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**IMPACT OF THE THREE-LOOP CORRECTIONS  
ON THE QCD ANALYSIS  
OF THE DEEP-INELASTIC-SCATTERING DATA**

Protvino 2001

**Abstract**

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We perform the analysis of the existing inclusive deep inelastic scattering (DIS) data within NNLO QCD approximation. The parton distributions functions (PDFs) and the value of strong coupling constant  $\alpha_s(M_Z) = 0.1164 \pm 0.0013$  (exp) are obtained. The sensitivity of the PDFs to the uncertainty in the value of the NNLO corrections to the splitting functions is analyzed. It is shown that the PDFs errors due to this uncertainty is generally less than the experimental uncertainty in PDFs through the region of  $x$  spanned by the existing DIS data.

**Аннотация**

Алехин С.И. Влияние трёхпетлевых поправок КХД на анализ данных по глубоко-неупругому рассеянию: Препринт ИФВЭ 2001-30. – Протвино, 2001. – 8 с., 4 рис., 2 табл., библиогр.: 19.

Приведены результаты анализа существующих данных по глубоко-неупругому рассеянию (ГНР) в рамках трёхпетлевого приближения КХД. Получены партонные распределения и величина постоянной сильного взаимодействия  $\alpha_s(M_Z) = 0.1164 \pm 0.0013$  (эксп). Проанализирована чувствительность партонных распределений к неопределённостям трёхпетлевых поправок к функциям расщепления и показано, что ошибки партонных распределений, связанные с этими неопределённостями, не превышают экспериментальных ошибок партонных распределений во всём диапазоне, перекрываемом существующими данными по ГНР.

1. The account of higher-order corrections in an analysis based on the QCD perturbative expansions is very important. For the relevant processes measured to the moment the typical value of the strong coupling constant  $\alpha_s$  is  $O(0.1)$  and the convergence of series in  $\alpha_s$  is slow. For the deep-inelastic-scattering (DIS) process this problem is especially important for the largest and the lowest  $x$  regions, where the coefficients of the series contain the terms proportional to “large logarithms”. Meanwhile due to great technical difficulties the progress in calculation of the higher-order QCD corrections is not so fast. In particular for the case of the inclusive DIS structure functions only the two-loop QCD corrections have been calculated completely [1]. The three-loop (NNLO) case coefficient functions are known exactly [2], while for the corrections to the splitting functions only the even Mellin moments up to 8 and some asymptotes were known to the recent time [3,4].

An attempt to combine all available information about splitting functions in order to obtain reasonable approximation to the exact expressions was done in Refs. [5,6]. The result of this study is the set of approximate NNLO splitting functions in the  $x$  space supplied by the estimate of their possible variation due to effect of the highest moments. These approximate splitting functions have been used in the analysis of Ref. [7] aimed to estimate the effect of the NNLO QCD corrections on the shape of the parton distributions functions (PDFs) extracted from the global fit. Meanwhile the gluon distribution obtained in this analysis turned out to be sensitive to the uncertainties of the NNLO splitting functions given in Refs. [5,6]. In particular at  $x \sim 10^{-4}$  and  $Q^2 = 20 \text{ GeV}^2$  the error on the gluon distribution due to this uncertainty is about 35%, which is much larger than the experimental error on the gluon distribution obtained in the two-loop analysis of the existing DIS data [8].

Fortunately the Mellin moments of the splitting functions up to 12 were calculated recently in Ref. [9] that allowed to elaborate new set of the approximate splitting functions with much narrower uncertainty range [10]. In this paper we describe the results of our analysis of the existing DIS data with account of the NNLO QCD corrections. The analysis is based on the recent splitting functions given in Ref. [10]. Our main aim is to study the effect of NNLO corrections on the PDFs and the value of  $\alpha_s$  extracted from the data with a particular attention paid on the errors due to the remaining uncertainty of the NNLO splitting functions.

