Yu.F. Pirogov[1] and O.V. Zenin[2]

UNCERTAINTY OF THE TWO-LOOP RG UPPER BOUND ON THE HIGGS MASS

[1] E-mail: pirogov@mx.ihep.su
[2] E-mail: zenin_o@mx.ihep.su

Abstract

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A modified criterion of the SM perturbative consistency is proposed. It is based on the analytic properties of the two-loop SM running couplings. Under the criterion adopted, the Higgs mass up to 380 GeV might not give rise to the strong coupling prior to the Planck scale. This means that the light Higgs boson is possibly preferred for reasons other than the SM perturbative consistency, i.e. for reasons beyond the SM.

Аннотация

Пирогов Ю.Ф., Зенин О.В. Неопределенность двухпетлевого ренормгруппового ограничения сверху на массу бозона Хиггса: Препринт ИФВЭ 2002–21. – Протвино, 2002. – 7 с., 3 рис., библиогр.: 10.

Предложен модифицированный критерий пертурбативной самосогласованности СМ, использующий аналитические свойства ее двухпетлевых бегущих констант. С учетом данного критерия масса бозона Хиггса, меньшая 380 ГэВ, может не приводить к возникновению режима сильной связи вплоть до планковского масштаба. Это означает, что легкий бозон Хиггса, возможно, предпочтителен по причинам, не связанным с пертурбативной самосогласованностью СМ, т.е. по причинам, лежащим вне СМ.

1. Introduction

The current experimental data restrict the Higgs mass in the Standard Model (SM) within the range 114.1 GeV$< M_H < 194$ GeV. The lower bound on M_H comes from the absence of the Higgs production signal at LEP II at the 95% CL [1]. The upper bound is derived at the same CL from the fit to the precision electroweak data [2]. On the other hand, the upper bound on the Higgs mass can be obtained from the requirement of the SM perturbative consistency up to a cutoff energy scale Λ at which the SM might get into the strong coupling regime. The two-loop renormalization group (RG) gives typical upper bounds $M_H < 200$ GeV at $\Lambda = M_{GUT} = 10^{14}$ GeV and $M_H < 180$ GeV at $\Lambda = M_{Pl} = 10^{19}$ GeV (see, e.g., [3]). Thus both the electroweak precision data and the SM perturbative consistency up to the GUT scale exclude the Higgs mass $M_H \geq 200$ GeV. This could be interpreted as though the Higgs should be light due to the self-suppression of the strong coupling in the SM. But the question is to what extent the Higgs upper bound from the SM perturbative consistency is reliable?

A clear-cut criterion of the strong coupling in the Higgs sector of the SM exists only in one loop. In this case, the one-loop quartic coupling λ develops the Landau pole at a finite energy scale Λ. In two loops, the pole is compensated but λ becomes large, $\lambda/4\pi^2 \simeq 1$, nearly at the same energy scale Λ. Taken alone, this does not give the unambiguous criterion of the nonperturbative regime any more. In the conventional assumption that the higher loops become comparable with the first and second ones at the same scale Λ, the results of [3], [4] follow (see also [5] for a review). On the other hand the contributions of the higher loops might be either small, or large but mutually compensated. This would not change drastically the two-loop running of λ and may relax the conventional upper bound on the Higgs mass.

Presently, the full set of the SM β functions is known up to the two loops only. This forces one to study the reliability of the self-consistency criterion of the two-loop RG approximation in the SM. This is the purpose of the present paper. The method proposed in the paper relies on the subtracted RG and the analytic properties of the running couplings. It is similar in spirit to methods applied to resolve the Landau singularity problem in QED [6] and, later, to improve the infrared behaviour of the QCD running coupling $\alpha_S(\mu^2)$ [7], [8].

2. Subtracted finite-loop RG

Let us consider the system of the SM two-loop RG equations (RGE):

$$\mu^2 \frac{da_i(\mu^2)}{d\mu^2} = \beta_i\left(\{a_j(\mu^2)\}\right) \ . \tag{1}$$

Here and in what follows $a_i(\mu^2)$ are the SM running couplings vs. the energy squared scale μ^2, and β_i are the respective β functions calculated at the given number of loops. We disregard the

mass effects here. Conventionally, the system (1) is integrated numerically along the real axis $\operatorname{Re}\mu^2 < 0$:

$$a_i(\mu^2) = a_i(\mu_0^2) + \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \beta_i\left(\left\{a_j(\mu'^2)\right\}\right) \ , \tag{2}$$

where $\mu_0^2 < 0$ is a reference point, $|\mu_0| \sim M_Z$. The β functions can now be defined as the functions of the real negative μ^2:

$$\beta_i(\mu^2) \equiv \beta_i\left(\left\{a_j(\mu^2)\right\}\right) \ . \tag{3}$$

