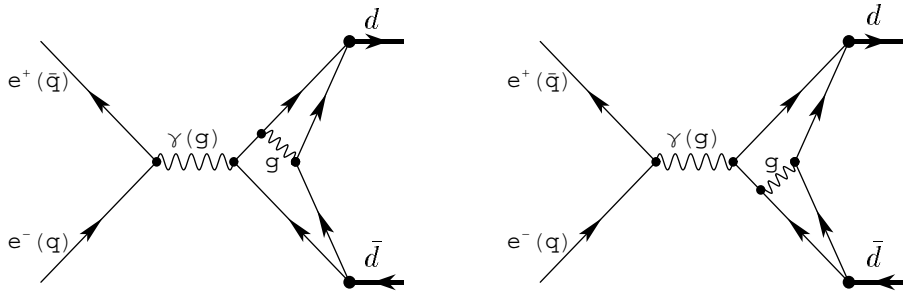

Fig. 1. The diagrams of e^+e^- $(q\bar{q})$ annihilation into the pair of doubly heavy diquarks due to the single photon (gluon) exchange.

3.1. Annihilation into pairs of scalar diquarks

The matrix element for the pair production of scalar diquarks may be written as follows:

$$\mathcal{M}_{ss} = -i\frac{64\pi^2}{3}\,\frac{f_{ss}}{s^2}\,\delta_{ij}\,|\Psi_s(0)|^2\,(P'_\mu - P_\mu)l_\mu \tag{5}$$

where f_{ss} is equal to

$$\begin{aligned} f_{ss} \;=\; & M\left(\alpha_s\Big(\frac{m_1^2}{M^2}s\Big)\cdot\frac{q_2}{m_1^2} + \alpha_s\Big(\frac{m_2^2}{M^2}s\Big)\cdot\frac{q_1}{m_2^2}\right)\alpha_{em}\,(s) - \\ & -\frac{2M^3}{s}\left(\alpha_s\Big(\frac{m_1^2}{M^2}s\Big)\cdot\frac{q_2 m_2}{m_1^3} + \alpha_s\Big(\frac{m_2^2}{M^2}s\Big)\cdot\frac{q_1 m_1}{m_2^3}\right)\alpha_{em}\,(s)\,, \end{aligned} \tag{6}$$

and l_μ denotes the leptonic vector current, $\Psi_s(0)$ is the wave function of scalar diquark at the origin. The charges of the quarks Q_1 and Q_2 are equal to q_1 and q_2. P', P are the four-momenta of scalar diquark and antidiquark, correspondingly.

After simple algebraic calculations for the differential cross section $d\sigma_{ss}/\,d\cos\theta$, we get the following expression:

$$\frac{d\sigma_{ss}}{d\cos\theta} = 64\pi^3\frac{f_{ss}^2}{3s^3}\,|\Psi_s(0)|^4\left(1-\frac{4M^2}{s}\right)^{3/2}(1-\cos^2\theta), \tag{7}$$

where θ is the angle between the momenta of lepton and diquark.

The total cross section for the exclusive production of heavy scalar diquark pairs in e^+e^- annihilation is equal to

$$\sigma_{ss} = 256\pi^3\frac{f_{ss}^2}{9s^3}\,|\Psi_s(0)|^4\left(1-\frac{4M^2}{s}\right)^{3/2}. \tag{8}$$

3.2. Annihilation into axial and scalar diquarks

The matrix element for the pair production of axial antidiquark and scalar diquark may be written as follows:

$$\mathcal{M}_{as} = -\frac{128\pi^2}{3s^3}\delta_{ij}f_{as}\Psi_s^*(0)\Psi_a(0)\epsilon_{\mu\alpha\beta\gamma}e_\alpha P_\beta q_\gamma l_\mu, \tag{9}$$

where f_{as} is equal to

$$f_{as} = M^3 \left(\alpha_s\Big(\frac{m_1^2}{M^2}s\Big) \cdot \frac{q_2}{m_1^3} - \alpha_s\Big(\frac{m_2^2}{M^2}s\Big) \cdot \frac{q_1}{m_2^3} \right) \alpha_{em}\,(s)\,.$$

The $\Psi_a(0)$ is the wave function of axial diquark at the origin, $q = P + P'$.