2. Our theoretical ansatz and the fitting procedure are the same as in our previous analysis of Ref. [8], except that now we use the NNLO QCD approximation both for the splitting functions and the leading twist (LT) coefficient functions of DIS. Impact of the NNLO corrections to the coefficient functions on the values of  $F_2$  and  $F_L$  is illustrated in Fig.1, where we give the ratios of these structure functions in the NNLO approximation to the ones in the NLO approximation.

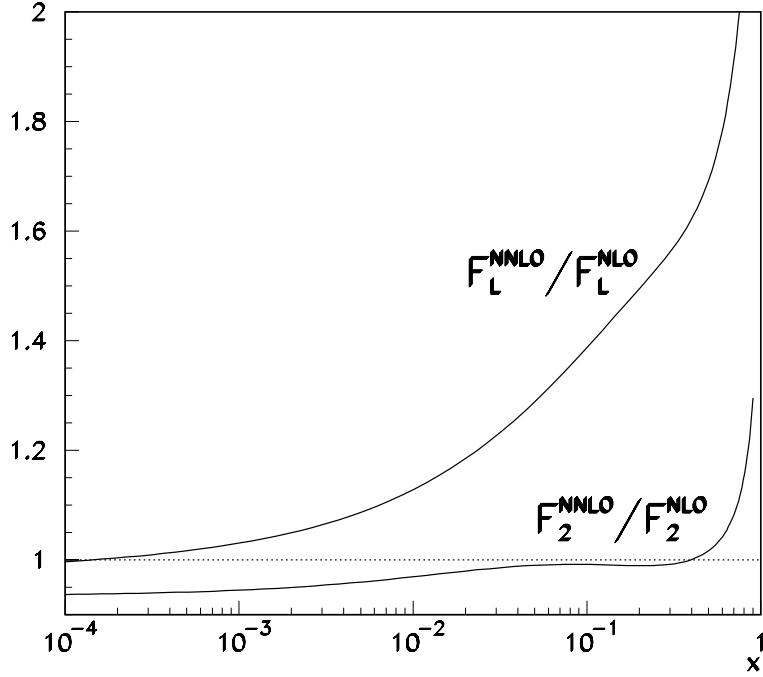


Fig. 1. The ratios of the leading twist structure functions  $F_{2,L}$  calculated in the NNLO and the NLO approximations.

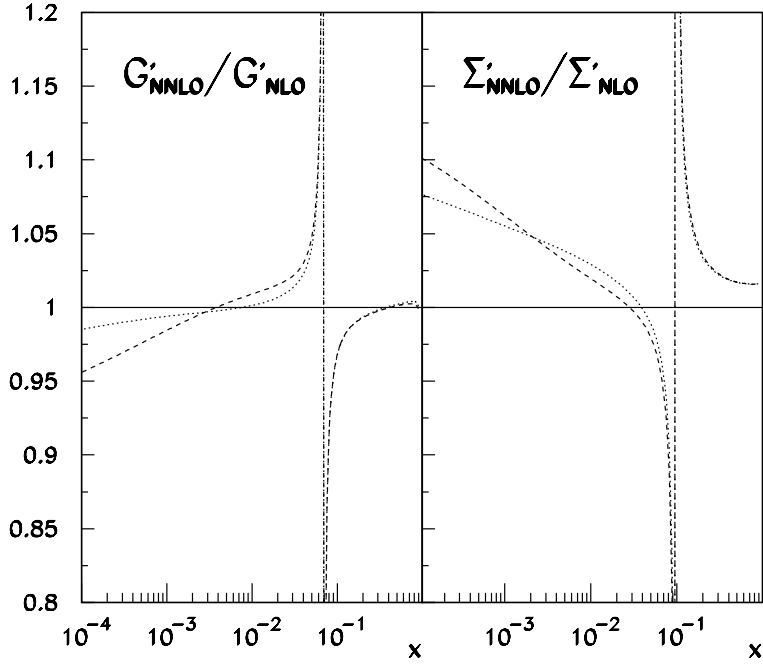


Fig. 2. Ratio of the logarithmic derivatives of the gluon distribution  $G' = d \ln G / d \ln Q$  calculated in the NNLO and the NLO approximations (left); the same for the singlet distribution (right). The dotted curves correspond to the choice A and the dashed curves – to the choice B for the splitting functions approximations of Ref. [10].

