O.V. Zenin, V.V. Ezhela, S B. Lugovsky<br>COMPAS Group, IHEP, Protvino, Russia<br>M.R. Whalley<br>HEPDATA Group, Durham University, Durham, UK<br>K. Kang ${ }^{\mathbf{a}, \mathbf{b}}$ and S.K. Kang ${ }^{\mathbf{a}}$<br>${ }^{\text {a }}$ Korea Institute for Advanced Study, ${ }^{\mathbf{b}}$ Brown University

# A compilation of total cross section data on $e^{+} e^{-} \rightarrow$ hadrons and pQCD tests 

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#### Abstract

Zenin O.V. et al. A Compilation of Total Cross-Section Data on $\boldsymbol{e}^{+} \boldsymbol{e}^{-} \boldsymbol{\rightarrow}$ Hadrons and pQCD Tests: IHEP Preprint 2001-25. - Protvino, 2001. - p. 24, figs. 9, tables 3, refs.: 115.

All available data on the total cross sections and $\boldsymbol{R}$-ratio of $\boldsymbol{e}^{+} \boldsymbol{e}^{-} \rightarrow$ hadrons are compiled from the PPDS(DataGuide), PPDS(ReacData) and HEPDATA(Reaction) databases and transformed to a compilation of data on the $\boldsymbol{e}^{+} \boldsymbol{e}^{-} \rightarrow$ quark $\overline{\text { quark }} \rightarrow$ hadrons in the continuum region which can be used in tests of the parton model and pQCD calculations. This evaluated data compilation is made available in PPDS system and is accessible through the Web. It is shown that current predictions from the parton model and PQCD are well supported by this world "continuum" data compilation which can then be used in future refinements of the $\boldsymbol{\alpha}_{\boldsymbol{s}}\left(\boldsymbol{Q}^{2}\right)$ as well as $\boldsymbol{\alpha}_{\boldsymbol{Q E D}}\left(\boldsymbol{Q}^{2}\right)$ evolution forms.


## Аннотация

Зенин О.В. и др. Компиляция экспериментальных данных по полному сечению аннигиляции $e^{+} e^{-} \rightarrow$ hadrons, оцененных для проверки партонной модели с КХД поправками: Препринт ИФВЭ 2001-25. - Протвино, 2001. - 24 с., 9 рис., 3 табл., библиогр.: 115.

Сформированы компиляции мировых оцененных экспериментальных данных по полному сечению и $\boldsymbol{R}$-отношению в реакции $\boldsymbol{e}^{+} \boldsymbol{e}^{-} \rightarrow$ hadrons. Данные преобразованы в интегрированные компиляции данных по полному сечению и $\boldsymbol{R}$-отношению в "каскаде" $e^{+} e^{-} \rightarrow$ quark $\overline{q u a r k} \rightarrow$ hadrons, которые предназначены для настройки вариантов партонной модели с КХД-поправками, а также для уточнения эволюции параметров $\alpha_{\boldsymbol{s}}\left(\boldsymbol{Q}^{2}\right)$ и $\boldsymbol{\alpha}_{\boldsymbol{Q} \boldsymbol{E D}}\left(\boldsymbol{Q}^{2}\right)$. Компиляции сформированы на основе баз данных DG и RD системы PPDS с использованием данных из базы Reaction системы HEPDATA для проверки полноты и достоверности данных. Компиляции реализованы в базе данных CS системы PPDS.

## Motivation

Despite the tremendous efforts of experimentalists and phenomenologists devoted to the investigation of hadron production in $\boldsymbol{e}^{+} \boldsymbol{e}^{-}$collisions there is no real integrated view on the experimental situation even for one of the main observables, the total cross section for the reaction $\boldsymbol{e}^{+} \boldsymbol{e}^{-} \rightarrow$ hadrons and the ratio

$$
R=\frac{\sigma\left(e^{+} e^{-} \rightarrow \text { hadrons }\right)}{\sigma\left(e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}\right)_{Q E D}}
$$

Some data are published only as cross sections, $\boldsymbol{\sigma}$, and others only as $\boldsymbol{R}$. The situation is more complicated as QED radiative effects to the data have been corrected for in different ways, and to different degrees, in the various data sets. For example, in constructing $\boldsymbol{R}$ experimentalists have used two forms of theoretical calculations, one with $\boldsymbol{\alpha}_{\boldsymbol{Q E D}}(\mathbf{0})$, or another taking into account the then currently accepted form of evolution of $\boldsymbol{\alpha}_{\boldsymbol{Q E D}}\left(\boldsymbol{Q}^{2}\right)$. Furthermore, some data were never published in numerical form and have been read by compilers from graphs. The current experimental situation (March 2001) is summarized in Figs. 3 and 4.

This aim of this work is firstly to prepare a comprehenesive compilation of $\boldsymbol{\sigma}$ and $\boldsymbol{R}$ transformed, wherever possible, to a unique, and meaningful, style of implementing the pure QED radiative corrections. The second aim is to construct a procedure to extract "continuum data sets" where the direct comparison of the parton model with different variants of the pQCD corrections will be conclusive. In selecting data for the compilation we use expert assessments from recent reviews and works dealing with precision estimation of running $\boldsymbol{\alpha}_{\boldsymbol{Q E D}}\left(\boldsymbol{Q}^{2}\right)$ and $\boldsymbol{\alpha}_{\boldsymbol{S}}\left(\boldsymbol{Q}^{2}\right)$ (see [6]-[14]).

## 1. Data selection and normalizations

### 1.1. General overview

The data, which have been published in [24]-[112], were extracted in numerical form from the PPDS RD [113] database and from the Reaction database of HEPDATA system [114]. The following criteria were then applied to define suitable data sets:

- All preliminary data are excluded;
- Data obtained under incomplete kinematical conditions, and not extrapolated to the complete kinematic region, are excluded;
- Data in the form of dense energy scans are omitted if the authors published in addition the data averaged over wider energy bins.

The data sets remaining after the above exclusion criteria have been applied, are subdivided into the following six cathegories:
$\boldsymbol{\sigma}_{1}^{\boldsymbol{s} \boldsymbol{d}}$.(Refs. $\left.[60]-[67]\right)$ Cross section data corrected by their authors for the contributions of two-photon exchange diagrams, the initial state bremsstrahlung and for the initial state vertex loops, but not corrected for the vacuum polarization contribution to the running $\boldsymbol{\alpha}_{\boldsymbol{Q E D}}$. These "semi-dressed" cross sections correspond to the contribution of the diagram shown on Fig. 1. The full $\gamma / \boldsymbol{Z}$ propagator, taking into account all vacuum polarization effects, is denoted on the figure by the bold waved line.
$\boldsymbol{\sigma}_{2}^{\boldsymbol{s} \boldsymbol{d}}$.(Refs. [68] - [70]) Cross section data radiatively corrected by their authors according to the procedure of Bonneau and Martin [1]. This procedure included radiative corrections for the initial state radiation, electronic vertex correction, and the correction for the electronic loop in the photon propagator. Thus it partially took into account the vacuum polarization effects also. To obtain from these cross sections the "semi-dressed" ones (corresponding to the diagram on Fig. 1) we rescale $\sigma_{2}^{s d}$ by a factor $1 /\left(1-\Delta \alpha_{Q E D}^{e}(s)\right)^{2}$.

We suggest that all the cross sections published before 1978 were radiatively corrected according to this procedure.


