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# PROTON-PROTON TOTAL CROSS SECTIONS
# FROM THE WINDOW OF COSMIC RAY EXPERIMENTS

**Abstract**

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The importance of cosmic-ray experimental measurements of proton-proton total cross sections to understand the underlying fundamental dynamics is discussed. It is shown that early discovered global structure of proton-proton total cross section [5,17] is completely compatible with the values obtained from cosmic-ray experiments.

**Аннотация**

Архипов А.А. Полные протон-протонные сечения и данные экспериментальных наблюдений в космических лучах: Препринт ИФВЭ 2001-23. – Протвино, 2001. – 10 с., 7 рис., библиогр.: 23.

В работе обсуждается важность экспериментальных измерений полных протон-протоных сечений при сверхвысоких энергиях, доступных в экспериментах с космическими лучами, для понимания фундаментальной динамики. Показано, что ранее обнаруженная глобальная структура полных протон-протоных сечений [5,17] превосходно согласуется с имеющимися экспериментальными данными, полученными из наблюдений в космических лучах.

# Introduction

It is a well known fact that at energies above $\sqrt{s} \sim 20\,GeV$ all hadronic total cross sections rise with the growth of energy. In 1970 the experiments at the Serpukhov accelerator revealed that the $K^+ p$ total cross section increased with energy [1]. Increase of the $pp$ total cross section has been discovered at the CERN ISR [2] and then the effect of rising $p\bar{p}$ total cross sections was confirmed at the Fermilab accelerator [3] and CERN $Sp\bar{p}S$ [4].

Although nowadays we have in the framework of local quantum field theory a gauge model of strong interactions formulated in terms of the known QCD Lagrangian its relations to the so called "soft" (interactions at large distances) hadronic physics are far from desired. In spite of 30 years after the formulation of QCD, we cannot still obtain from the QCD Lagrangian the answer to the question why and how all hadronic total cross sections grow with energy. We cannot predict total cross sections in an absolute way starting from the fundamental QCD Lagrangian as well mainly because it is not a perturbative problem.

The behaviour of hadronic total cross sections at high energies is a wide and much discussed topic in high-ehergy physics community; see e.g. the proceedings of famous Blois Workshops. At present time there are a lot of different models which provide different energy dependencies of hadronic total cross sections at high energies.

All different phenomenological models can conditionally be separated into two groups in according to two forms of strong interaction dynamics used: t-channel form and s-channel one [5]. The first group contains the Regge-type models with power-like, $s^{\alpha_P(0)-1}$, behaviour of hadronic total cross sections. Here $\alpha_P(0)$ is an intercept of the supercritical Pomeron trajectory: $\alpha_P(0) - 1 = \Delta << 1$, $\Delta > 0$ is responsible for the growth of hadronic cross sections with energy; see recent paper [6] and references therein. There are a lot of people who works with such a type of Regge-pole models.

However some part of scientific community works in the field related to $s$-channel form of strong interaction dynamics and elaborates the impact picture or geometrical models [7,8,9,10, 11,12,13], which exhibit an $ln^2 s$ high-energy dependence and therefore it asymptotically appears a saturation of the Froissart bound [14] in these models.

In our opinion the second group of the models is more preferable than the first one from many points of view (see discussion in [5]). Moreover, careful analysis of the experimental data on hadronic total cross sections and comparative study of two above mentioned characteristic asymptotic parameterizations have shown that statistically a "Froissart-like" type parameterization for hadronic total cross sections is strongly favoured [15,16].

On the other hand if we suppose that unitarity saturation of fundamental forces takes place at super-high energies then the energy dependence of hadronic total cross sections can be derived and investigated independently of phenomenological models but using only general principles of relativistic quantum theory, such as analyticity and unitarity, together with dynamic apparatus of single-time formalism in QFT [5,17].

The experimental information on the behaviour of hadronic total cross sections at ultrahigh energies can be obtained from cosmic ray experiments. In this respect, analysis of extensive air showers observations provides a unique source of such information. In fact, the ultrahigh energy hadronic interactions occur when a primary cosmic ray proton collides on air nucleus and as a result the extensive air showers are produced by hadronic cascade in the atmosphere. The primary cosmic ray protons with energy of $10^{18}\,eV$ have been observed in the Utah "Fly's Eye" detector. This energy significantly exceeds the energy available at now working accelerators and LHC in the near future as well. That is why, the cosmic-ray data on hadronic total cross sections are most important.