The NNLO contributions to the coefficient functions in the form given in Ref. [5] were used in the calculations. The input for both NLO and NNLO calculations was chosen the same as in Ref. [5]: The gluon distribution  $xG(x) = x^{-0.37}(1-x)^5$ , the total singlet distribution  $x\Sigma(x) = 0.6x^{-0.3}(1-x)^{3.5}(1+5x^{0.8})$ , the number of flavors  $N_f = 4$ , and the value of  $\alpha_s = 0.2$ . The largest effect of the NNLO corrections to the coefficient functions on the values of  $F_{2,L}$  is the rise of  $F_L$  at large  $x$ . Nevertheless for the analysis of existing data this rise is not so important due to sensitivity of the data to variation of  $F_L$  at large  $x$  is rather poor. Much more important is suppression of the structure function  $F_2$  by  $\sim 5\%$  at small  $x$  since the precision of existing data on  $F_2$  is  $O(1\%)$  in this region. Effect of the NNLO corrections on the splitting functions is demonstrated in Fig.2, where the ratios of the logarithmic derivatives of the gluon and the singlet distributions calculated in the NNLO and the NLO approximations are plotted. We used in the calculations the approximations of the splitting functions from Ref. [10] and the input distributions from Ref. [5]. Different curves correspond to the two choices of the splitting functions which give the range of the uncertainty of the latter. One can see that the NNLO corrections to the splitting functions change the “speed” of evolution moderately: At the scale of  $Q \sim 10$  GeV the derivatives change by  $\lesssim 10\%$  at smallest  $x$  and even less at the largest  $x$  in question (the spike at  $x \sim 0.1$  is just due to the QCD evolution has crossover point here and the derivatives are very small in this region). As a result the main effect of the NNLO corrections is due to corrections to the coefficient function for  $F_2$ . Figs.1,2 may be used for the benchmark of our NNLO evolution code as well. For this purpose one can compare these figures with Fig.10 of Ref. [5] and Fig.4 of Ref. [10] correspondingly and convince that the agreement of both codes is perfect<sup>1</sup>

The boundary LT PDFs fitted to the data were parameterized within the scheme with fixed number of flavors at  $N_f = 3$ . At our starting value of the QCD evolution  $Q_0^2 = 9$  GeV<sup>2</sup> they read

$$\begin{aligned}
xu_V(x, Q_0) &= \frac{2}{N_u^V} x^{a_u} (1-x)^{b_u} (1 + \gamma_2^u x), & xd_V(x, Q_0) &= \frac{1}{N_d^V} x^{a_d} (1-x)^{b_d}, \\
xu_S(x, Q_0) &= \frac{A_S}{N_S} \eta_u x^{a_{su}} (1-x)^{b_{su}}, & xd_S(x, Q_0) &= \frac{A_S}{N_S} x^{a_{sd}} (1-x)^{b_{sd}}, & xs_S(x, Q_0) &= \frac{A_S}{N_S} \eta_s x^{a_{ss}} (1-x)^{b_{ss}}, \\
xG(x, Q_0) &= A_G x^{a_G} (1-x)^{b_G} (1 + \gamma_1^G \sqrt{x} + \gamma_2^G x), & & & & (1)
\end{aligned}$$

where  $u, d, s, G$  are the up, down, strange quarks, and gluons distributions respectively; the indices  $V$  and  $S$  correspond to the valence and sea quarks. The parameters  $N_u^V, N_d^V$  and  $A_G$  were not fitted, instead they were calculated from the other parameters using the conservation of the partons momentum and the fermion number. The normalization parameter  $N^S$  is also calculated from the other parameters in such way that the normalization parameter  $A_s$  correspond to the total momentum carried by the sea quarks. The parameter  $\eta_s$  was fixed at 0.42 and the other sea distributions parameters were constrained as  $a_{su} = a_{sd} = a_{ss}$ ,  $b_{ss} = (b_{su} + b_{sd})/2$ .