Fig. 1

hadrons

Fig. 2
$\boldsymbol{R}_{3}^{\boldsymbol{b a r e}}$. (Ref. [59]). Data on $\boldsymbol{R}$ obtained from measurements of the "semi-dressed" ( $\boldsymbol{\sigma}_{2}^{\boldsymbol{s} \boldsymbol{d}}$ ) cross-section divided, by their authors, by the point-like muonic cross section with fixed $\boldsymbol{\alpha}_{\boldsymbol{Q E D}}=$ $\alpha_{Q E D}(0)$ We rescale these data by the factor $\left(\alpha_{Q E D}(0) / \alpha_{Q E D}(s)\right)^{2} /\left(1-\Delta \alpha_{Q E D}^{e}(s)\right)^{2}$, to obtain the "bare" $\boldsymbol{R}$-parameter described by the diagram shown on the Fig. 2. The tree-level $\gamma / Z$ propagator is denoted here by the waved line.
$\boldsymbol{R}_{4}^{\boldsymbol{b a r e}}$. (Refs. [24]-[58]). Data on $\boldsymbol{R}$ obtained from measurements of the "semi-dressed" $\left(\boldsymbol{\sigma}_{1}^{s d}\right)$ cross section divided, by their authors, by the point-like muonic cross section, but with the running $\boldsymbol{\alpha}_{\boldsymbol{Q E D}}(s)$ thus obtaining the "bare" $\boldsymbol{R}$-ratio, which corresponds to the Fig. 2 diagram again.
$\boldsymbol{\sigma}_{5}^{s \boldsymbol{d}}$. (Refs. [71] - [81]). LEP I cross section data at the $\boldsymbol{Z}$ peak not corrected for initial state radiation and electronic QED vertex loops, and LEP II - III cross section data at $\sqrt{s}>\mathbf{1 3 0} \mathrm{GeV}$ with a cut $s^{\prime} / s=1-(\sqrt{s} / \mathbf{2})(\mathbf{1} \mathbf{- 0 . 7 2 2 5})$. One can interprete $\sqrt{s^{\prime}}$ as an effective mass of
the propagator after the initial state radiation has reduced the $\boldsymbol{e}^{+} \boldsymbol{e}^{-}$pair center-of-mass energy. Indeed, the definition of $s^{\prime}$ depends on the assumptions about the initial and final state radiation interference. These data were rescaled to the "semi-dressed" cross section, $\boldsymbol{\sigma}_{1}^{s d}$, defined by the contribution of the Fig. 1 diagram as follows. First, the theoretical cross sections ${ }^{1} \boldsymbol{\sigma}_{\text {cut }}^{\text {th }}$ and $\boldsymbol{\sigma}_{\text {Born }}^{\text {th }}$ were calculated using the ZFITTER 6.30 package $^{2}$ [23] assuming the recommended by PDG [2] values of the standard model parameters. $\boldsymbol{\sigma}_{\text {cut }}^{t h}$ means here the cross section measured with cuts applied in the particular experiment and calculated by ZFITTER. $\boldsymbol{\sigma}_{\boldsymbol{B o r n}}^{\boldsymbol{t h}}$ denotes theoretical cross section, calculated by ZFITTER in the Improved Born Approximation (IBA), that corresponds to the Fig 1 diagram. Then the experimental cross section was multiplied by the factor $\boldsymbol{\sigma}_{\boldsymbol{B o r n}}^{\boldsymbol{t h}} / \boldsymbol{\sigma}_{\boldsymbol{c u t}}^{\boldsymbol{t} \boldsymbol{h}}$. The results of this rescaling procedure are enlisted in the Tables 3a, 3b.
$\boldsymbol{\sigma}_{\mathbf{6}}^{\boldsymbol{s} \boldsymbol{d}}$. Low energy data $\left(\mathbf{2} \boldsymbol{m}_{\boldsymbol{\pi}}<\boldsymbol{E}_{\boldsymbol{c m}}<\mathbf{2} \mathrm{GeV}\right)$. The treatment of these data is discussed in the following section.

### 1.2. Low energy data treatment ( $2 m_{\pi}<E_{c m}<2 \mathrm{GeV}$ )

Our treatment of the cross section, $\boldsymbol{\sigma}_{6}^{s \boldsymbol{d}}$, data in low energy range $\mathbf{2} \boldsymbol{m}_{\boldsymbol{\pi}}<\sqrt{s}<\mathbf{2} \mathrm{GeV}$ requires special consideration.

All the cross sections $\boldsymbol{\sigma}\left(e^{+} e^{-} \rightarrow \boldsymbol{h a d r o n s}\right)$ in this range are obtained via exclusive channel summation and therefore give only a lower estimate of the total hadronic cross section. Below 1 GeV we summed up the $\mathbf{2 \pi}$ and $\mathbf{3} \boldsymbol{\pi}$ channels from the references [82] - [95] using a linear interpolation of the individual data sets within specific energy regions and combined them to give the total hadron cross sections in these regions. The errors were calculated according to these interpolation and summation procedures.

Such an approach works well if all the data are evenly distributed over $\sqrt{s}$ and have comparable errors. Otherwise if in the given $\sqrt{s}$ interval there are few points of the leading channel with large errors, and this interval is filled more densely by the points of less contributing channels, resonance-like false structures in the total cross section may arise.

The possibility of such false structures is likely for the exclusive channel data in the range $1.4<\sqrt{s}<2 \mathrm{GeV}$, where we summed the contributions from the channels $\boldsymbol{\pi}^{+} \boldsymbol{\pi}^{-}, \boldsymbol{K}^{+} \boldsymbol{K}^{-}$, $\boldsymbol{K}_{\boldsymbol{S}} \boldsymbol{K}_{\boldsymbol{L}}$, and $\geq \mathbf{3}$ hadrons. To avoid these false structures we have determined the sum of exclusive channels not for all points, where at least one channel is measured, but in a fewer number of points, in which the channels yielding the major contributions are measured. In other aspects the summation procedure was the same as the one used for the data below 1 GeV . The exclusive channel data for $\mathbf{1 . 4}<\sqrt{s}<\mathbf{2} \mathrm{GeV}$ are [96] - [112]. The exclusive sum data in the $\mathbf{0 . 8 1}<\sqrt{s}<\mathbf{1 . 4} \mathrm{GeV}$ region are taken from [112]. The data used for the exclusive channel summation around $\phi$ resonance $(0.997<\sqrt{s}<1.028 \mathrm{GeV})$ were corrected by their authors for the initial state radiation, electronic vertex loops and leptonic 1-loop insertions insertions into the $\gamma$ propagator. We have properly rescaled these data points to the Fig 1 diagram. In the remaining 0.81-1.4 GeV data their authors applied QED-corrections using the Bonneau and Martin [1] prescription and all these data points were rescaled as in the $\boldsymbol{\sigma}_{2}^{s d}$-case.

[^0]
## 2. Data compilations on $\sigma^{s d}$ and $R^{b a r e}$

After the data selection and rescaling, described in Section 1, we assemble the complete data set of the total hadronic cross sections $\boldsymbol{\sigma}^{\boldsymbol{s d}}$, normalized to the contribution of the Fig. 1 diagram in accordance with the symbolic relation

$$
\sigma^{s d}=\sigma_{1}^{s d} \cup \sigma_{2}^{s d} \cup \sigma_{5}^{s d} \cup \sigma_{6}^{s d} \cup\left[\left(R_{3}^{\text {bare }} \cup R_{4}^{b a r e}\right) \cdot \sigma_{Q E D \text { pole, running } \alpha_{Q E D}}^{\mu \mu}\right]
$$

and the complete data set for the $\boldsymbol{R}$-ratios

$$
R^{\text {bare }}=\frac{\sigma\left(e^{+} e^{-} \rightarrow \text { hadrons }\right)_{\text {Fig. } 1 \text { diagram }}}{\sigma\left(e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}\right)_{Q E D \text { pole, running } \alpha_{Q E D}}} .
$$

As decribed above, in creating compilations which are uniform in the sense of a standardized implementation of pure QED radiative corrections to the "raw" published experimental data, the values of the running $\alpha_{Q E D}(s)$ and $\Delta \alpha_{Q E D}^{e}(s)$ were used. The procedures for obtaining their numerical values and estimates for their uncertainties are described in the Appendix. Both compilations are stored and maintained in the PPDS CS database. Each record in the compilation corresponds to the record of the original data but contains the rescaled data points. Brief descriptions of the applied conversion procedures are stored in the special comment in each record. The $\boldsymbol{\sigma}$ and $\boldsymbol{R}^{\text {bare }}$ compilations obtained by this procedure are shown in the Figs. 5 and 6 , respectively.

## 3. The continuum regions

To be able to pick out the region where the parton model with QCD corrections can be tested we need to consider the relevant theoretical formulae.

### 3.1. Theoretical relations.

In the parton model, before QCD corrections are applied, the $\boldsymbol{R}$-ratio is given by

$$
\begin{equation*}
R=3 \sum_{q} R_{q}^{0}=3 \sum_{q}\left[\beta_{q}\left(1+\frac{1}{2}\left(1-\beta_{q}^{2}\right)\right) \cdot R_{q}^{V V}+\beta_{q}^{3} R_{q}^{A A}\right] \tag{1}
\end{equation*}
$$

where

$$
\begin{align*}
R_{q}^{V V} & =e_{e}^{2} e_{q}^{2}+2 e_{e} e_{q} \bar{v}_{e} \bar{v}_{q} \operatorname{Re} \chi+\left(\bar{v}_{e}^{2}+\bar{a}_{e}^{2}\right) \bar{v}_{q}^{2}|\chi|^{2} \\
R_{q}^{A A} & =\left(\bar{v}_{e}^{2}+\bar{a}_{e}^{2}\right) \bar{a}_{q}^{2}|\chi|^{2} \tag{2}
\end{align*}
$$

with

$$
\begin{align*}
\chi & =\frac{1}{16 \bar{s}^{2} \bar{c}^{2}} \frac{s}{\left(s-M_{Z}^{2}+i M_{Z} \Gamma_{Z}\right)} \\
\bar{v} & =\sqrt{\rho}\left(T_{3}-2 Q \bar{s}^{2}\right) \\
\bar{a} & =\sqrt{\rho} T_{3} \\
\rho & =1+\frac{3 \sqrt{2} G_{F}}{16 \pi^{2}} m_{t}^{2}+\cdots \tag{3}
\end{align*}
$$

Here $\overline{\boldsymbol{s}}^{2}, \overline{\boldsymbol{c}}^{2}$ are an effective $\sin ^{2} \boldsymbol{\theta}_{\boldsymbol{W}}, \cos ^{2} \boldsymbol{\theta}_{\boldsymbol{W}}$ defined through renormalized couplings at $s=$ $M_{Z}^{2}$ :

$$
\begin{equation*}
M_{Z}^{2}=\frac{\pi \cdot \alpha_{Q E D}\left(M_{Z}^{2}\right)}{\sqrt{2} G_{F} \cdot \rho \cdot \bar{s}^{2} \cdot \bar{c}^{2}} . \tag{4}
\end{equation*}
$$

The dominant correction term in $\boldsymbol{\rho}$ originates from $\boldsymbol{t}$-quark loops in $\boldsymbol{W}$ and $\boldsymbol{Z}$ propagators which result in an $\boldsymbol{S U ( 2 )}$ violation due to the large mass splitting between the $\boldsymbol{b}$ and $\boldsymbol{t}$ quarks [15].