## 1. Cosmic-ray experiments and phenomenology

Recently we have two sets of data on proton-proton total cross sections extracted from cosmic ray air showers observations [18,19], including one point at $\sqrt{s} = 30\,TeV$ from Fly's Eye Collaboration experiment [18] and six points up to $\sqrt{s} = 24\,TeV$ from AGASA Collaboration experiment [19].

It is well known fact that extracting proton-proton total cross sections from cosmic ray extensive air showers observations is not so straightforward. The physical description of extensive air showers created by hadronic cascade in the atmosphere depends significantly on the fundamental dynamics for hadron-nucleus and nucleus-nucleus interactions at ultrahigh energies. Moreover, a procedure of extracting any information about basic hadronic interactions requires, as a rule, some model, which relates, for example, proton-nucleus inelastic (absorption) cross section to the proton-proton total cross section. Having in the hands such reliable relation we could to discriminate which of the different models for the high-energy behaviour of the proton-proton total cross sections are consistent with cosmic ray data at ultrahigh energies and which may be ruled out.

The procedure generally used to relate the proton-air inelastic cross section to the proton-proton total cross section is the Glauber multiple-scattering approach [7]. However, at present time we can find in the literature some debate concerning the procedure of extracting the $pp$ total cross sections from cosmic ray experimental data. We shortly present here the basic conclusions of these polemics.

It was pointed out in paper [20] (hereafter referred to as NNN) that Glauber method establishes the relationship between proton-nucleus absorption cross section and proton-proton inelastic cross section, which we write in the form

$$\sigma_{abs}^{p-air} = G[\sigma_{inel}^{pp}, B_{el}^{pp}, \rho^{nucl}], \tag{1}$$

where $\rho^{nucl}$ is nuclear matter density, $B_{el}^{pp}$ is the slope of $pp$ differential elastic scattering cross section

$$B_{el}^{pp} = [\frac{d}{dt}(\ln \frac{d\sigma_{el}^{pp}}{dt})]_{t=0}, \tag{2}$$

$\sigma_{inel}^{pp}$ is $pp$ inelastic cross section

$$\sigma_{inel}^{pp} = \sigma_{tot}^{pp} - \sigma_{el}^{pp}, \tag{3}$$

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$\sigma_{abs}^{p-air}$ is $p-air$ absorption cross section

$$\sigma_{abs}^{p-air} = \sigma_{tot}^{p-air} - \sigma_{el}^{p-air} - \sigma_{q-el}^{p-air}, \qquad (4)$$

$\sigma_{q-el}^{p-air}$ is quasi-elastic $p-air$ cross section corresponding to the intermediate excited states of air nucleus, $G$ is some known functional of the quantities $\sigma_{inel}^{pp}$, $B_{el}^{pp}$ and $\rho^{nucl}$.

The same Glauber transformation between inelastic proton-nucleus cross section and proton-proton total cross section is true as well [20]

$$\sigma_{inel}^{p-air} = G[\sigma_{tot}^{pp}, B_{el}^{pp}, \rho^{nucl}], \qquad (5)$$

where $G$ is the same functional as in Eq. 1, and

$$\sigma_{inel}^{p-air} = \sigma_{tot}^{p-air} - \sigma_{el}^{p-air}. \qquad (6)$$

If we additionally suppose the geometric scaling in the form $\sigma_{inel}^{pp} \sim B_{el}^{pp}$, then Glauber formula (1) can be used for extraction $\sigma_{inel}^{pp}$ from measured values of $\sigma_{abs}^{p-air}$. In that case the total cross section $\sigma_{tot}^{pp}$ is obtained from that extracted inelastic $pp$ cross section $\sigma_{inel}^{pp}$ by adding the elastic cross section: $\sigma_{tot}^{pp} = \sigma_{inel}^{pp} + \sigma_{el}^{pp}$. That is why, Nikolaev argued that the underestimated values of $\sigma_{tot}^{pp}$ were inferred in Akeno Collaboration paper [19]. A reanalysis of the Akeno data made in NNN paper gives a proton-proton total cross section about $30\,mb$ larger than found in [19], its the main conclusion presented in NNN paper [20].