Carying out algebraic calculations for the differential cross section $d\sigma_{as}/d\cos\theta$, we find the following expression:

$$\frac{d\sigma_{as}}{d\cos\theta} = 64\pi^3\,\frac{f_{as}^2}{3s^4}\,|\Psi_s(0)\Psi_a(0)|^2 \left(1 - \frac{4M^2}{s}\right)^{3/2} (2 - \sin^2\theta). \tag{10}$$

The total cross section for the exclusive production of heavy scalar diquark and axial antidiquark pairs in e^+e^- annihilation is given by the following formula:

$$\sigma_{as} = 512\pi^3\,\frac{f_{as}^2}{9s^4}\,|\Psi_s(0)\Psi_a(0)|^2 \left(1 - \frac{4M^2}{s}\right)^{3/2}. \tag{11}$$

In the above formulae the difference between the masses of axial and scalar diquarks is neglected.

3.3. Annihilation into pairs of axial diquarks

The matrix element for the exclusive pair production of two axial diquarks may be represented as follows:

$$\mathcal{M}_{aa} = -i\frac{128\pi^2}{3s^3}\delta_{ij}\,|\Psi_a(0)|^2 \left(f_{aa}^{[1]}(P'_\mu - P_\mu)(e'^*e) + f_{aa}^{[2]}((e'^*q)e_\mu - (eq)e'^*_\mu) \right) l_\mu, \tag{12}$$

where $f_{aa}^{[1]}$ and $f_{aa}^{[2]}$ are equal to

$$f_{aa}^{[1]} = M^3 \left(\alpha_s\Big(\frac{m_1^2}{M^2}s\Big) \cdot \frac{q_2 m_2}{m_1^3} + \alpha_s\Big(\frac{m_2^2}{M^2}s\Big) \cdot \frac{q_1 m_1}{m_2^3} \right) \alpha_{em}\,(s)\,, \tag{13}$$

$$f_{aa}^{[2]} = M^4 \left(\alpha_s\Big(\frac{m_1^2}{M^2}s\Big) \cdot \frac{q_2}{m_1^3} + \alpha_s\Big(\frac{m_1^2}{M^2}s\Big) \cdot \frac{q_1}{m_2^3} \right) \alpha_{em}\,(s)\,. \tag{14}$$

For the differential cross section $d\sigma_{aa}/d\cos\theta$ we calculate the following expression:

$$\frac{d\sigma_{aa}}{d\cos\theta} = \frac{512\pi^3}{3s^5}\,|\Psi_a(0)|^4 \left(1 - \frac{4M^2}{s}\right)^{3/2} (\mathcal{A} - \mathcal{B}\cos^2\theta),$$

where $\mathcal{A}$ and $\mathcal{B}$ have the form

$$\mathcal{A} = (f_{aa}^{[1]})^2(8 + (\eta - 2)^2) - 2f_{aa}^{[1]}f_{aa}^{[2]}\eta(\eta - 2) + (f_{aa}^{[2]})^2(\eta^2 + 2\eta),$$

$$\mathcal{B} = (f_{aa}^{[1]})^2(8 + (\eta - 2)^2) - 2f_{aa}^{[1]}f_{aa}^{[2]}\eta(\eta - 2) + (f_{aa}^{[2]})^2(\eta^2 - 2\eta),$$

and $\eta = s/M^2$.

The total cross section for the exclusive production of heavy axial diquarks pairs in e^+e^- annihilation is given by

$$\sigma_{aa} = \frac{1024\pi^3}{9s^5}\,|\Psi_a(0)|^4 \left(1 - \frac{4M^2}{s}\right)^{3/2} (3\mathcal{A} - \mathcal{B}). \tag{15}$$

Actually, the behaviour of this cross section at large s is $\sigma_{aa} \sim 1/s^3$.

3.4. The diquark containing two equivalent particles

If the diquarks are composed of identical particles, the above formulae must be changed. Evidently, we have to take into account the production of axial diquark, only. Since the smallest angular momentum for the scalar diquark is equal to unity, it does not contribute in the leading order of $1/m$ expansion. In contrast, the angular momentum of axial diquark can be equal to zero.

All the formulae writen down for the annihilation into two axial diquarks remain valid except for the two form factors $f_{aa}^{[1]}, f_{aa}^{[2]}$. They must be transformed in the following way:

$$f_{aa}^{[1]} = 2q\alpha_s(M^2)M\ \alpha_{em}\,(s)\,, \tag{16}$$

$$f_{aa}^{[2]} = 4q\alpha_s(M^2)M\ \alpha_{em}\,(s)\,, \tag{17}$$

where q is the quark charge.

4. Amplitudes and cross sections for the $q\bar{q}$ annihilation

In this section we consider the $q\bar{q}$ annihilation into the diquark and antidiquark pairs. The diagrams, which can contribute to the process in the leading order, are shown in Figs.1, 2, and 3, but the diagram shown in Fig.2 reduces to zero in our approach, when the quarks are on the mass shells.