Including QCD loops the expression for $\boldsymbol{R}$ is now (see, e.g. [21], [22]):

$$
\begin{align*}
R & =3 \sum R_{q} \\
& =3 \sum\left[R_{q}^{0}+\left(\frac{\alpha_{s}}{\pi}+\left(\frac{\alpha_{s}}{\pi}\right)^{2}\left(1.9857-0.1153 N_{f}\right)\right.\right. \\
& \left.-\left(\frac{\alpha_{s}}{\pi}\right)^{3}\left(6.6368+1.2001 N_{f}+0.0052 N_{f}^{2}+1.2395\left(\sum Q_{q}\right)^{2}\right)\right) \times  \tag{5}\\
& \left.\times\left(f_{1} R_{q}^{V V}+f_{2} R_{q}^{A A}\right)\right]
\end{align*}
$$

The coefficients $f_{1}$ and $f_{2}$ depend only on the quark velocity $\beta=\left(1-4 M_{q}^{2} / s\right)^{1 / 2}$, where $\boldsymbol{M}_{\boldsymbol{q}}$ is the flavour production threshold mass of the quark $\boldsymbol{q}$. Schwinger [17] calculated $\boldsymbol{f}_{1}$ in the QED context and parameterised it in a form which is accurate enough for our purposes:

$$
\begin{equation*}
f_{1}=\beta_{q}\left(1+\frac{1}{2}\left(1-\beta_{q}^{2}\right)\right) \frac{4 \pi}{3}\left(\frac{\pi}{2 \beta_{q}}-\frac{3+\beta_{q}}{4}\left(\frac{\pi}{2}-\frac{3}{4 \pi}\right)\right) . \tag{6}
\end{equation*}
$$

Here $\boldsymbol{N}_{\boldsymbol{f}}$ is a number of active quark flavours. The coefficient $\boldsymbol{f}_{\mathbf{2}}$ (which have no counterpart in QED) were calculated by Jersak et al [16]. As there is no compact analytical expression for $f_{2}$, we parameterised it as

$$
\begin{equation*}
f_{2}=a_{4} \beta_{q}^{4}+a_{3} \beta_{q}^{3}+\left(1-a_{4}-a_{3}\right) \beta_{q}^{2} \tag{7}
\end{equation*}
$$

with $\boldsymbol{a}_{\mathbf{4}}=-\mathbf{1 6}, \boldsymbol{a}_{\mathbf{3}}=\mathbf{1 7}$. >From equations (1), (2), (5) one can easily estimate that even assuming a $100 \%$ error for $\boldsymbol{f}_{\mathbf{2}}$, the relative error of $\boldsymbol{R}$ is less than $\mathbf{1 0}^{\mathbf{- 7}}$ at $\sqrt{\boldsymbol{s}}=\mathbf{1 0} \mathrm{GeV}$ and less than $0.5 \%$ at the $\boldsymbol{Z}$ pole. As we did not used $\boldsymbol{Z}$ pole data in our fits, it could not be a source of large theoretical errors.

In the massless quark limit, $f_{1}=f_{2}=1$. However, at $\sqrt{s}=\mathbf{3 5} \mathrm{GeV}$ for $b$ quarks one has $\beta=0.963$ whence $f_{1} \simeq \mathbf{1 . 3}$ and $f_{2} \simeq \mathbf{1 . 7}$. Indeed, such a parameterisation of mass effects is valid in QCD only for $\mathcal{O}\left(\boldsymbol{\alpha}_{\boldsymbol{s}}\right)$ order. Now the correct parameterisation is known upto $\mathcal{O}\left(\boldsymbol{\alpha}_{\boldsymbol{s}}^{3}\right)$ order $([21],[22])$, but it was not implemented yet into our programme. It is likely that it results in large enough discrepancies just in the region where $\boldsymbol{\alpha}_{\boldsymbol{s}} / \boldsymbol{\pi}$ becomes large.

The following three-loop parameterisation of $\boldsymbol{\alpha}_{\boldsymbol{s}}$ was chosen here (see, e.g., [18]):

$$
\begin{align*}
\ln \left(Q^{2} / \Lambda \frac{2}{M S}\right)=\frac{4 \pi}{\beta_{0} \alpha_{s}} & -\frac{1}{2} \frac{\beta_{1}}{\beta_{0}^{2}} \ln \left[\left(\frac{4 \pi}{\beta_{0} \alpha_{s}}\right)^{2}+\frac{\beta_{1}}{\beta_{0}^{2}}\left(\frac{4 \pi}{\beta_{0} \alpha_{s}}\right)+\frac{\beta_{2}}{\beta_{0}^{3}}\right] \\
& -\frac{1}{\Delta}\left(\frac{\beta_{1}}{\beta_{0}^{2}}\right) \tan ^{-1}\left[\frac{1}{\Delta}\left(\frac{\beta_{1}}{\beta_{0}^{2}}+\frac{2 \beta_{2}}{\beta_{0}^{3}}\left(\frac{\beta_{0} \alpha_{s}}{4 \pi}\right)\right)\right] \tag{8}
\end{align*}
$$

with

$$
\begin{align*}
\beta_{0} & =11-2 N_{f} / 3 \\
\beta_{1} & =2\left(51-19 N_{f} / 3\right) \\
\beta_{2} & =\left(2857-5033 N_{f} / 9+325 N_{f}^{2} / 27\right) / 2 \\
\Delta & =\sqrt{4 \beta_{2} / \beta_{0}^{3}-\beta_{1}^{2} / \beta_{0}^{4}} \tag{9}
\end{align*}
$$

Here $\boldsymbol{N}_{\boldsymbol{f}}$ is a number of quark flavours, contributing to the $\boldsymbol{\alpha}_{\boldsymbol{s}}$ evolution. The onset of a new flavour $\boldsymbol{q}$ takes place at $\boldsymbol{Q}^{2}=\boldsymbol{\mu}_{\boldsymbol{q}}^{\mathbf{2}}=\mathbf{4} \boldsymbol{m}_{\boldsymbol{q}}^{2}$, where, in general, $\boldsymbol{m}_{\boldsymbol{q}} \neq \boldsymbol{M}_{\boldsymbol{q}}$. Here are five different $\boldsymbol{\Lambda}_{\overline{M \boldsymbol{S}}}\left(\boldsymbol{N}_{f}\right)$ corresponding to $\boldsymbol{N}_{f}=\mathbf{2}, \mathbf{3}, \mathbf{4}, \mathbf{5}, \mathbf{6}$ above the appropriate quark thresholds. To sew up $\boldsymbol{\alpha}_{\boldsymbol{s}}$ at the thresholds an apparent matching condition for $\boldsymbol{\Lambda}$ 's was used. Thus at any $\boldsymbol{Q}^{\mathbf{2}}$ $\boldsymbol{\alpha}_{\boldsymbol{s}}\left(Q^{2}\right)$ is determined by, say, $\boldsymbol{\alpha}_{\boldsymbol{s}}\left(M_{Z}^{2}\right)$, which has been used as a parameter for fits.

We solved this equation at given $Q^{2}$ numerically by rewriting it in the form $\boldsymbol{x}=\boldsymbol{f}(\boldsymbol{x})$, where $\boldsymbol{x}=4 \boldsymbol{\pi} /\left(\boldsymbol{\beta}_{0} \alpha_{\boldsymbol{s}}\right)$. It enabled us to obtain correct $\alpha_{\boldsymbol{s}}$ even at $Q^{2} / \boldsymbol{\Lambda}^{\mathbf{2}} \sim \mathbf{1}$, where the expansion by powers of leading logarithms (see, e.g.,[13]) is no more valid. At $\boldsymbol{Q}^{2} / \boldsymbol{\Lambda}^{2} \gg 1$ both methods yield the same result. (See Fig. 8).