A quite opposite conclusion we found in paper [21] (hereafter referred to as BHS). Block et al. faced with the problem to predict proton-air and proton-proton cross sections at energies near $\sqrt{s} = 30\,TeV$ using Glauber approach and their QCD-ispired parameterization of all accelerator data on forward proton-proton and proton-antiproton scattering amplitudes. When BHS confronted their predictions of $p-air$ cross sections $\sigma_{inel}^{p-air}$ as a function of energy with published cross section measurements of the Fly's Eye and AGASA groups, they found that the predictions systematically are about one standard deviation below the published cosmic ray values [18,19]; see Fig. 1.

To overcome these troubles Block et al. emphasized that the measured quantity in cosmic ray experiment is the shower attenuation length or the mean free path for development of air showers $\Lambda$ which is not only sensitive to the interaction length of the protons in the atmosphere (mean free path) $\lambda_{p-air}$ but also depends on the inelasticity parameter $k$, which determines the rate at which the energy of the primary proton is dissipated into electromagnetic shower energy observed in the experiment [21]

$$\Lambda = k\lambda_{p-air} = k\frac{14.5m_p}{\sigma_{inel}^{p-air}}. \qquad (7)$$

The rate of shower development and its fluctuations are the origin of the deviation of $k$ from unity in Eq. (7). The value of $k$ is model dependently estimated through Monte Carlo simulations, its predicted values range from 1.5 for a model where the inclusive cross section exhibits Feynman scaling to 1.1 for the models with large scaling violations. Akeno Collaboration used $k = 1.5$ and this value of $k$ was obtained with the assumption that there is no significant break of Feynman scaling in the fragmentation region ($x \geq 0.05$) and that the multiplicity increases as $ln^2 s$ [19]. If we assume a breakdown of scaling in the fragmentation region, a smaller value of $k$ is expected. Fly's Eye Collaboration [18] used $k = 1.6$ with uncertainty of $10\%$.

The extraction of the $pp$ cross section from the cosmic ray data is a two stage procedure. First, from measured value of $\Lambda$ and fixed value of $k$ one calculates the $p - air$ inelastic cross section inferred in Eq. (7), where [1]

$$\sigma_{inel}^{p-air} = \sigma_{tot}^{p-air} - \sigma_{el}^{p-air} - \sigma_{q-el}^{p-air}.$$

This step neglects the possibility that $k$ may have a weak energy dependence over the range measured.

In the next step the Glauber formula (1) transforms the value of $\sigma_{inel}^{p-air}$ into a proton-proton inelastic cross section $\sigma_{inel}^{pp}$. Here all the necessary steps are calculable in the framework of Glauber theory, but depend sensitively on a knowledge of the slope $B_{el}^{pp}$ as it was mentioned above.

Block et al. decided to let $k$ be a free parameter and to make a global fit to the accelerator and cosmic ray data using the QCD-inspired parameterization of the forward proton-proton and proton-antiproton scattering amplitudes. So, in they global fit, all 4 quantities, $\sigma_{tot}^{pp}$, $B_{el}^{pp}$, $\rho = Re/Im$ and $k$ were simultaneously fitted. The fit also neglected the energy dependence of $k$. It was found that the accelerator and cosmic ray $pp$ cross sections are readily reconcilable using a value of $k = 1.349 \pm 0.045 \pm 0.028$, where the quoted errors are statistical and systematic ones respectively. They concluded that this determination of $k$ severely constrains any model of high energy hadronic interactions.

At the LHC ($\sqrt{s} = 14\,TeV$), they predicted $\sigma_{tot}^{pp} = 107.9 \pm 1.2\,mb$ for the $pp$ total cross section, $B_{el}^{pp} = 19.59 \pm 0.11\,(GeV/c)^{-2}$ for the elastic slope and $\rho = 0.117 \pm 0.001$ for the ratio $Re/Im$, where the quoted errors are due to the statistical errors of the fitting parameters.