The LT structure functions  $F_{2,L}$  obtained using these PDFs evolved using the GLAPD equations [11] were corrected for the target-mass correction and the high-twist contribution as well as it was done in our earlier analysis of Ref. [8]. The result was substituted to the regular expression for the inclusive DIS cross section, which was fitted to the existing data varying parameters of

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<sup>1</sup>The extensive cross-check of the different NNLO evolution codes is underway now and the results will be released at the WWW page of the Les Houches workshop “Physics at TeV colliders” (<http://pdf.fnal.gov/LesHouches.htm>).

PDFs, the value of  $\alpha_s$ , and the high-twist contributions to  $F_{2,L}$ . For our nominal fit we use the NNLO splitting functions obtained as the average of the variants A and B given in Ref. [10].

We used for the analysis the data on DIS of charged leptons off the proton and deuterium targets. The data set coincides in part with the one used in Ref. [8]. The difference from that analysis is that now we include in the fit the H1 data of Ref. [12] and the ZEUS data of Ref. [13] collected in 1996-97 instead of earlier data of these collaborations. Besides, we drop the data from FNAL-E-665 experiment [14] since they have no impact on the results of the analysis due to large experimental errors. It was also checked that inclusion of the high  $Q^2$  data of Ref. [15] does not decrease the experimental errors in the fitted values of PDFs and  $\alpha_s$  and for this reason they were not included in our analysis. The same is valid for the ZEUS data of Ref. [13] with  $Q^2 > 300 \text{ GeV}^2$  and we also discarded these data points from the analyzed data set in order to escape the region where the corrections due to the  $Z$ -boson contribution should be taken into account. The data points with  $Q^2 < 2.5 \text{ GeV}^2$  and  $x > 0.75$  were cut in order to improve the perturbative stability of the results and to minimize the effect of nuclear corrections correspondingly. The resulting data set outlined in Table 1 spans the region of  $x = 5 \cdot 10^{-5} \div 0.75$  and  $Q^2 = 2.5 \div 250 \text{ GeV}^2$ .

**Table 1.** The numbers of data points (NDP) and the  $\chi^2$  values for the separate experimental data sets used in the analysis.

Experiment	NDP		$\chi^2/\text{NDP}$
	proton	deuterium	
SLAC-E-49A	58	58	0.66
SLAC-E-49B	144	135	1.37
SLAC-E-87	90	90	1.08
SLAC-E-89A	66	59	1.46
SLAC-E-89B	79	62	1.13
SLAC-E-139	–	16	0.59
SLAC-E-140	–	26	0.71
BCDMS	351	254	1.16
NMC	245	245	1.30
H1(96-97)	122	–	1.16
ZEUS(96-97)	161	–	1.18
TOTAL	1316	945	1.13

The statistic and systematic errors in the experimental data were combined in the minimized  $\chi^2$  using the covariance matrix approach as well as it was done in our earlier analysis. The normalization factors for all experiments excluding the old SLAC ones were also included into the covariance matrix, while for the latter the fitted re-normalization factors were introduced (see Ref. [8] for details). In Table 1 we give the value of  $\chi^2$  obtained after the fit and the contributions of each separate experiment to its total value. One can see that the value of  $\chi^2$  reduced to the number of data points (NDP) is about unity for the total data set. The deviation of the values of  $\chi^2/\text{NDP}$  off unity for some separate experiments can be attributed to the statistical fluctuations in the most cases. This allows one to conclude that in good approximation the data can be described by the statistical model with the Gaussian probability functions for all errors including the systematic ones and hence the errors in the fitted parameters are also Gaussian distributed. The obtained values of the fitted parameters with their errors are given in Table 2.