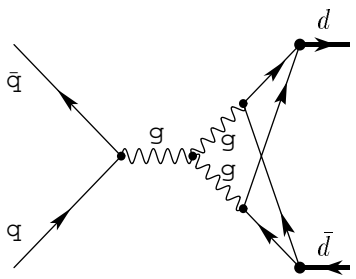

Fig. 2. The $q\bar{q}$ annihilation into the diquarks due to the three-gluon interaction.

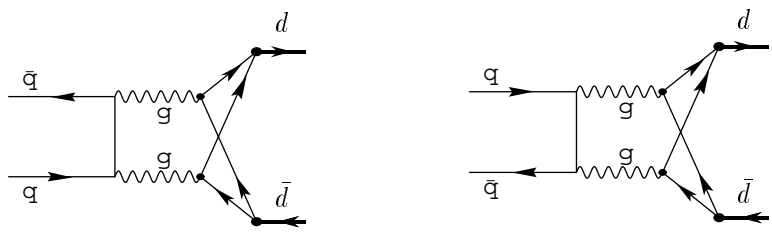

Fig. 3. The diagrams of double gluon annihilation of light quarks into the pair of doubly heavy diquarks.

The colour factor for the diagrams shown in Fig.1 is equal to

$$\mathrm{Colour}^{[1]}_{(ij)(lm)} = \frac{1}{3} t^a_{ij} t^a_{lm},$$

where m, l are the colour indices of annihilating quark and antiquark, correspondingly. It is easy to notice that this is a colour octet state. The colour factor for the diagrams shown in Fig.3 is equal to

$$\mathrm{Colour}^{[2]}_{(ij)(lm)} = \frac{5}{12} t^a_{ij} t^a_{lm} - \frac{1}{9} \delta_{ij} \delta_{lm},$$

which appears to be a mixture of octet and singlet colour states.

4.1. Annihilation into pairs of scalar diquarks

The matrix element for the exclusive pair production of two scalar diquarks may be written as follows:

$$\mathcal{M}_{ss} = \frac{32\pi^2 i}{s^2} \, |\Psi_s(0)|^2 \left(\tilde{f}^{[1]}_{ss} \frac{2 t^a_{if} t^a_{lm}}{3} P'_\mu - \tilde{f}^{[2]}_{ss} \left(\frac{5 t^a_{if} t^a_{lm}}{6} - \frac{2\delta_{if}\delta_{lm}}{9} \right) \frac{(p, P' - P)}{s} P_\mu \right) l_\mu, \tag{18}$$

where $\tilde{f}^{[1]}_{ss}$ and $\tilde{f}^{[2]}_{ss}$ are equal to

$$\tilde{f}^{[1]}_{ss} = M \left(\frac{\alpha_s\left(\frac{m_1^2}{M^2} s\right)}{m_1^2} + \frac{\alpha_s\left(\frac{m_2^2}{M^2} s\right)}{m_2^2} \right) \alpha_s(s) -$$

$$- \frac{2M^3}{s} \left(\alpha_s\left(\frac{m_1^2}{M^2} s\right) \cdot \frac{m_2}{m_1^3} + \alpha_s\left(\frac{m_2^2}{M^2} s\right) \cdot \frac{m_1}{m_2^3} \right) \alpha_s(s), \tag{19}$$

$$\tilde{f}^{[2]}_{ss} = \frac{M^5}{m_1^3 m_2^3} \alpha_s\left(\frac{m_1^2}{M^2} s\right) \alpha_s\left(\frac{m_2^2}{M^2} s\right), \tag{20}$$

and p is the four-momentum of annihilating quark.

For the differential cross section $d\sigma_{ss}/d\cos\theta$ we get the following expression:

$$\frac{d\sigma_{ss}}{d\cos\theta} = \frac{8\pi^3}{81 s^3} \, |\Psi_s(0)|^4 \left(\left(2\tilde{f}^{[1]}_{ss} + \frac{5}{4} \tilde{f}^{[2]}_{ss} \sqrt{1 - \frac{4M^2}{s}} \cos\theta \right)^2 + \frac{(\tilde{f}^{[2]}_{ss})^2}{2} (1 - \frac{4M^2}{s}) \cos^2\theta \right) \cdot$$

$$\cdot \left(1 - \frac{4M^2}{s} \right)^{3/2} (1 - \cos^2\theta). \tag{21}$$

The total cross section for the exclusive production of two heavy scalar diquarks in the $q\bar{q}$ annihilation is given by the formula

$$\sigma_{ss} = \frac{8\pi^3}{81 s^3} \, |\Psi_s(0)|^4 \left(1 - \frac{4M^2}{s} \right)^{3/2} \left(\frac{16}{3} (\tilde{f}^{[1]}_{ss})^2 + \frac{11}{20} \left(1 - \frac{4M^2}{s} \right) (\tilde{f}^{[2]}_{ss})^2 \right). \tag{22}$$