### 3.2. Preliminary fit results

In order to check the consistency of the description of the data on

$$
\boldsymbol{R}=\sigma\left(e^{+} e^{-} \rightarrow \text { hadrons }\right) / \sigma\left(e^{+} e^{-} \rightarrow \mu^{+} \mu^{-}\right)
$$

with the theoretical $\boldsymbol{R}$-ratio expression we have performed several fits using a sub-set of the total compiled data set of $\boldsymbol{R}$ which is applicable to the perturbative domain. This sub-set of the data is defined by the the following steps:

1. The low energy region $2 m_{\boldsymbol{\pi}}<\sqrt{s}<\mathbf{2} \mathrm{GeV}$ is completely excluded.
2. All the data above 70 GeV are excluded, as we did not attempt to perform any fits of $\boldsymbol{Z}$ pole parameters $\boldsymbol{M}_{\boldsymbol{Z}}, \boldsymbol{\Gamma}_{\boldsymbol{Z}}, \bar{s}_{\boldsymbol{W}}^{\boldsymbol{W}}$ and $\boldsymbol{M}_{\boldsymbol{t o p}}$. This exclusion justified because these data slightly depend on $\boldsymbol{\alpha}_{\boldsymbol{s}}\left(\boldsymbol{M}_{\boldsymbol{Z}}\right)$ and quark masses and thus their influence to the fits of these parameters would be rather negligable.
3. Data located closer than 20 Breit-Wigner widths to the narrow hadronic $\mathbf{1}^{--}$resonances $J / \psi(1 S), \psi(2 S), \psi(3770), \Upsilon(1 S), \Upsilon(2 S), \Upsilon(3 S), \Upsilon(4 S), \Upsilon(10860)$ and $\Upsilon(11020)$ are completely excluded.
4. We also completely exclude SLAC-SPEAR-MARK-1 data [41] in the $\mathbf{2 . 6} \mathrm{GeV}<\sqrt{s}<\mathbf{7 . 8}$ GeV range. These data systematically lie $\simeq 15 \%$ above other experiments in the same interval.

Some remarks on our treatment of the quark masses should be made here. We distinguished the $\boldsymbol{\gamma}^{*} \rightarrow \boldsymbol{q} \overline{\boldsymbol{q}}$ production threshold masses $\boldsymbol{M}_{\boldsymbol{q}}$ and the QCD quark masses $\boldsymbol{m}_{\boldsymbol{q}}$. The $\boldsymbol{M}_{\boldsymbol{q}}$ 's should be set actually equal to the masses of the lightest mesons, above which pair production threshold the onset of the new flavour $\boldsymbol{q}$ gives rise to the continuum cross section $\sigma\left(e^{+} e^{-} \rightarrow\right.$ hadrons $)$. The latters are just $\boldsymbol{m}_{\boldsymbol{q}}=\boldsymbol{\mu}_{\boldsymbol{q}} / \mathbf{2}$, where $\boldsymbol{\mu}_{\boldsymbol{q}}$ is the energy scale at which the onset of the new flavour $\boldsymbol{q}$ in the evolution of $\boldsymbol{\alpha}_{\boldsymbol{s}}$ takes place. We fitted only $\boldsymbol{M}_{\boldsymbol{c}}$ and $\boldsymbol{M}_{\boldsymbol{b}}$, the other $\boldsymbol{M}_{\boldsymbol{q}}$ and $\boldsymbol{m}_{\boldsymbol{q}}$ were fixed at their central PDG values.

The standard $\chi^{2}$ of the least squares method with weights as inverse squared total experimental errors (neglecting correlations in data) is used with the standard MINUIT[115] package.

After performing the exclusions 1) - 4) we fixed $\boldsymbol{\alpha}_{\boldsymbol{s}}\left(\boldsymbol{M}_{\boldsymbol{Z}}\right), \boldsymbol{M}_{\boldsymbol{Z}}, \boldsymbol{\Gamma}_{\boldsymbol{Z}}, \overline{\boldsymbol{s}}_{\boldsymbol{W}}^{2}, \boldsymbol{M}_{\boldsymbol{q}}(\boldsymbol{q}=\boldsymbol{u}, \boldsymbol{d}, \boldsymbol{s}, \boldsymbol{t})$, $\boldsymbol{m}_{\boldsymbol{q}}(\boldsymbol{q}=\boldsymbol{u}, \boldsymbol{d}, \boldsymbol{s}, \boldsymbol{c}, \boldsymbol{b}, \boldsymbol{t})$ at the values shown in the Table 1, leaving as free parameters just $\boldsymbol{M}_{\boldsymbol{c}}$ and $\boldsymbol{M}_{\boldsymbol{b}}$. Another essential feature of our fit (I) was that we retained as a free parameters left and right boundaries of the interval $\left[\sqrt{s_{1}}, \sqrt{s_{2}}\right], \mathbf{3 . 0} \leq \sqrt{s_{1}} \leq \mathbf{3 . 6 7 0} \mathrm{GeV}, \mathbf{3 . 8 7 0} \leq \sqrt{s_{2}} \leq \mathbf{5 . 0}$ GeV , which was to be excluded from the data set in the process of $\chi^{2} /$ dof minimization itself. ${ }^{3}$ Thus, the number of degrees of freedom was also variable during this fit. Such a $\chi^{2} / \boldsymbol{d o f}$ minimization procedure cancelled possible arbitrarities in the exclusions of broad resonances in the region above $\boldsymbol{c} \overline{\boldsymbol{c}}$ threshold.

In the fit (II) we released $\boldsymbol{\alpha}_{\boldsymbol{s}}\left(\boldsymbol{M}_{\boldsymbol{Z}}\right)$, fixing at the same time the excluded region $\sqrt{\boldsymbol{s}}=$ $\mathbf{3 . 0 9} \div 4.44 \mathrm{GeV} . \boldsymbol{M}_{\boldsymbol{c}}, \boldsymbol{M}_{\boldsymbol{b}}$ were fixed at their fitted values obtained in the fit (I). Fit (II) results in too high value $\boldsymbol{\alpha}_{\boldsymbol{s}}\left(\boldsymbol{M}_{Z}\right)=\mathbf{0 . 1 2 8} \pm \mathbf{0 . 0 3 2}$.

Preliminary fit parameter settings and results are enlisted in the Table 1. The reduced data set is shown on the Fig. 7. The references used for our fit and the list of excluded $\sqrt{s}$ regions containing hadronic resonances are quoted in the Table 2.

## 4. Summary. Assessed Data Compilations

In summary:

- We have created two complementary computerized "raw" data compilations on $\boldsymbol{\sigma}$ and $\boldsymbol{R}$ with data presented as in the original publications in all cases except the low energy subsample. In the low energy region, where there are no direct measurements of the total cross section, we obtain estimates of the total cross section either as the sum of exclusive channels or as the sum of the two-body exclusive channels with the data on $e^{+} e^{-} \rightarrow \geq 3$ hadrons.
- On the basis of above two data sets we have created compilations of data on $\boldsymbol{\sigma}^{\text {sd }}$ (corrected to the level of Fig 1) and $\boldsymbol{R}^{\text {bare }}$ (corrected to the level of Fig. 2) with one-to-one correspondence between the data points. Where necessary the data have been rescaled to the standard style of implementing the pure QED radiative corrections to the initial state and to photonic propagator to produce a data set which is suitable for tests of the parton model with pQCD corrections for $\boldsymbol{R}^{\text {bare }}$, and to be able to obtain more reliable estimates for $\Delta \alpha_{Q E D}^{h a d}\left(Q^{2}\right)$.
- From the total data compilation on $\boldsymbol{R}^{\text {bare }}$ we have defined a "continuum" data compilation sub-set which can be used in conjunction with other data in the refinements of the $\boldsymbol{\alpha}_{\boldsymbol{s}}\left(\boldsymbol{Q}^{2}\right)$ evolution and possibly in the global fits of the Standard Model.
All data files are accessible by:

> http://wwwppds.ihep.su:8001/comphp.html
and will be accessible from the PDG site and its mirrors soon.