## 2. Cosmic-ray experiments and theory

Recently a simple theoretical formula describing the global structure of $pp$ and $p\bar{p}$ total cross sections in the whole range of energies available at now working accelerators has been derived [5,17]. The fit to the accelerators experimental data with the formula has been made and it was shown that there is a very good correspondence of the theoretical formula to the existing experimental data. In Figs. 2,3, we have presented the fit results. It was also demonstrated in papers [5,17] (see Figs. 8,7 there), that experimental point from cosmic ray Fly's Eye Collaboration do not contradict to the theoretical predictions made from the fit to the accelerator data only. Unfortunately, we did not have in the hands at that time the numerical experimental values from cosmic ray experiment of AGASA Collaboration. Now these values are available in the database of Particle Data Group [22] and we can compare our theoretical predictions with all existing cosmic-ray data on proton-proton total cross sections.

The comparison is shown in Figs. 4,5. As is seen from the Figures there is very good correspondence of the theory to all existing cosmic ray experimental data without any reanalysis of the data. What can we learn from this very nice, at least for us, fact and what really could it mean?

To understand it more clearly we plotted in Fig. 6 an error band where upper and lower curves correspond to one deviation in the fitting parameter $a_2$ which controls the high energy asymptotic in the total cross section [17]. As one can see from this Figure the error band is

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[1]It should be born in mind that the different notations for one and the same quantity have been used in NNN and BHS papers: $\sigma_{abs}^{p-air}(NNN) \equiv \sigma_{inel}^{p-air}(BHS)$.

narrow enough so, there is no a large room for the experimental uncertainties. In this respect a more precise total cross section $\sigma_{tot}^{pp}$ measurements at cosmic ray energies are very desirable. Anyway, we would like to emphasize that we faced here a happy case when the predicted values for $\sigma_{tot}^{pp}$ obtained from theoretical description of all existing accelerators data are completely compatible with the values obtained from cosmic ray experiments. Here we confront with the conclusion made in paper [23]. The best fit of accelerators data made in paper [16] and they predictions up to cosmic-ray energies are close to our theory predictions within error band but a little bit lower though. At the LHC we predict

$$\sigma_{tot}^{pp}(\sqrt{s} = 14\,TeV) = 116.53 \pm 3.52\,mb, \tag{8}$$

which is in $3\sigma$ higher than the BHS prediction. Our estimated value $\sigma_{tot}^{pp}(\sqrt{s} = 40\,TeV) = 142.46\,mb$ is significantly lower than the value predicted by NNN; see Fig. 7.

Let's try to explain these discrepancies. In reanalysis of cosmic ray experimental data made by NNN the formula

$$\sigma_{in}^{pp} = [\sigma_{abs}^{p-air}/507\,mb]^{1.89} \cdot 100\,mb \tag{9}$$

has been used. It has been argued by NNN that this formula is valid to a few percent accuracy at $\sigma_{abs}^{p-air} > 300\,mb$ and/or $\sigma_{in}^{pp} > 37\,mb$. Using cosmic ray experimental data identified with $\sigma_{abs}^{p-air}$ Nikolaev obtained $\sigma_{in}^{pp}$ with the help of formula (9) and after that the total cross section $\sigma_{tot}^{pp}$ was obtained by adding the elastic cross section: $\sigma_{tot}^{pp} = \sigma_{in}^{pp} + \sigma_{el}^{pp}$. However, it should be pointed out that Eq. 9 is not a theoretically well grounded formula but a pure phenomenological one. That is why, the further theoretical study of multiparticle dynamics is needed.

Concerning the BHS analysis we might apply the arguments of NNN and say that, in fact, BHS extracted $\sigma_{in}^{pp}$ from cosmic ray experimental measurements of $\sigma_{inel}^{p-air}$ but not $\sigma_{tot}^{pp}$ as it has been argued by BHS. Therefore to obtain $\sigma_{tot}^{pp}$ we have to add $\sigma_{el}^{pp}$ to the values extracted by BHS, and in that case we would come to the agreement with the values published by cosmic-ray experimental groups. It should be noted that BHS did not cite the paper of NNN.

## 3. Conclusion

In conclusion, we would like to hope that in the near future it would be possible to repeat the cosmic ray experiments to measure the proton-proton total cross sections with a higher accuracy.