4.2. Annihilation into axial and scalar diquarks

The matrix element for the pair production of axial antidiquark and scalar diquark may be represented in the following way:

$$\mathcal{M}_{as} = \frac{32\pi^2}{s^3}\Psi_s^*(0)\Psi_a(0)\left(-\tilde{f}_{as}^{[1]}\frac{2t_{if}^a t_{lm}^a}{3}\,\epsilon_{\mu\alpha\beta\gamma}P_\beta e_\alpha q_\gamma - \tilde{f}_{as}^{[2]}\left(\frac{5t_{if}^a t_{lm}^a}{6} - \frac{2\delta_{if}\delta_{lm}}{9}\right)\epsilon_{\mu\nu\alpha\beta}q_\alpha e_\beta p_\nu\right) l_\mu, \tag{23}$$

where $\tilde{f}_{as}^{[1]}$ and $\tilde{f}_{as}^{[2]}$ are equal to

$$\tilde{f}_{as}^{[1]} = M^3\left(\frac{\alpha_s\left(\frac{m_1^2}{M^2}s\right)}{m_1^3} - \frac{\alpha_s\left(\frac{m_2^2}{M^2}s\right)}{m_2^3}\right)\alpha_s(s), \tag{24}$$

$$\tilde{f}_{as}^{[2]} = \alpha_s\Big(\frac{m_1^2}{M^2}s\Big)\alpha_s\Big(\frac{m_2^2}{M^2}s\Big)\frac{M^5(m_2-m_1)}{m_1^3 m_2^3}. \tag{25}$$

The differential cross section $d\sigma_{as}/d\cos\theta$ are given by

$$\frac{d\sigma_{as}}{d\cos\theta} = \frac{64\pi^3}{81s^4}\,|\Psi_s(0)\Psi_a(0)|^2\sqrt{1-\frac{4M^2}{s}}\cdot\left(\frac{(\tilde{f}_{as}^{[1]})^2}{2}\left(1-\frac{4M^2}{s}\right)(1+\cos^2\theta)\right. \tag{26}$$
$$\left. +\frac{33(\tilde{f}_{as}^{[2]})^2}{16}\left(1+\frac{s-4M^2}{8M^2}\sin^2\theta\right) + \frac{5}{2}\tilde{f}_{as}^{[1]}\tilde{f}_{as}^{[2]}\sqrt{1-\frac{4M^2}{s}}\cos\theta\right).$$

The total cross section for the exclusive production of heavy axial antidiquark and scalar diquark pairs in the $q\bar{q}$ annihilation equals

$$\sigma_{as} = \frac{64\pi^3}{243s^4}\,|\Psi_s(0)\Psi_a(0)|^2\sqrt{1-\frac{4M^2}{s}}\left(4(\tilde{f}_{as}^{[1]})^2\left(1-\frac{4M^2}{s}\right) + \frac{33(\tilde{f}_{as}^{[2]})^2}{8}\left(2+\frac{s}{4M^2}\right)\right). \tag{27}$$

Here we have also neglected the difference between masses of axial and scalar diquarks.

4.3. Annihilation into pairs of axial diquarks

The amplitude for the exclusive pair production of two axial diquarks in the process may be represented as follows:

$$\begin{aligned}\mathcal{M}_{aa} = {} & \frac{32\pi^2 i}{3}\frac{\alpha_s(4m_1^2)\alpha_s(4m_2^2)}{m_1^3 m_2^3 s^3}\,M^5\,|\Psi_a(0)|^2\Big\{\Big(\,(-Pe'^*)\cdot\Big((pP')(le)+(lP')(pe)\Big) + \\ & +(-P'e)\cdot\Big((Pp)(le'^*)+(lP)(pe'^*)\Big) + (ee'^*)\cdot\Big((lP)(pP')+(lP')(pP)\Big) \\ & +\frac{s}{2}\cdot\Big((pe)(le'^*)+(le)(pe'^*)\Big)\,\Big)\cdot\Big(t_{if}^a t_{lm}^a - \frac{4}{15}\delta_{if}\delta_{lm}\Big) \\ & +\Big(\tilde{f}_{aa}^{[2]}\Big((el)(Pe'^*)-(P'e)(e'^*l)\Big) + \tilde{f}_{aa}^{[1]}(Pl)(ee'^*)\Big)\cdot\Big(t_{if}^a t_{lm}^a\Big)\Big\}.\end{aligned} \tag{28}$$