[^1]
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## Appendix. Running $\alpha_{Q E D}$ and $\Delta \alpha_{Q E D}^{h a d}$

As mentioned above, the authors of the original papers have published data in different forms which required the calculation of several types of factors, containing $\boldsymbol{\alpha}_{\boldsymbol{Q E D}}(s)$, to rescale the data to, say, "bare" $\boldsymbol{R}$-ratios. $\boldsymbol{\alpha}_{\boldsymbol{Q E D}}(s)$ can be expressed in the form (see: [3], [4], [5], [7])

$$
\begin{align*}
\alpha_{Q E D}(s) & =\frac{\alpha(0)}{1-\Delta \alpha(s)} \\
\Delta \alpha(s) & =\Delta \alpha^{\text {had }}(s)+\Delta \alpha^{l e p}(s) \tag{10}
\end{align*}
$$

where $\boldsymbol{\Delta} \boldsymbol{\alpha}^{\boldsymbol{h a d}}(s), \boldsymbol{\Delta} \boldsymbol{\alpha}^{\boldsymbol{l e p}}(s)$ are hadronic and leptonic contributions to the QED vacuum polarization, respectively.
$\boldsymbol{\Delta} \boldsymbol{\alpha}^{\boldsymbol{l e p}}(s)$ is well determined in perturbative QED as a sum of leptonic loop contributions,

$$
\begin{equation*}
\Delta \alpha^{l e p}(s)=\sum_{l=e, \mu, \tau} \frac{\alpha(0)}{3 \pi}\left[\ln \frac{s}{m_{l}^{2}}-\frac{5}{3}+\mathcal{O}\left(\frac{m_{l}^{2}}{s}\right)\right] \tag{11}
\end{equation*}
$$

The situation with $\boldsymbol{\Delta} \boldsymbol{\alpha}^{\boldsymbol{h a d}}(s)$ is more complicated due to an essentially non-perturbative character of the strong interaction at low energy scales. Using the unitarity condition and the analyticity of the scattering amplitudes, one can express $\boldsymbol{\Delta} \boldsymbol{\alpha}^{\boldsymbol{h a d}}(s)$ in the form of a subtracted dispersion relation (see, e.g., [5], [7])

$$
\begin{equation*}
\Delta \alpha^{h a d}(s)=-\frac{\alpha(0) s}{3 \pi} \int_{4 m_{\pi}^{2}}^{\infty} \frac{R\left(s^{\prime}\right) d s^{\prime}}{s^{\prime}\left(s^{\prime}-s-i 0\right)} \tag{12}
\end{equation*}
$$

where $\boldsymbol{R}(\boldsymbol{s})$ is the "bare" hadronic $\boldsymbol{R}$-ratio.
This relation allows to effectively combine for its evaluation the $\boldsymbol{p} \boldsymbol{C C D} \boldsymbol{R}$-ratios in the continuum $\sqrt{s}$ intervals and experimental $\boldsymbol{R}$ data in the non-perturbative ones.

We have evaluated the dispersion integral as

$$
\begin{equation*}
\Delta \alpha^{h a d}(s)=-\frac{\alpha(0) s}{3 \pi}\left[\int_{4 m_{\pi}^{2}}^{\sqrt{s_{c u t}}} \frac{R_{b a r e}^{\text {data }}\left(s^{\prime}\right) d s^{\prime}}{s^{\prime}\left(s^{\prime}-s-i 0\right)}+\int_{\sqrt{s_{c u t}}}^{\infty} \frac{3 \sum_{q} Q_{q}^{2} d s^{\prime}}{s^{\prime}\left(s^{\prime}-s-i 0\right)}\right] \tag{13}
\end{equation*}
$$

where the first integral was calculated numerically as a trapezoidal sum over the weighted average of experimental $R$ points in the range $2 m_{\pi}<\sqrt{s^{\prime}}<\sqrt{s_{\text {cut }}}=19.5 \mathrm{GeV}^{4}$

[^2]Our numerical evaluation procedure is in general similar to the one applied in Ref. [7], except that we performed the trapezoidal integration over all resonances, except $\boldsymbol{\phi}(\mathbf{1 0 2 0}){ }^{5}$

We obtain here $\Delta \alpha^{\text {had }}\left(M_{Z}^{2}\right)=0.0271 \pm 0.0004($ exp. $)$, being consistent with the latest known to us results $\mathbf{0 . 0 2 7 3 8 2} \pm \mathbf{0 . 0 0 0 1 9 7}$ and $\mathbf{0 . 0 2 7 6 1 2} \pm \mathbf{0 . 0 0 0 2 2 0}$, obtained with two methodically different low energy data sets in Ref. [20].

The behaviour of $\boldsymbol{\Delta} \boldsymbol{\alpha}^{\boldsymbol{l e p}}, \boldsymbol{\Delta} \boldsymbol{\alpha}_{(1 \text {-loop }}^{\boldsymbol{h a d}} \boldsymbol{q E D}_{)}$and $\boldsymbol{\Delta} \boldsymbol{\alpha}_{(\text {dispersion })}^{\text {had }}$ is depicted in Fig. 9.
Table 1. Fit results in the defined "continuum" regions. Adjustable parameters are shown in bold.

| Parameter | I | II | III |
| :---: | :---: | :---: | :---: |
| $\alpha_{s}\left(M_{Z}^{2}\right)$ | 0.1181 | 0.128(32) | 0.126(37) |
| $M_{Z}$ | 91.187 | 91.187 | 91.187 |
| $\Gamma_{Z}$ | 2.4944 | 2.4944 | 2.4944 |
| $\bar{s}^{2}$ | 0.23117 | 0.23117 | 0.23117 |
| $M_{t}$ | 174.3 | 174.3 | 174.3 |
| $M_{u}$ | 0.140 | 0.140 | 0.140 |
| $M_{\text {d }}$ | 0.140 | 0.140 | 0.140 |
| $M_{s}$ | 0.492 | 0.492 | 0.492 |
| $M_{c}$ | 1.500(18) | 1.5 | 1.9710(1) |
| $M_{b}$ | $5.23 \pm 1.16$ | 5.23 | $6.0 \pm 1.4$ |
| $m_{u}$ | 0.003 | 0.003 | 0.003 |
| $m_{d}$ | 0.006 | 0.006 | 0.006 |
| $m_{s}$ | 0.120 | 0.120 | 0.120 |
| $m_{c}$ | 1.2 | 1.2 | 1.2 |
| $m_{b}$ | 4.2 | 4.2 | 4.2 |
| $m_{t}$ | 174.3 | 174.3 | 174.3 |
| Excluded $\sqrt{S}$ <br> intervals | $\begin{gathered} \hline \hline 2 \boldsymbol{m}_{\pi} \div 2.0 \\ 3.093 \div 3.113 \end{gathered}$ | $2 \boldsymbol{m}_{\boldsymbol{\pi}} \div 2.0$ | $\begin{gathered} 2 \boldsymbol{m}_{\pi} \div 2.0 \\ 3.093 \div 3.113 \\ 3.684 \div 3.688 \\ 3.670 \div 3.870 \end{gathered}$ |
|  | $\begin{aligned} & 3.175(247) \div \\ & 4.319(\mathbf{1 0 5}) \end{aligned}$ | $3.09 \div 4.44$ | $4.000 \div 4.400$ |
|  | 9.450 $\div 9.470$ | $9.450 \div 9.470$ | $9.450 \div 9.470$ |
|  | $10.000 \div 10.025$ | $10.000 \div 10.025$ | $10.000 \div 10.025$ |
|  | $10.34 \div 10.37$ | $10.34 \div 10.37$ | $10.34 \div 10.37$ |
|  | $10.52 \div 10.64$ | $10.52 \div 10.64$ | $10.52 \div 10.64$ |
|  | $10.75 \div 10.97$ | $10.75 \div 10.97$ | $10.75 \div 10.97$ |
|  | $11.00 \div 11.20$ | $11.00 \div 11.20$ | $11.00 \div 11.20$ |
|  | $70 \div 188.7$ | $70 \div 188.7$ | $70 \div 188.7$ |
| $\chi^{2} /$ dof | 0.690 | 0.665 | 0.822 |

[^3]Table 2. References of preliminary candidate data to form the continuum domain.