Eqs. (1) – (3) preserve their meaning for the complex μ^2 as well. But the numerical solution obtained says nothing about the analytic properties of the running couplings with respect to μ^2. In two loops, despite the absence of the real singularities of the Higgs quartic coupling λ there could be the complex ones. They influence the strong coupling regime $\lambda/4\pi^2 \geq 1$ at large enough real μ^2. The extension of the two-loop RG analysis onto the complex μ^2 plane allows one to find the position of the singularities implicitly.
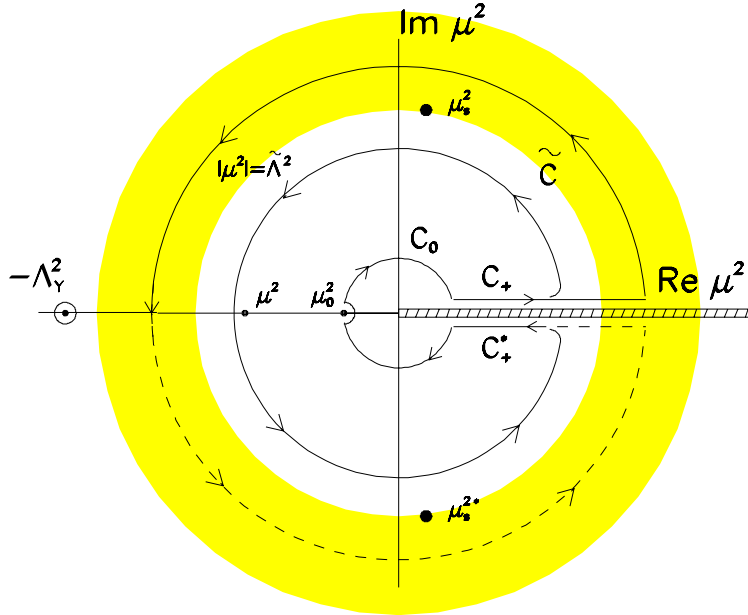
Fig. 1: The integration contour C and the generic complex-conjugate singularity points μ_s^2, μ_s^{2*} with $|\mu_s^2| = \Lambda_s^2$. The real point $(-\Lambda_Y^2)$ corresponds to the $U(1)_Y$ singularity. All the complex singularities are assumed to reside within the shadowed area at $\Lambda_s^2 \leq |\mu^2| \leq \Lambda_Y^2$. The hatched line designates the physical cut.

To this end, let us continue analytically the β functions and running couplings onto the complex μ^2 plane with the cut along the real axis $\operatorname{Re}\mu^2 > 0$ (Fig. 1). The cut is chosen so that $-\pi < \operatorname{Im}\ln\left(-\mu^2\right) < \pi$. All the running couplings are assumed to satisfy the hermiticity condition $a_i(\mu^{2*}) = a_i^*(\mu^2)$. Let us first choose the closed contour $C = C_0 \cup C_+ \cup \tilde{C} \cup C_+^*$ (Fig. 1) so that C encircles the given point μ^2, and all the singularities of the running couplings $a_i(\mu^2)$ reside outside C. Then $\beta_i(\mu^2)$ satisfy the identity

$$\beta_i(\mu^2) \equiv \frac{1}{2\pi i} \int_C \frac{\beta_i(s)\ ds}{s - \mu^2} \ , \tag{4}$$

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where $\beta_i(s) \equiv \beta_i(\{a_j(s)\})$. Substituting Eq. (4) into Eq. (2) one gets

$$a_i(\mu^2) = a_i(\mu_0^2) + \frac{1}{2\pi i} \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \int_C \frac{\beta_i(s)\ ds}{s - \mu'^2}\ , \qquad (5)$$

where the integration path between points μ_0^2 and μ^2 should lie inside C. In what follows, the square root $\tilde{\Lambda}$ of the radius of the outer contour $\tilde{C}$ is referred to as the modification radius.

Now let us spread the outer contour $\tilde{C}$ so that at least a part of the implicit singularities of $a_i(\mu^2)$ gets located inside C. In general, the identity (5) ceases to be valid. Moreover, the integration of the RGE system (1) from the reference point μ_0^2 to the real point $(-\tilde{\Lambda}^2)$, $\tilde{\Lambda} > \Lambda_s$, along the real axis and the upper half of the contour C (shown in solid in Fig. 1) does not give the identical results. Remarkably, in the latter case the couplings $a_i(-\tilde{\Lambda}^2)$ acquire the nonzero complex parts while in the former case they are real by construction. This discrepancy reflects the contribution of the implicit complex singularities. The minimal radius Λ_s^2 of the external contour $\tilde{C}$ at which all these irregularities take place gives the estimate of the upper range of the reliability of the RG in the given loops. The value of Λ_s corresponds to crossing the nearest singularities of $a_i(\mu^2)$. At scales larger than Λ_s, the original finite-loop approximation is definitely unreliable. It is at $|\mu^2| \geq \Lambda_s^2$, where the contributions of higher loops are needed to improve the analytic properties of the conventional running couplings $a_i(\mu^2)$.