The differential cross section $d\sigma_{aa}/d\cos\theta$ is given by the following expression:

$$\begin{aligned}\frac{d\sigma_{aa}}{d\cos\theta} = {} & \frac{200\pi^3}{81}\alpha_s^2(4m_1^2)\alpha_s^2(4m_2^2)\frac{M^{10}}{m_1^6 m_2^6 s^7}|\Psi_a(0)|^4\sqrt{1-\frac{4M^2}{s}} \\ & \cdot(a_4\cos^4\theta + a_3\cos^3\theta + a_2\cos^2\theta + a_1\cos\theta + a_0).\end{aligned} \tag{29}$$

The total cross section for the exclusive production of heavy axial diquark pairs in the $q\bar{q}$ annihilation has the form

$$\sigma_{aa} = \frac{200\pi^3}{81}\alpha_s^2(4m_1^2)\alpha_s^2(4m_2^2)\frac{M^{10}}{m_1^6 m_2^6 s^7}|\Psi_a(0)|^4\sqrt{1-\frac{4M^2}{s}}\left(\frac{2}{5}a_4+\frac{2}{3}a_2+2a_0\right), \qquad (30)$$

where we have used the following definitions:

$$\tilde{f}_{aa}^{[1]} = -\frac{\alpha_s(s)}{\alpha_s(4m_1^2)\alpha_s(4m_2^2)}\frac{8m_1^3m_2^3}{5M^2}\left(\frac{\alpha_s(4m_1^2)m_2}{m_1^3}+\frac{\alpha_s(4m_2^2)m_1}{m_2^3}\right), \qquad (31)$$

$$\tilde{f}_{aa}^{[2]} = \frac{\alpha_s(s)}{\alpha_s(4m_1^2)\alpha_s(4m_2^2)}\frac{4m_1^3m_2^3}{5M}\left(\frac{\alpha_s(4m_1^2)}{m_1^3}+\frac{\alpha_s(4m_2^2)}{m_2^3}\right), \qquad (32)$$

$$a_4 = \frac{99}{400}s^2(s-4M^2)^2, \qquad (33)$$

$$a_3 = \frac{1}{8}\frac{s^3}{M^2}\left(2\tilde{f}_{aa}^{[2]}s+\tilde{f}_{aa}^{[1]}(6M^2+s)\right)\left(1-4\frac{M^2}{s}\right)^{3/2}, \qquad (34)$$

$$a_2 = \frac{1}{16M^4}s(s-4M^2)\Big\{4\tilde{f}_{aa}^{[1]}\tilde{f}_{aa}^{[2]}s(s-2M^2)+(\tilde{f}_{aa}^{[1]})^2(12M^4-4M^2s+s^2)$$
$$+s\Big(\frac{33}{25}(12M^6-M^4s)+4(\tilde{f}_{aa}^{[2]})^2(s-2M^2)\Big)\Big\}, \qquad (35)$$

$$a_1 = -\frac{s^2}{8M^2}\sqrt{1-4\frac{M^2}{s}}\Big(2\tilde{f}_{aa}^{[2]}s(s+4M^2)+\tilde{f}_{aa}^{[1]}(-24M^4+2M^2s+s^2)\Big), \qquad (36)$$

$$a_0 = -\frac{s}{16M^4}\Big(4\tilde{f}_{aa}^{[1]}\tilde{f}_{aa}^{[2]}s\Big(8M^4-6M^2s+s^2\Big)+(\tilde{f}_{aa}^{[1]})^2\Big(-48M^6+28M^4s-8M^2s^2+s^3\Big)$$
$$+2s\Big(\frac{33}{25}M^4s(4M^2+s)+\tilde{f}_{aa}^{[2]}(-16M^4-4M^2s+2s^2)\Big)\Big). \qquad (37)$$

4.4. The diquark with two identical particles

In this section we consider the $q\bar{q}$ annigilation into the pair of diquarks composed of two identical heavy quarks. So, as in e^+e^- annihilation, we have to take into account the axial diquark production, only.