| Ref. No. | Data type | $\boldsymbol{N}_{\boldsymbol{p o i n t s}}$ | $\boldsymbol{E}_{\boldsymbol{m i n}}$ | $\boldsymbol{E}_{\boldsymbol{m a x}}$ |
| :---: | :---: | :--- | :--- | :--- |
| $[58]$ | R | 85 | 2.00 | 4.80 |
| $[69]$ | $\boldsymbol{\sigma}$ | 1 | 2.23 | 2.23 |
| $[70]$ | $\boldsymbol{\sigma}$ | 11 | 2.40 | 5.00 |
| $[57]$ | R | 6 | 2.60 | 5.00 |
| $[33]$ | R | 2 | 3.598 | 3.886 |
| $[59]$ | R | 33 | 3.6025 | 5.1950 |
| $[27]$ | R | 27 | 3.878 | 4.496 |
| $[30]$ | R | 15 | 5.00 | 7.40 |
| $[49]$ | R | 31 | 7.30 | 10.29 |
| $[39]$ | R | 3 | 7.440 | 9.415 |
| $[42]$ | $\boldsymbol{\sigma}$ | 13 | 9.30 | 9.48 |
| $[48]$ | R | 1 | 9.36 | 9.36 |
| $[46]$ | R | 1 | 9.39 | 9.39 |
| $[56]$ | R | 1 | 9.51 | 9.51 |
| $[37]$ | R | 1 | 10.04 | 10.04 |
| $[38]$ | R | 1 | 10.43 | 10.43 |
| $[44]$ | R | 1 | 10.49 | 10.49 |
| $[50]$ | R | 12 | 12.00 | 41.40 |
| $[36]$ | R | 7 | 12.00 | 31.25 |
| $[40]$ | R | 14 | 12.00 | 36.00 |
| $[35]$ | R | 12 | 12.00 | 35.80 |
| $[43]$ | R | 20 | 12.00 | 46.47 |
| $[52]$ | R | 18 | 12.00 | 46.47 |
| $[24]$ | R | 9 | 14.00 | 46.60 |
| $[31]$ | R | 4 | 14.03 | 43.70 |
| $[47]$ | R | 1 | 29.00 | 29.00 |
| $[26]$ | R | 1 | 29.00 | 29.00 |
| $[34]$ | R | 1 | 31.57 | 31.57 |
| $[55]$ | R | 1 | 34.85 | 34.85 |
| $[32]$ | R | 2 | 34.86 | 42.72 |
| $[51]$ | R | 2 | 41.45 | 44.20 |
| $[28]$ | R | 12 | 50.00 | 61.40 |
| $[25]$ | R | 2 | 50.00 | 52.00 |
| $[29]$ | R | 13 | 50.00 | 61.40 |
| $[66]$ | $\boldsymbol{\sigma}$ | 9 | 57.37 | 59.84 |
| $[67]$ | $\boldsymbol{\sigma}$ | 1 | 57.77 | 57.77 |
| $[45]$ | R | 2 | 63.60 | 64.00 |
|  |  |  |  |  |

Table 3a. ZFITTER rescaling results for the $\boldsymbol{Z}$ pole.

| $\sqrt{s}, \mathrm{GeV}$ | $\sigma_{\text {cut }}^{\text {th }}$ [nb] | $\sigma_{\text {born }}^{t h}[n b]$ | $\sigma^{\boldsymbol{e x p}}$ [mb] | $\sigma^{\boldsymbol{e x p}} \cdot\left(\sigma_{\text {born }}^{\text {th }} / \sigma_{\text {cut }}^{\text {th }}\right)[\mathrm{mb}]$ |
| :---: | :---: | :---: | :---: | :---: |
| 88.22300 | 0.44967E+01 | $0.60846 \mathrm{E}+01$ | $0.44800 \mathrm{E}-05$ | $0.60619 \mathrm{E}-05$ |
| 88.22300 | $0.44967 \mathrm{E}+01$ | $0.60846 \mathrm{E}+01$ | $0.46100 \mathrm{E}-05$ | $0.62378 \mathrm{E}-05$ |
| 88.22400 | $0.44992 \mathrm{E}+01$ | 0.60881E+01 | $0.46300 \mathrm{E}-05$ | $0.62651 \mathrm{E}-05$ |
| 88.23101 | $0.45168 \mathrm{E}+01$ | $0.61131 \mathrm{E}+01$ | 0.44600E-05 | $0.60363 \mathrm{E}-05$ |
| 88.27800 | $0.46369 \mathrm{E}+01$ | $0.62843 \mathrm{E}+01$ | 0.50400E-05 | $0.68306 \mathrm{E}-05$ |
| 88.46400 | $0.51600 \mathrm{E}+01$ | $0.70320 \mathrm{E}+01$ | 0.51500E-05 | $0.70183 \mathrm{E}-05$ |
| 88.46400 | $0.51600 \mathrm{E}+01$ | $0.70320 \mathrm{E}+01$ | 0.54700E-05 | $0.74544 \mathrm{E}-05$ |
| 88.48001 | $0.52091 \mathrm{E}+01$ | $0.71020 \mathrm{E}+01$ | $0.52200 \mathrm{E}-05$ | $0.71169 \mathrm{E}-05$ |
| 88.48101 | $0.52121 \mathrm{E}+01$ | $0.71064 \mathrm{E}+01$ | $0.53500 \mathrm{E}-05$ | $0.72944 \mathrm{E}-05$ |
| 89.21701 | 0.83923E+01 | $0.11703 \mathrm{E}+02$ | 0.84100E-05 | $0.11728 \mathrm{E}-04$ |
| 89.22200 | 0.84219E+01 | $0.11746 \mathrm{E}+02$ | 0.84800E-05 | $0.11827 \mathrm{E}-04$ |
| 89.22600 | $0.84458 \mathrm{E}+01$ | $0.11781 \mathrm{E}+02$ | 0.84300E-05 | $0.11759 \mathrm{E}-04$ |
| 89.23600 | $0.85058 \mathrm{E}+01$ | 0.11868E+02 | 0.85200E-05 | $0.11888 \mathrm{E}-04$ |
| 89.28300 | $0.87956 \mathrm{E}+01$ | 0.12290E+02 | 0.96800E-05 | $0.13526 \mathrm{E}-04$ |
| 89.45500 | 0.99723E+01 | $0.14007 \mathrm{E}+02$ | 0.99900E-05 | $0.14032 \mathrm{E}-04$ |
| 89.45500 | $0.99723 \mathrm{E}+01$ | 0.14007E+02 | 0.10010E-04 | $0.14060 \mathrm{E}-04$ |
| 89.47000 | 0.10084E+02 | 0.14170E+02 | 0.10150E-04 | $0.14262 \mathrm{E}-04$ |
| 89.47200 | 0.10099E+02 | 0.14192E+02 | 0.10130E-04 | $0.14235 \mathrm{E}-04$ |
| 90.20800 | $0.17921 \mathrm{E}+02$ | 0.25591E+02 | 0.17860E-04 | $0.25505 \mathrm{E}-04$ |
| 90.21200 | 0.17977E+02 | $0.25672 \mathrm{E}+02$ | 0.18230E-04 | $0.26034 \mathrm{E}-04$ |
| 90.21701 | 0.18047E+02 | $0.25773 \mathrm{E}+02$ | 0.18000E-04 | $0.25706 \mathrm{E}-04$ |
| 90.21701 | 0.18047E+02 | 0.25773E+02 | 0.18590E-04 | $0.26549 \mathrm{E}-04$ |
| 90.22600 | 0.18173E+02 | $0.25956 \mathrm{E}+02$ | 0.18740E-04 | $0.26765 \mathrm{E}-04$ |
| 90.22700 | 0.18188E+02 | $0.25976 \mathrm{E}+02$ | 0.18320E-04 | $0.26165 \mathrm{E}-04$ |
| 90.22800 | $0.18202 \mathrm{E}+02$ | $0.25996 \mathrm{E}+02$ | 0.18210E-04 | $0.26008 \mathrm{E}-04$ |
| 90.23800 | $0.18343 \mathrm{E}+02$ | $0.26201 \mathrm{E}+02$ | 0.18680E-04 | $0.26681 \mathrm{E}-04$ |
| 90.24000 | 0.18372E+02 | 0.26242E+02 | 0.18830E-04 | $0.26896 \mathrm{E}-04$ |
| 90.28400 | $0.19005 \mathrm{E}+02$ | 0.27152E+02 | 0.19560E-04 | $0.27945 \mathrm{E}-04$ |
| 91.03400 | $0.29394 \mathrm{E}+02$ | $0.40836 \mathrm{E}+02$ | 0.29940E-04 | $0.41594 \mathrm{E}-04$ |
| 91.20700 | $0.30374 \mathrm{E}+02$ | $0.41415 \mathrm{E}+02$ | 0.30590E-04 | $0.41710 \mathrm{E}-04$ |
| 91.20800 | $0.30377 \mathrm{E}+02$ | $0.41414 \mathrm{E}+02$ | 0.30100E-04 | $0.41037 \mathrm{E}-04$ |
| 91.21500 | $0.30396 \mathrm{E}+02$ | $0.41404 \mathrm{E}+02$ | 0.30460E-04 | $0.41491 \mathrm{E}-04$ |
| 91.21701 | $0.30401 \mathrm{E}+02$ | $0.41400 \mathrm{E}+02$ | $0.30540 \mathrm{E}-04$ | $0.41590 \mathrm{E}-04$ |
| 91.22200 | $0.30413 \mathrm{E}+02$ | $0.41391 \mathrm{E}+02$ | 0.30330E-04 | $0.41278 \mathrm{E}-04$ |
| 91.22300 | $0.30415 \mathrm{E}+02$ | $0.41389 \mathrm{E}+02$ | $0.30190 \mathrm{E}-04$ | $0.41082 \mathrm{E}-04$ |
| 91.22300 | $0.30415 \mathrm{E}+02$ | $0.41389 \mathrm{E}+02$ | 0.30480E-04 | $0.41477 \mathrm{E}-04$ |
| 91.23001 | $0.30431 \mathrm{E}+02$ | $0.41373 \mathrm{E}+02$ | $0.30440 \mathrm{E}-04$ | $0.41386 \mathrm{E}-04$ |