Table 2. The values of the fitted parameters of the leading twist PDFs and the strong coupling constant.

		NLO	NNLO	
			(A+B)/2	B
Valence	$a_u$	$0.709 \pm 0.027$	$0.712 \pm 0.030$	$0.711 \pm 0.030$
	$b_u$	$3.911 \pm 0.051$	$4.025 \pm 0.050$	$4.028 \pm 0.050$
	$\gamma_2^u$	$1.06 \pm 0.35$	$1.21 \pm 0.40$	$1.22 \pm 0.41$
	$a_d$	$0.706 \pm 0.073$	$0.739 \pm 0.071$	$0.749 \pm 0.071$
	$b_d$	$4.95 \pm 0.12$	$5.12 \pm 0.11$	$5.13 \pm 0.12$
Glue	$a_G$	$-0.145 \pm 0.019$	$-0.134 \pm 0.026$	$-0.092 \pm 0.026$
	$b_G$	$8.2 \pm 1.3$	$9.7 \pm 1.6$	$10.2 \pm 1.4$
	$\gamma_1^G$	$-3.79 \pm 0.45$	$-3.01 \pm 0.73$	$-3.59 \pm 0.60$
	$\gamma_2^G$	$7.7 \pm 1.7$	$7.0 \pm 2.5$	$7.9 \pm 2.0$
Sea	$A_S$	$0.165 \pm 0.011$	$0.170 \pm 0.010$	$0.1694 \pm 0.0096$
	$a_{sd}$	$-0.1961 \pm 0.0048$	$-0.2134 \pm 0.0047$	$-0.2117 \pm 0.0045$
	$b_{sd}$	$4.7 \pm 1.3$	$4.9 \pm 1.2$	$5.0 \pm 1.1$
	$\eta_u$	$1.16 \pm 0.11$	$1.11 \pm 0.10$	$1.11 \pm 0.10$
	$b_{su}$	$10.42 \pm 0.86$	$9.99 \pm 0.89$	$9.99 \pm 0.88$
	$\alpha_s(M_Z)$	$0.1171 \pm 0.0015$	$0.1164 \pm 0.0013$	$0.1165 \pm 0.0014$

To examine the sensitivity of our results to the specific choice of the NNLO splitting functions we repeated the fit with the choice B for splitting functions of Ref. [10]. The results of this fit are compared to the ones of the nominal fit in Table 2 and in Fig.3. One can see that the difference of the PDFs values obtained in the fits with different choices of the NNLO splitting functions is largest for the gluon distribution at small  $x$  and anyway does not exceed the experimental uncertainties in the PDFs through the whole region of  $x$  in question.

We also performed fit to the same data within the NLO approximation in order to check the perturbative stability of our analysis. The comparison of the results of this fit with the NNLO ones is given in Table 2. One can see that the main difference between the NLO and NNLO results is for the parameters describing the sea and the gluon distributions at small  $x$ . Nevertheless, as one can see from Fig.4, even in this region the shift of the NNLO gluon distribution as compared to the NLO one is smaller than their experimental errors in the wide range of  $Q$ . The same is valid for the sea distribution and other distributions are even less sensitive to the inclusion of the NNLO corrections. The N<sup>3</sup>LO QCD corrections as they were estimated in Ref. [16] should have smaller effect than the NNLO ones. The reasonable conclusion based on these considerations is that the perturbative stability of the obtained NNLO PDFs is better than their experimental uncertainties at  $x \gtrsim 0.0001$ . A particular feature of our analysis is that our gluon distribution is positive up to  $Q \sim 1 \text{ GeV}^2$  in the region of  $x \gtrsim 0.0001$ , in contrast with the gluon distribution obtained in the analysis of Ref. [7].