The amplitude has the form

$$\mathcal{M}_{aa} = \frac{512\pi^2 i}{3}\frac{\alpha_s^2(M^2)}{Ms^3}|\Psi_a(0)|^2\Big\{\Big((-Pe'^*)\cdot\Big((pP')(le)+(lP')(pe)\Big)$$
$$+(-P'e)\cdot\Big((Pp)(le'^*)+(lP)(pe'^*)\Big)+(ee'^*)\cdot\Big((lP)(pP')+(lP')(pP)\Big)$$
$$+\frac{s}{2}\cdot\Big((pe)(le'^*)+(le)(pe'^*)\Big)\Big)\cdot\Big(t_{if}^a t_{lm}^a-\frac{4}{15}\delta_{if}\delta_{lm}\Big)$$
$$+\Big(\tilde{f}_{aa}^{[2]}\Big((el)(Pe'^*)-(P'e)(e'^*l)\Big)+\tilde{f}_{aa}^{[1]}(Pl)(ee'^*)\Big)\cdot\Big(t_{if}^a t_{lm}^a\Big)\Big\}. \qquad (38)$$

The differential cross section $d\sigma_{aa}/d\cos\theta$ equals

$$\frac{d\sigma_{aa}}{d\cos\theta} = 2^{11}\frac{25\pi^3}{81}\frac{\alpha_s^4(M^2)}{M^2s^7}|\Psi_a(0)|^4\sqrt{1-\frac{4M^2}{s}}(a_4\cos^4\theta+a_3\cos^3\theta+a_2\cos^2\theta+a_1\cos\theta+a_0). \qquad (39)$$

The total cross section for the exclusive production is given by

$$\sigma_{aa} = 2^{11}\frac{25\pi^3}{81}\frac{\alpha_s^4(M^2)}{M^2 s^7}|\Psi_a(0)|^4\sqrt{1-\frac{4M^2}{s}}\left(\frac{2}{5}a_4+\frac{2}{3}a_2+2a_0\right), \tag{40}$$

where we have used the following definitions:

$$\tilde{f}_{aa}^{[1]} = -\frac{\alpha_s(s)}{\alpha_s(M^2)}\frac{M^2}{5}, \tag{41}$$

$$\tilde{f}_{aa}^{[2]} = \frac{\alpha_s(s)}{\alpha_s(M^2)}\frac{M^2}{5}. \tag{42}$$

5. Numerical estimates

The cross section ratios of the exclusive heavy diquark pair production to the respective heavy quark cross section in e^+e^- annihilation,

$$\sigma(e^+e^- \to Q\bar{Q}) = \frac{4\pi\alpha_{em}^2 q_Q^2}{s}\sqrt{1-\frac{4m_Q^2}{s}}\left(1+\frac{2m_Q^2}{s}\right),$$

are shown in Figs.4a, 4b.

We see that the most optimistic expectations for the pair production is given for the cc-diquarks at B-factories, when the high luminosities can yield several thousand pairs of doubly charmed baryons.

The cross section ratios of the exclusive heavy diquark pair production to the respective heavy quark cross section in $q\bar{q}$ annihilation,

$$\sigma(q\bar{q} \to Q\bar{Q}) = \frac{8\pi\alpha_s^2}{27s}\sqrt{1-\frac{4m_Q^2}{s}}\left(1+\frac{2m_Q^2}{s}\right)$$

are shown in Figs.4c, 4d.

Figs. 4a,4c represent the case of diquark composed of different quarks and Figs.4b, 4d give the case of diquark composed of identical quarks. We put

$$\begin{aligned} m_c &= 1.55 \text{ GeV}, \\ m_b &= 4.9 \text{ GeV}, \\ \Lambda_{QCD} &= 0.2 \text{ GeV}. \end{aligned} \tag{43}$$

The values of diquark wave functions at the origin have been taken from [1].

In the quark-antiquark annihilation the pair production of doubly charmed baryons gives about 10^{-5} fraction of charm yield at $\sqrt{s} < 100$ GeV in the hadron collisions, while at higher energies the gluon-gluon fusion dominates. In experiments at a fixed target the threshold behaviour results in an additional suppression.

The inclusive production of doubly heavy baryons was considered in [4], so we see that the pair production results in the suppression factor about 0.1.