## Acknowledgments

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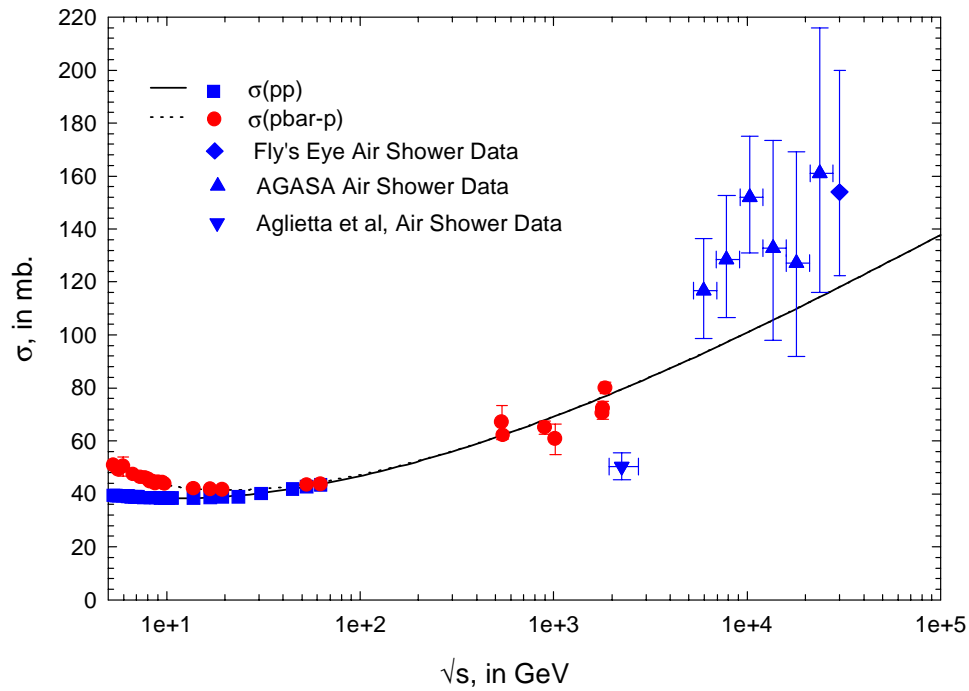
Fig. 1.   A plot of the QCD-inspired fit of the nucleon-nucleon total cross section extracted from paper [21].
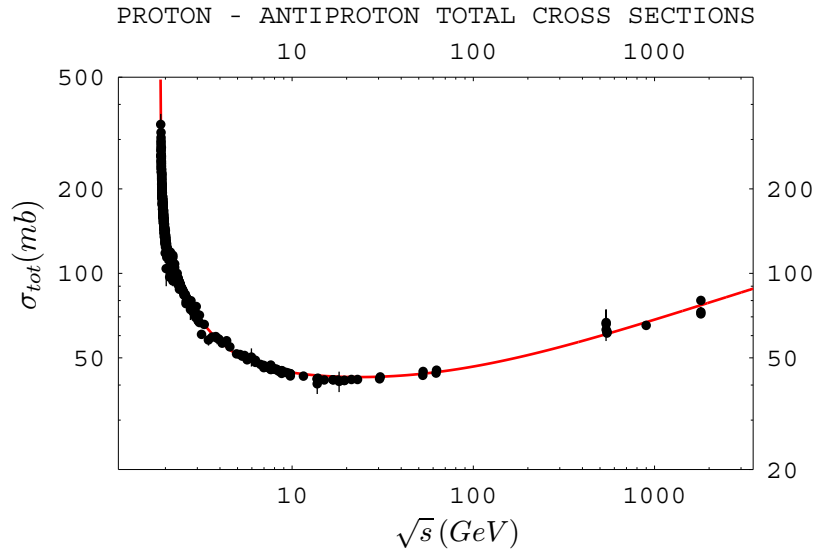
Fig. 2. The proton-antiproton total cross sections versus $\sqrt{s}$ compared with the theory. Solid line represents our fit to the data [5,17]. Statistical and systematic errors added in quadrature.
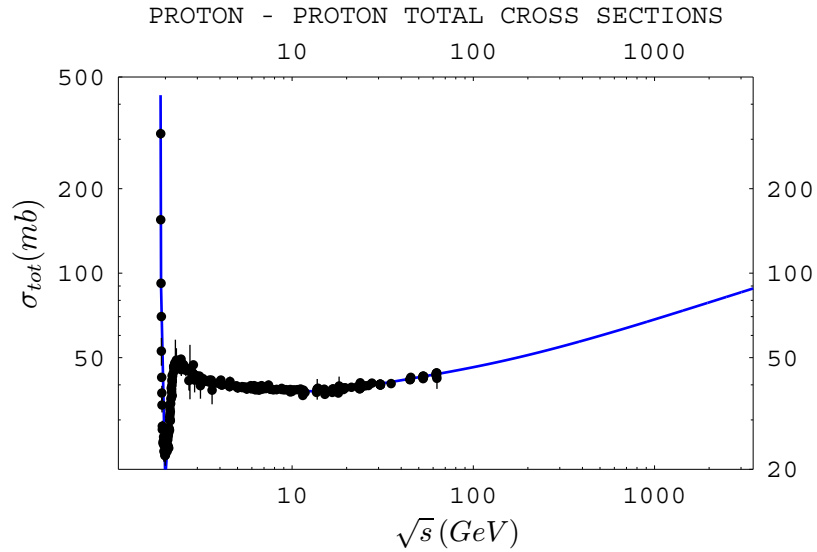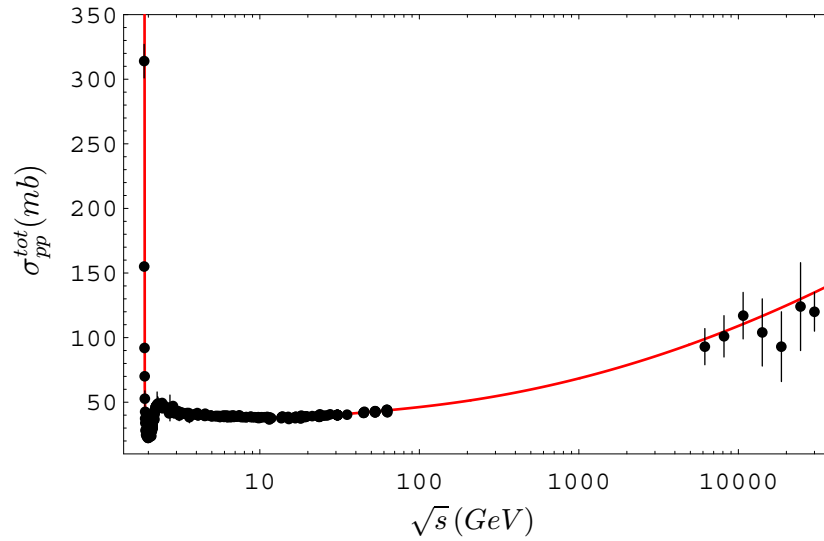