Table 3a (continued). ZFITTER rescaling results for the $\boldsymbol{Z}$ pole.

| $\sqrt{s}, \mathrm{GeV}$ | $\sigma_{\text {cut }}^{\text {th }}$ [nb] | $\sigma_{\text {born }}^{\text {th }}[\underline{n b}]$ | $\boldsymbol{\sigma}^{\boldsymbol{e x p}}$ [mb] | $\sigma^{\boldsymbol{e x p}} \cdot\left(\sigma_{\text {born }}^{\text {th }} / \sigma_{\text {cut }}^{\text {th }}\right)[\mathrm{mb}]$ |
| :---: | :---: | :---: | :---: | :---: |
| 91.23800 | 0.30447E+02 | $0.41352 \mathrm{E}+02$ | 0.30630E-04 | $0.41601 \mathrm{E}-04$ |
| 91.23900 | $0.30448 \mathrm{E}+02$ | $0.41349 \mathrm{E}+02$ | $0.29960 \mathrm{E}-04$ | $0.40686 \mathrm{E}-04$ |
| 91.25400 | $0.30472 \mathrm{E}+02$ | $0.41300 \mathrm{E}+02$ | $0.30450 \mathrm{E}-04$ | $0.41270 \mathrm{E}-04$ |
| 91.25400 | $0.30472 \mathrm{E}+02$ | $0.41300 \mathrm{E}+02$ | $0.30460 \mathrm{E}-04$ | $0.41284 \mathrm{E}-04$ |
| 91.28000 | $0.30494 \mathrm{E}+02$ | $0.41187 \mathrm{E}+02$ | $0.30440 \mathrm{E}-04$ | $0.41113 \mathrm{E}-04$ |
| 91.28900 | $0.30497 \mathrm{E}+02$ | $0.41139 \mathrm{E}+02$ | $0.30860 \mathrm{E}-04$ | $0.41629 \mathrm{E}-04$ |
| 91.29400 | 0.30497E+02 | $0.41111 \mathrm{E}+02$ | $0.30450 \mathrm{E}-04$ | $0.41048 \mathrm{E}-04$ |
| 91.52900 | $0.29641 \mathrm{E}+02$ | $0.38504 \mathrm{E}+02$ | $0.29210 \mathrm{E}-04$ | $0.37945 \mathrm{E}-04$ |
| 91.95201 | $0.25217 \mathrm{E}+02$ | $0.30140 \mathrm{E}+02$ | $0.25310 \mathrm{E}-04$ | $0.30251 \mathrm{E}-04$ |
| 91.95301 | $0.25205 \mathrm{E}+02$ | $0.30119 \mathrm{E}+02$ | $0.24780 \mathrm{E}-04$ | $0.29611 \mathrm{E}-04$ |
| 91.96701 | $0.25030 \mathrm{E}+02$ | $0.29821 \mathrm{E}+02$ | $0.24640 \mathrm{E}-04$ | $0.29356 \mathrm{E}-04$ |
| 91.96900 | $0.25005 \mathrm{E}+02$ | $0.29779 \mathrm{E}+02$ | $0.24690 \mathrm{E}-04$ | $0.29403 \mathrm{E}-04$ |
| 92.20700 | $0.22038 \mathrm{E}+02$ | $0.24922 \mathrm{E}+02$ | $0.21830 \mathrm{E}-04$ | $0.24686 \mathrm{E}-04$ |
| 92.20900 | $0.22014 \mathrm{E}+02$ | $0.24883 \mathrm{E}+02$ | $0.21570 \mathrm{E}-04$ | $0.24382 \mathrm{E}-04$ |
| 92.21500 | $0.21941 \mathrm{E}+02$ | $0.24768 \mathrm{E}+02$ | $0.21220 \mathrm{E}-04$ | $0.23954 \mathrm{E}-04$ |
| 92.22600 | $0.21807 \mathrm{E}+02$ | $0.24558 \mathrm{E}+02$ | $0.22010 \mathrm{E}-04$ | $0.24786 \mathrm{E}-04$ |
| 92.28200 | $0.21135 \mathrm{E}+02$ | $0.23509 \mathrm{E}+02$ | $0.21240 \mathrm{E}-04$ | $0.23626 \mathrm{E}-04$ |
| 92.56200 | $0.18034 \mathrm{E}+02$ | $0.18865 \mathrm{E}+02$ | $0.16660 \mathrm{E}-04$ | $0.17428 \mathrm{E}-04$ |
| 92.95201 | $0.14545 \mathrm{E}+02$ | $0.14006 \mathrm{E}+02$ | $0.14590 \mathrm{E}-04$ | $0.14049 \mathrm{E}-04$ |
| 92.95301 | $0.14537 \mathrm{E}+02$ | $0.13995 \mathrm{E}+02$ | $0.14120 \mathrm{E}-04$ | $0.13594 \mathrm{E}-04$ |
| 92.96601 | $0.14437 \mathrm{E}+02$ | $0.13862 \mathrm{E}+02$ | $0.14440 \mathrm{E}-04$ | $0.13865 \mathrm{E}-04$ |
| 92.96800 | $0.14421 \mathrm{E}+02$ | $0.13841 \mathrm{E}+02$ | $0.14110 \mathrm{E}-04$ | $0.13542 \mathrm{E}-04$ |
| 93.20800 | $0.12739 \mathrm{E}+02$ | $0.11648 \mathrm{E}+02$ | $0.12580 \mathrm{E}-04$ | $0.11503 \mathrm{E}-04$ |
| 93.20900 | 0.12733E+02 | $0.11640 \mathrm{E}+02$ | $0.12480 \mathrm{E}-04$ | $0.11409 \mathrm{E}-04$ |
| 93.22000 | $0.12663 \mathrm{E}+02$ | $0.11551 \mathrm{E}+02$ | $0.12330 \mathrm{E}-04$ | $0.11248 \mathrm{E}-04$ |
| 93.22800 | $0.12612 \mathrm{E}+02$ | $0.11487 \mathrm{E}+02$ | $0.12380 \mathrm{E}-04$ | $0.11276 \mathrm{E}-04$ |
| 93.28601 | $0.12254 \mathrm{E}+02$ | $0.11035 \mathrm{E}+02$ | $0.11770 \mathrm{E}-04$ | $0.10599 \mathrm{E}-04$ |
| 93.70100 | 0.10097E+02 | $0.84120 \mathrm{E}+01$ | $0.10200 \mathrm{E}-04$ | $0.84981 \mathrm{E}-05$ |
| 93.70201 | 0.10092E+02 | $0.84067 \mathrm{E}+01$ | $0.10070 \mathrm{E}-04$ | $0.83883 \mathrm{E}-05$ |
| 93.71601 | $0.10030 \mathrm{E}+02$ | $0.83341 \mathrm{E}+01$ | $0.10100 \mathrm{E}-04$ | $0.83921 \mathrm{E}-05$ |
| 93.71701 | $0.10026 \mathrm{E}+02$ | $0.83289 \mathrm{E}+01$ | $0.99500 \mathrm{E}-05$ | $0.82660 \mathrm{E}-05$ |
| 94.20201 | $0.82093 \mathrm{E}+01$ | $0.62771 \mathrm{E}+01$ | $0.78200 \mathrm{E}-05$ | $0.59795 \mathrm{E}-05$ |
| 94.20201 | $0.82093 \mathrm{E}+01$ | $0.62771 \mathrm{E}+01$ | $0.79900 \mathrm{E}-05$ | $0.61094 \mathrm{E}-05$ |
| 94.21900 | $0.81557 \mathrm{E}+01$ | $0.62189 \mathrm{E}+01$ | $0.78800 \mathrm{E}-05$ | $0.60087 \mathrm{E}-05$ |
| 94.22300 | $0.81432 \mathrm{E}+01$ | $0.62053 \mathrm{E}+01$ | $0.80600 \mathrm{E}-05$ | $0.61420 \mathrm{E}-05$ |
| 94.27700 | $0.79772 \mathrm{E}+01$ | $0.60261 \mathrm{E}+01$ | $0.75900 \mathrm{E}-05$ | $0.57336 \mathrm{E}-05$ |
| 95.03601 | $0.61405 \mathrm{E}+01$ | $0.41416 \mathrm{E}+01$ | $0.64400 \mathrm{E}-05$ | $0.43436 \mathrm{E}-05$ |

Table 3b. ZFITTER rescaling results for the LEP II-III data.