Our value of

$$\alpha_s^{\text{NNLO}}(M_Z) = 0.1164 \pm 0.0013 \text{ (exp)} \quad (2)$$

is in agreement with the value of

$$\alpha_s^{\text{NNLO}}(M_Z) = 0.1166 \pm 0.0009 \text{ (exp)} \quad (3)$$

obtained in the analysis of the similar data set performed in Ref. [17]. Meanwhile we should underline some differences of those results with ours. Contrary to the results of Ref. [17], we observe sizeable decrease of the  $\alpha_s$  value under inclusion of the NNLO corrections (compare  $\alpha_s^{\text{NLO}}(M_Z) = 0.1171 \pm 0.0015$  (exp) in our analysis and  $\alpha_s^{\text{NLO}}(M_Z) = 0.1155 \pm 0.0014$  (exp) in Ref. [17]). In addition, we do not observe the sharp decrease of the error in  $\alpha_s^{\text{NNLO}}$  as compared with the error in  $\alpha_s^{\text{NLO}}$ . Among the most probable explanations of these discrepancies is the difference in treatment of the experimental data. For example one cannot compare the experimental errors in  $\alpha_s$  given in Eqn.(2) and in Eqn.(3) since the latter does not account for the systematic errors in data. Besides, the analysis of Ref. [17] was performed assuming that the contribution of the high-twist terms is zero, while we fitted this contribution together with other parameters. Evidently, since the high-twist contribution to  $F_2$  and the value of  $\alpha_s$  are strongly anti-correlated, this may take effect both on the central value and the error in  $\alpha_s$ . Nevertheless for the comprehensive clarification of the differences between our results and the ones of Ref. [17] a dedicated analysis is needed and we suppose to do it in future as well as the comparison with the earlier NNLO fit to the data on the neutrino DIS structure function  $F_3^{\nu N}$  (see Ref. [18] and references therein).