Fig. 3. The proton-proton total cross sections versus $\sqrt{s}$ compared with the theory. Solid line represents our fit to the data [5,17]. Statistical and systematic errors added in quadrature.

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Fig. 4.  The proton-proton total cross-section versus $\sqrt{s}$ with the cosmic-ray data points from Akeno Observatory and Fly's Eye Collaboration. Solid line corresponds to our theory predictions [5,17].
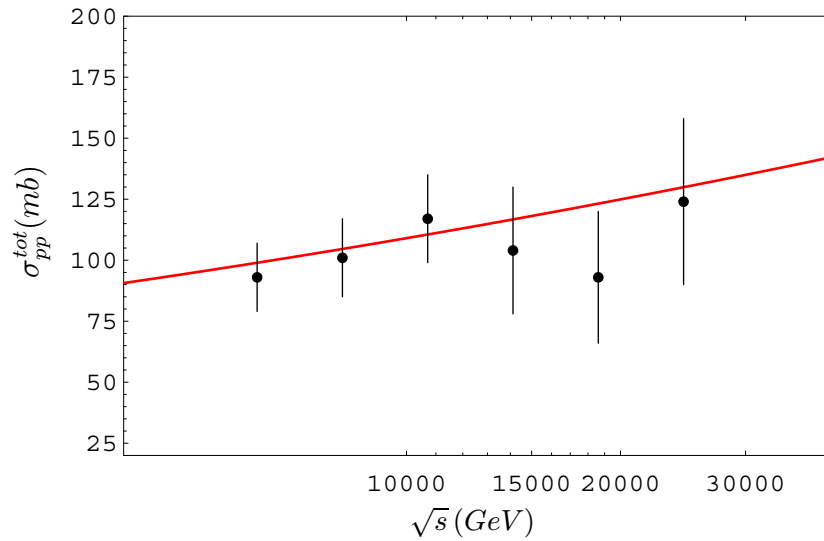