| $\sqrt{s}, \mathrm{GeV}$ | $\sigma_{c u t}^{t h}[n b]$ | $\sigma_{\text {born }}^{\text {th }}[n b]$ | $\boldsymbol{\sigma}^{\boldsymbol{e x p}}$ [mb] | $\sigma^{\boldsymbol{e x p}} \cdot\left(\sigma_{\text {born }}^{t h} / \sigma_{c u t}^{t h}\right)[\mathrm{mb}]$ |
| :---: | :---: | :---: | :---: | :---: |
| 130.00000 | 0.82691E-01 | 0.82628E-01 | 0.84200E-07 | $0.84136 \mathrm{E}-07$ |
| 130.11999 | $0.82294 \mathrm{E}-01$ | $0.82264 \mathrm{E}-01$ | $0.79700 \mathrm{E}-07$ | $0.79670 \mathrm{E}-07$ |
| 130.19999 | $0.82031 \mathrm{E}-01$ | 0.82022E-01 | $0.82100 \mathrm{E}-07$ | $0.82091 \mathrm{E}-07$ |
| 130.19999 | $0.82031 \mathrm{E}-01$ | 0.82022E-01 | $0.71600 \mathrm{E}-07$ | $0.71592 \mathrm{E}-07$ |
| 136.08000 | $0.66194 \mathrm{E}-01$ | $0.67219 \mathrm{E}-01$ | $0.66700 \mathrm{E}-07$ | $0.67732 \mathrm{E}-07$ |
| 136.10000 | $0.66150 \mathrm{E}-01$ | $0.67177 \mathrm{E}-01$ | $0.66600 \mathrm{E}-07$ | $0.67634 \mathrm{E}-07$ |
| 136.19999 | $0.65930 \mathrm{E}-01$ | $0.66967 \mathrm{E}-01$ | $0.65100 \mathrm{E}-07$ | $0.66124 \mathrm{E}-07$ |
| 136.19999 | 0.65930E-01 | $0.66967 \mathrm{E}-01$ | $0.58800 \mathrm{E}-07$ | $0.59725 \mathrm{E}-07$ |
| 161.30000 | 0.34218E-01 | $0.35777 \mathrm{E}-01$ | $0.37300 \mathrm{E}-07$ | $0.38999 \mathrm{E}-07$ |
| 161.30000 | $0.34218 \mathrm{E}-01$ | $0.35777 \mathrm{E}-01$ | $0.40900 \mathrm{E}-07$ | $0.42763 \mathrm{E}-07$ |
| 161.30000 | 0.34218E-01 | $0.35777 \mathrm{E}-01$ | $0.29940 \mathrm{E}-07$ | $0.31304 \mathrm{E}-07$ |
| 172.10000 | $0.28336 \mathrm{E}-01$ | $0.29787 \mathrm{E}-01$ | $0.30300 \mathrm{E}-07$ | $0.31852 \mathrm{E}-07$ |
| 172.10000 | 0.28336E-01 | $0.29787 \mathrm{E}-01$ | $0.26400 \mathrm{E}-07$ | $0.27752 \mathrm{E}-07$ |
| 172.30000 | $0.28244 \mathrm{E}-01$ | $0.29693 \mathrm{E}-01$ | $0.28200 \mathrm{E}-07$ | $0.29646 \mathrm{E}-07$ |
| 182.69000 | $0.24017 \mathrm{E}-01$ | 0.25342E-01 | $0.23700 \mathrm{E}-07$ | $0.25007 \mathrm{E}-07$ |
| 182.69999 | $0.24014 \mathrm{E}-01$ | 0.25338E-01 | $0.21710 \mathrm{E}-07$ | $0.22907 \mathrm{E}-07$ |
| 182.69999 | $0.24014 \mathrm{E}-01$ | 0.25338E-01 | $0.24700 \mathrm{E}-07$ | $0.26062 \mathrm{E}-07$ |
| 188.63000 | $0.22095 \mathrm{E}-01$ | 0.23352E-01 | $0.22100 \mathrm{E}-07$ | $0.23358 \mathrm{E}-07$ |
| 188.69999 | $0.22073 \mathrm{E}-01$ | $0.23330 \mathrm{E}-01$ | $0.23100 \mathrm{E}-07$ | $0.24415 \mathrm{E}-07$ |

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Fig. 3: World data on the total cross section of $\boldsymbol{e}^{+} \boldsymbol{e}^{-} \rightarrow$ hadrons as presented in the original publications.



Fig. 6: $\boldsymbol{R}$-ratio. Data set is the same as on the Fig. 5. Solid curve is the $\boldsymbol{R}$-ratio prediction obtained in the three-loop QCD approximation with the effect of non-zero quark masses taken into account. Dashed curve is a "naive" quark parton model prediction for the $\boldsymbol{R}$-ratio


 described in [?] (dashed curve).
0.1
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
-0.08
-0.1

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ГНЦ РФ Институт физики высоких энергий
142284 , Протвино Московской обл.

Индекс 3649

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[^0]:    ${ }^{1}$ Values for $\boldsymbol{\sigma}_{\boldsymbol{c u t}}^{\boldsymbol{t} \boldsymbol{h}}$ were calculated using ZFITTER subroutine ZUTHSM with argument settings according to the cuts applied in the LEP I-II-III measurements. Values for $\boldsymbol{\sigma}_{\boldsymbol{b o r} \boldsymbol{n} \boldsymbol{n}}^{\boldsymbol{t h}}$ were calculated by the same subroutine but with the special flag switching it to calculate the IBA cross sections
    ${ }^{2}$ ZFITTER 6.30 Fortran code is available at http://www.ifh.de/~riemann/Zfitter/zfitr630.uu .

[^1]:    ${ }^{3}$ Narrow $\boldsymbol{\psi}$ family resonances between 3 and 4 GeV were excluded by default. Somewhat controversial is the question, whether the gaps between $\boldsymbol{\psi}$ 's can be really treated as the continuum regions. $\boldsymbol{R}$ ratio demonstrates no step up until $\sqrt{\boldsymbol{s}}=\mathbf{3 . 9}-\mathbf{4 . 0} \mathrm{GeV}$, just at the left knee of the broad $\boldsymbol{\psi}(\mathbf{4 0 4 0})$ resonance, where the continuum approximation of $\boldsymbol{R}$ does not work. Nevertheless, this step results in the fitted $\boldsymbol{c}$-quark mass $\boldsymbol{M}_{\boldsymbol{c}}=\mathbf{1 . 9 7 1}$ (see fit (III) in the Table 1).

    In pQCD fits, performed in Refs. [19], [9] the interval $\mathbf{3 . 0}<\sqrt{\boldsymbol{s}}<\mathbf{4 . 0} \mathrm{GeV}$ with the resonances excluded was treated as a continuum.

[^2]:    ${ }^{4}$ Choosing $\boldsymbol{S}_{\boldsymbol{c u t}}$ we must satisfy at least the following two requirements:

    1) $\boldsymbol{S}_{\boldsymbol{c u t}}$ should lie well above all the hadronic resonances, i.e. 1-loop perturbative QED can be used to calculate hadronic vaquum polarization at $\boldsymbol{s}>\boldsymbol{s}_{\boldsymbol{c u t}}$;
    2) there should be enough densely distributed experimental points at $\boldsymbol{s}<\boldsymbol{s}_{\boldsymbol{c u t}}$ for the trapezoidal numerical evaluation of the dispersion integral over $2 \boldsymbol{m}_{\boldsymbol{\pi}}<\sqrt{\boldsymbol{s}}<\sqrt{\boldsymbol{s}_{\boldsymbol{c u t}}}$.

    As there are few experimental points in the interval $13<\sqrt{\boldsymbol{s}}<\mathbf{3 0} \mathrm{GeV}, \sqrt{\boldsymbol{s}_{\boldsymbol{c u t}}}=19.5 \mathrm{GeV}$ appears to be a compromise between these requirements.

[^3]:    ${ }^{5}$ As the calculation of $\boldsymbol{R}_{\boldsymbol{b a r e} \boldsymbol{e} \boldsymbol{d a t a}}^{\boldsymbol{d a t a}}$ from the original $\boldsymbol{\sigma}$ and $\boldsymbol{R}$ data (see subsection 1.1) in turn requires the evaluation of $\boldsymbol{\Delta} \boldsymbol{\alpha}^{\boldsymbol{h a d}}(\boldsymbol{s})$, we applied the following simple iterative procedure.

    Taking as a zero approximation $\boldsymbol{R}(\boldsymbol{s})$, obtained from the original data using 1-loop perturbative $\boldsymbol{\alpha}_{\boldsymbol{Q} \boldsymbol{E D}}(\boldsymbol{s})$, we evaluate $\boldsymbol{\Delta} \boldsymbol{\alpha}^{\boldsymbol{\operatorname { h a d } ( 1 )}}(\boldsymbol{s})$. Using the latter, we compute the next approximation of $\boldsymbol{R}(\boldsymbol{s})$ from the original data, the evaluate the integral again to obtain $\boldsymbol{\Delta} \boldsymbol{\alpha}^{\boldsymbol{h a d}(\mathbf{2})}(\boldsymbol{s})$, and so on. This process converges well even in resonance regions after 3 iterations. We restricted ourselves to 5 iterations. (For such a number the finite machine precision is likely to result in the error, much less than the error of the method itself).