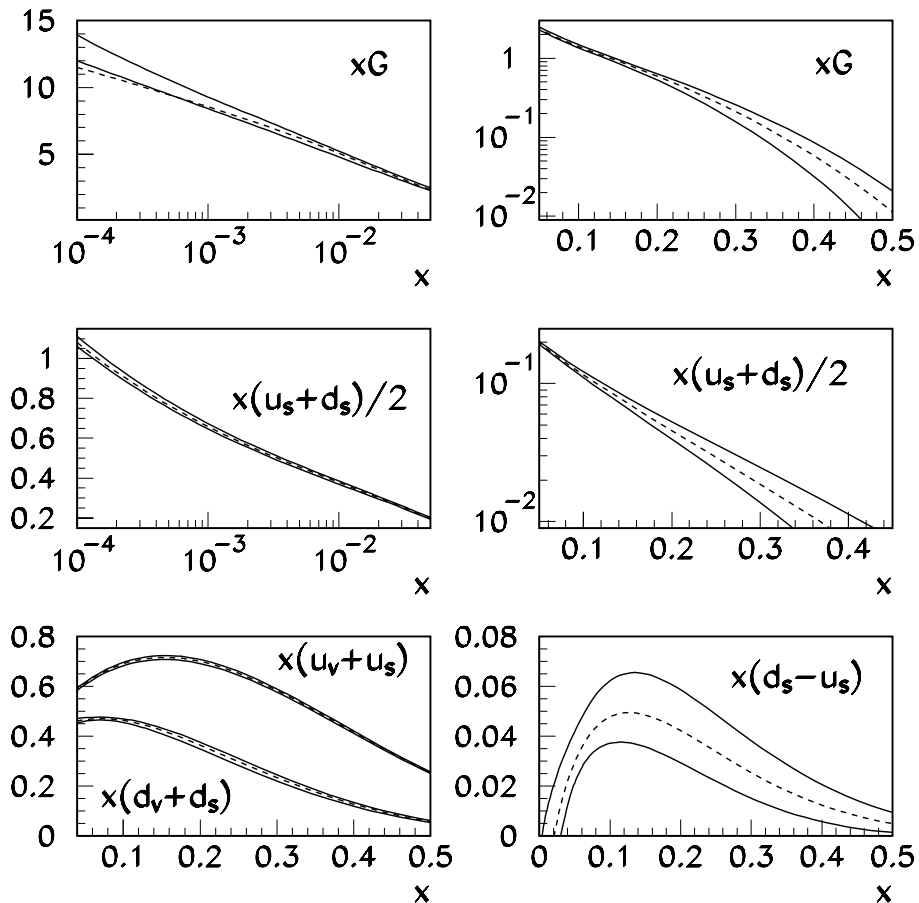


Fig. 3. The selected PDFs obtained from the NNLO fits with the different choice of the NNLO splitting functions. (Full curves: the  $1\sigma$  experimental bands for the fit with the splitting functions chosen as average of the variants A and B of Ref. [10]; dashed curves: the central values for the fit with variant B of Ref. [10]).



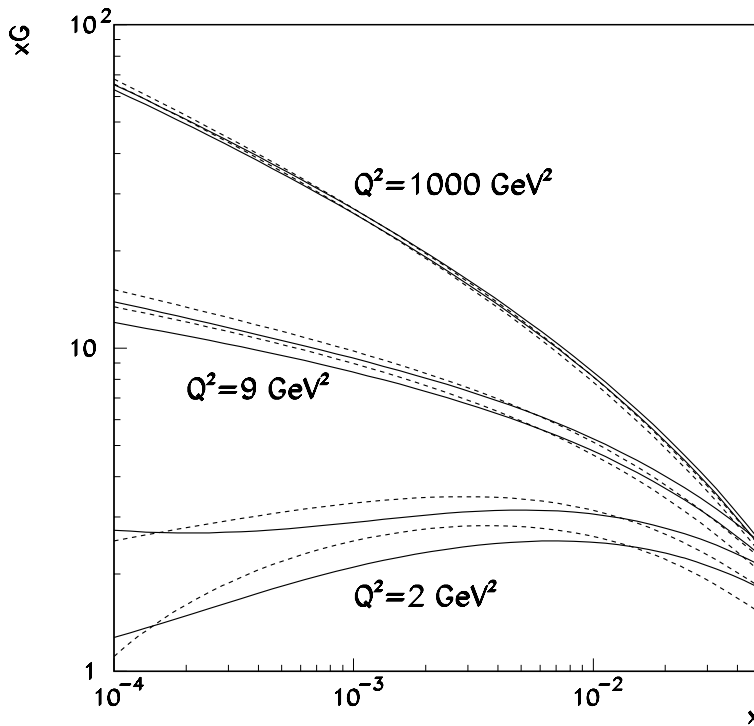


Fig. 4. The  $1\sigma$  bands for the gluon distributions obtained in the NNLO (full lines) and the NLO (dashed lines) analysis at different values of  $Q$ .

**3.** In summary, we performed the analysis of the existing inclusive DIS data within the NNLO QCD approximation. The PDFs and the value of strong coupling constant  $\alpha_s(M_Z) = 0.1164 \pm 0.0013$  (exp) are obtained. The sensitivity of the PDFs to the uncertainty in the value of the NNLO corrections to the splitting functions is analyzed and it is shown that the PDFs errors due to this uncertainty is generally less than the experimental uncertainty in PDFs through the region of  $x$  spanned by the existing DIS data. The obtained set of PDFs may be used to reduce the higher order QCD uncertainty in the predictions of the cross sections of the hard scattering processes in the hadron collisions. In particular this may be important for reliable estimation of the K-factor for the Higgs boson production (see Ref. [19] in this connection).

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С.И. Алёхин

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