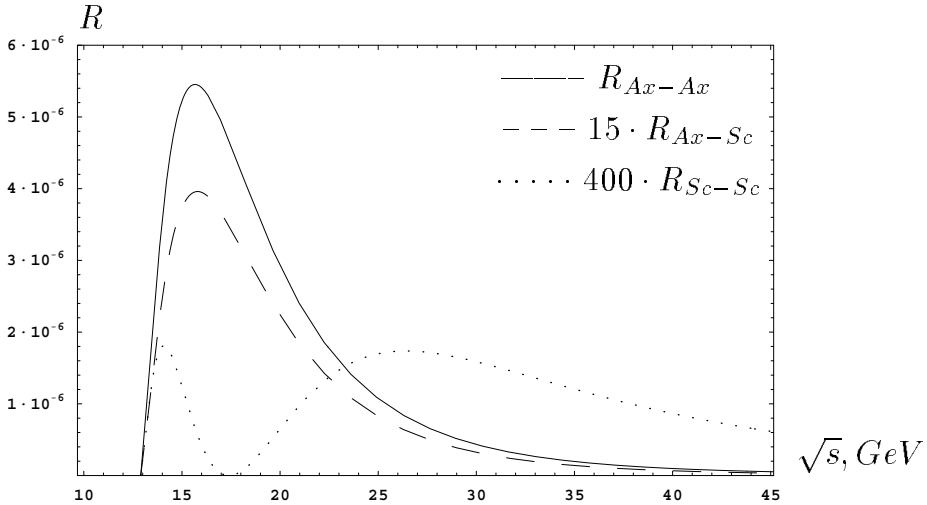

Fig. 4a. The ratios of total cross sections for the exclusive production of bc diquark pair and the $c\bar{c}$ production in e^+e^- annihilation for the axial-axial, axial-scalar and scalar scalar states.

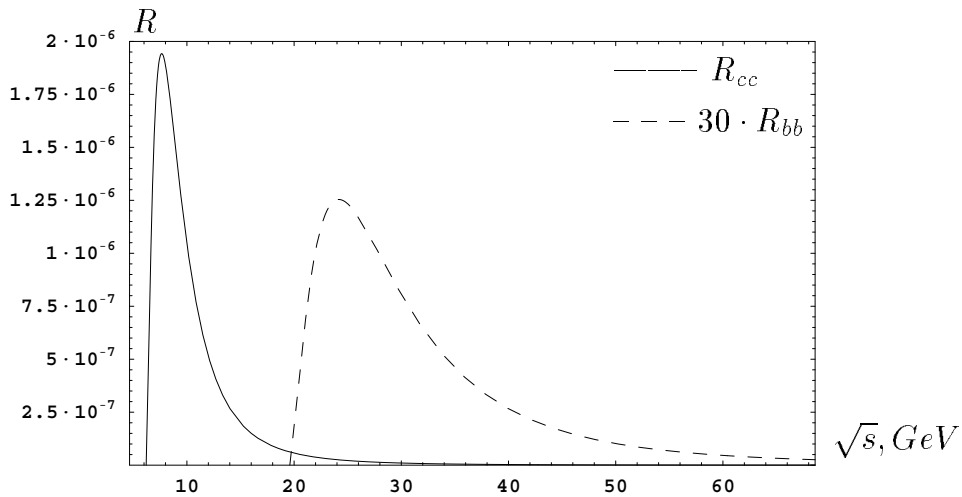

Fig. 4b. The ratios of cross sections for the exclusive production of bb and cc diquark pair and the respective $b\bar{b}$ and $c\bar{c}$ production in e^+e^- annihilation.

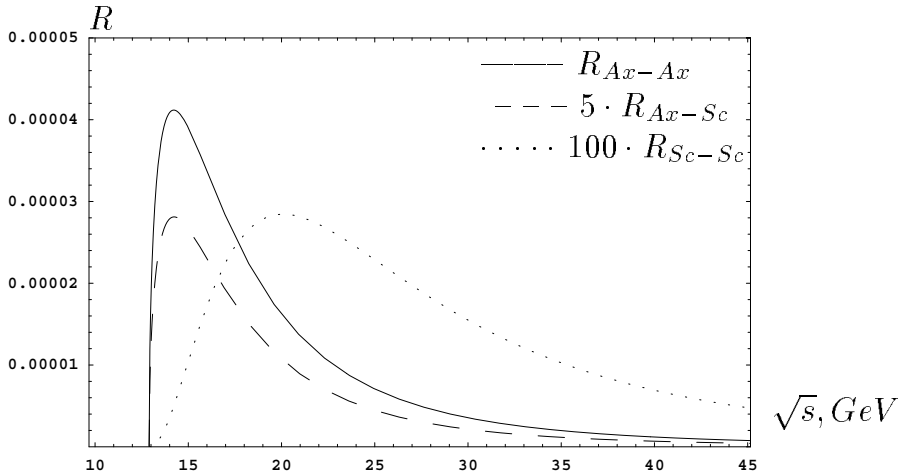

Fig. 4c. The ratios of cross sections for the exclusive production of bc diquark pair and the $c\bar{c}$ production in $q\bar{q}$ annihilation.