Fig. 5.  The proton-proton total cross-section versus $\sqrt{s}$ with the cosmic-ray data points from Akeno Observatory only. Solid line corresponds to our theory predictions [5,17].
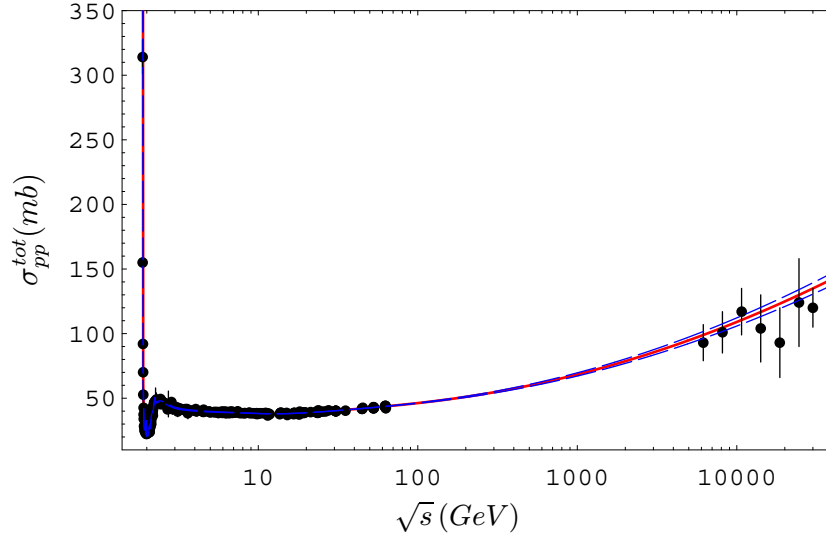
Fig. 6.    The proton-proton total cross-section versus $\sqrt{s}$ with the cosmic-ray data points from Akeno Observatory and Fly's Eye Collaboration.  Solid line corresponds to our theory predictions [5,17]. Upper and lower dashed lines show error band corresponding to one deviation in fitting parameter $a_2$ which controls the high-energy asymptotic in the total cross section.
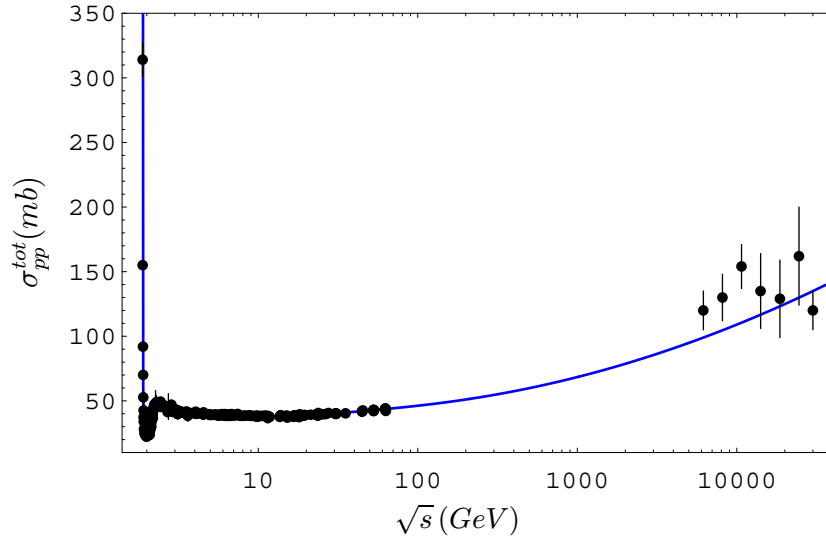


Fig. 7.    The proton-proton total cross-section versus $\sqrt{s}$ with the cosmic-ray data points extracted from NNN paper [20]. Solid line corresponds to our theory predictions [5,17].

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