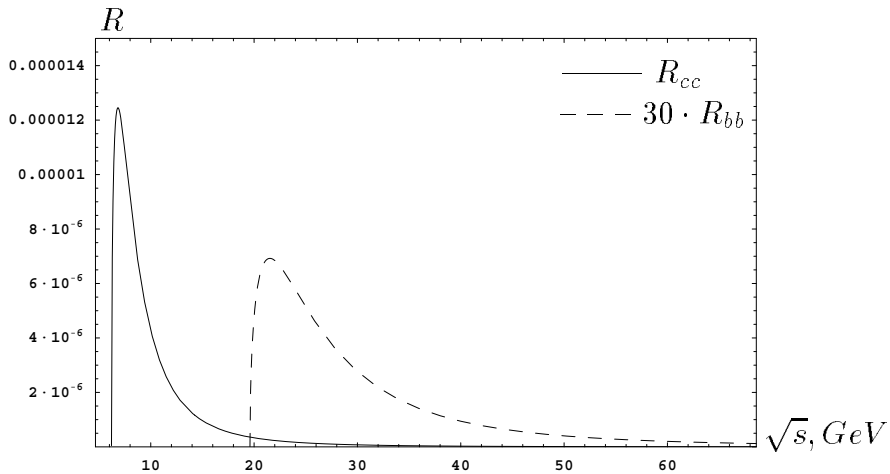

Fig. 4d. The ratios of cross sections for the the exclusive production of bb and cc diquark pair and the respective $b\bar{b}$ and $c\bar{c}$ production in $q\bar{q}$ annihilation.

Conclusion

In this paper we have considered the exclusive production of doubly heavy diquark pairs for the axial-axial, scalar-scalar and axial-scalar states in the framework of the constituent quark model. The matrix elements, differential and total cross sections for the processes of e^+e^- and $q\bar{q}$ annihilation were given. We calculated the pair production of diquarks composed of identical heavy quarks. The obtained formulae can be used in the calculation of cross sections for the diquark fragmentation into the doubly heavy baryons in the processes of e^+e^- annihilation and proton-antiproton inelastic collisions.

We found that the yield of doubly heavy baryon pairs could, in fact, reach $10^3 \div 10^6$ depending on the energies and luminosities of accelerators, e^+e^- colliders and fixed target experiments with hadron beams.

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References

[1] S.S.Gershtein, V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko, preprint IHEP 98-66 (1998) [hep-ph/9811212], Heavy Ion Phys **9**, 133 (1999), Mod. Phys. Lett. **A14**, 135 (1999), Phys. Atom. Nucl. **63** (2000) 274 [Yad. Fiz. **63** (2000) 334];
D.Ebert, R.N.Faustov, V.O.Galkin, A.P.Martynenko, V.A.Saleev, Z. Phys. C76 (1997) 111;
J.G.Körner, M.Krämer, D.Pirjol, Prog. Part. Nucl. Phys. 33 (1994) 787;
R.Roncaglia, D.B.Lichtenberg, E.Predazzi, Phys. Rev. D52 (1995) 1722.

[2] V.V.Kiselev, A.I.Onishchenko, hep-ph/9909337 (1999), to appear in Nucl. Phys. **B**;
V.V.Kiselev, A.E.Kovalsky, hep-ph/0005019 (2000);
E.Bagan, M.Chabab, S.Narison, Phys. Lett. **B306** (1993) 350;
E.Bagan at al., Z. Phys. **C64**, 57 (1994).

[3] V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko, Phys. Rev. **D60**, 014007 (1999), Phys. Atom. Nucl. **62** (1999) 1940 [Yad. Fiz. **62** (1999) 2095];
V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko, preprint DESY 98-212 (1999) [hep-ph/ 9901224], to appear in Eur. Phys. J. **C**;
B.Guberina, B.Melic, H.Stefancic, Eur. Phys. J. **C9**, 213 (1999).

[4] A.V.Berezhnoy, V.V.Kiselev, A.K.Likhoded, A.I.Onishchenko, Phys. Rev. D57 (1997) 4385;
A.V.Berezhnoy, V.V.Kiselev, A.K.Likhoded, Z.Phys. **A356**, 89 (1996), Phys. Atom. Nucl. **59**, 870 (1996) [Yad. Fiz. **59**, 909 (1996)];
S.P.Baranov, Phys. Rev. D 56, 3046 (1997);
V.V.Kiselev, A.K. Likhoded, M.V. Shevlyagin, Phys. Lett. B332, 411 (1994);
A.Falk et al., Phys. Rev. D49, 555 (1994).

[5] V.V.Kiselev, A.E.Kovalsky, hep-ph/9908321 (1999);
V.V.Kiselev, Phys. Atom. Nucl. **62** (1999) 300 [Yad. Fiz. **62** (1999) 335];
V.V.Kiselev, Phys. Rev. **D58** (1998) 054008.

[6] V.V.Kiselev, Int. J. Mod. Phys. **A10** (1995) 465.

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В.В.Брагута, А.Е.Чалов
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