The above procedure suffices to give the clear-cut numerical criterion of the self-consistency of the finite-loop RG. But to visualise, let us modify Eq. (5) and define the new running couplings $a_i^{(\tilde{\Lambda})}(\mu^2)$ as follows

$$a_i^{(\tilde{\Lambda})}(\mu^2) = a_i(\mu_0^2) + \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2} \beta_i^{(\tilde{\Lambda})}(\mu'^2)\ , \qquad (6)$$

with the once subtracted β functions

$$\beta_i^{(\tilde{\Lambda})}(\mu^2) \equiv \beta_i(\mu_0^2) + \frac{1}{2\pi i} \int_C ds\ \beta_i(s) \left(\frac{1}{s - \mu^2} - \frac{1}{s - \mu_0{}^2} \right)\ . \qquad (7)$$

Here the point μ_0^2 is shifted infinitesimally inside C and $\beta_i(s)$, restricted to contour C, are obtained by integrating the RGE system (1) along the contour C itself. By the very construction, the modified couplings $a_i^{(\tilde{\Lambda})}(\mu^2)$ exactly coincide with $a_i(\mu^2)$ at $|\mu| < \tilde{\Lambda}$ if the integration contour does not encompass the complex singularities, i.e. $\tilde{\Lambda} < \Lambda_s$. Due to hermiticity, the couplings are real at the real negative μ^2. If the complex singularities get inside the contour, the procedure is not uniquely defined. In particular $a_i^{(\tilde{\Lambda})}(\mu^2)$ cease generally to be hermitian. To improve this, we redefine the integral in Eq. (7) as the contribution of the upper half of the contour C minus the contribution of the symmetric lower half of the contour calculated in the similar manner. This does not change the results at $\tilde{\Lambda} < \Lambda_s$. So defined $a_i^{(\tilde{\Lambda})}(\mu^2)$ are regular and hermitian and differ from $a_i(\mu^2)$ by the contribution of singularities and normalization constants. The constants are chosen so that $\beta_i^{(\tilde{\Lambda})}(\mu_0^2) \equiv \beta_i(\mu_0^2)$ and hence $a_i^{(\tilde{\Lambda})}(\mu^2) = a_i(\mu^2) + \mathcal{O}((\mu^2 - \mu_0^2)^2)$ in a vicinity of μ_0^2 where the finite-loop RG is believed to be reliable. The large difference between the couplings arises as soon as the singular parts of $a_i(\mu^2)$ become large.

3. Modification of the SM two-loop couplings

The SM ultraviolet behaviour has been extensively studied by the conventional RG method up to the two loops [3] – [5]. An important outcome of this study is the range of the Higgs

mass for which the SM remains perturbatively consistent up to the given cutoff scale Λ. The consistency can be broken either by the heavy enough Higgs, whose quartic coupling λ "blows up" at the scale Λ, or by the light Higgs, whose coupling λ dumps below zero at the scale Λ^1. Thus, quite a narrow corridor is retained for the Higgs mass (see, e.g., Fig. 4 of Ref. [3]). These bounds are of special interest because the Higgs mass remains the last undetermined SM parameter.

In two loops, the Higgs quartic coupling λ, as well as the other SM couplings, develops no singularities prior to the Landau singularity of the $U(1)_Y$ gauge coupling at $\Lambda \geq 0.2 \cdot 10^{41}$ GeV, the latter corresponding to the Higgs mass $M_H \geq 114.1$ GeV [3]. The situation is obscured by the fact that the SM two-loop RG equations can be solved only numerically. The numerical solution vs. real μ^2 provides no information about the analytic properties of the SM two-loop running couplings.

The method of analytic modification studies the evolution of the running couplings vs. complex μ^2. The variation of the modification radius $\tilde{\Lambda}$ (Fig. 1) allows one to determine the two-loop singularity scale Λ_s without finding the unphysical singularities explicitly. Thus one can judge about the self-consistency of the two-loop RG at the given energy scale μ. It is sufficient to calculate the modified couplings $a_i^{(\tilde{\Lambda})}(\mu^2)$ and compare them to the conventional ones Eq. (2). This enables one to determine the radius Λ_s at which the singularity is located, making the numerical analysis rather productive. If $\tilde{\Lambda} < \Lambda_s$, then the conventional and the modified SM running couplings are identical within the routine accuracy, $a_i(\mu^2) \equiv a_i^{(\tilde{\Lambda})}(\mu^2)$, $|\mu| < \tilde{\Lambda}$. As soon as $\tilde{\Lambda}$ exceeds Λ_s, the modified couplings depart from the respective conventional ones.
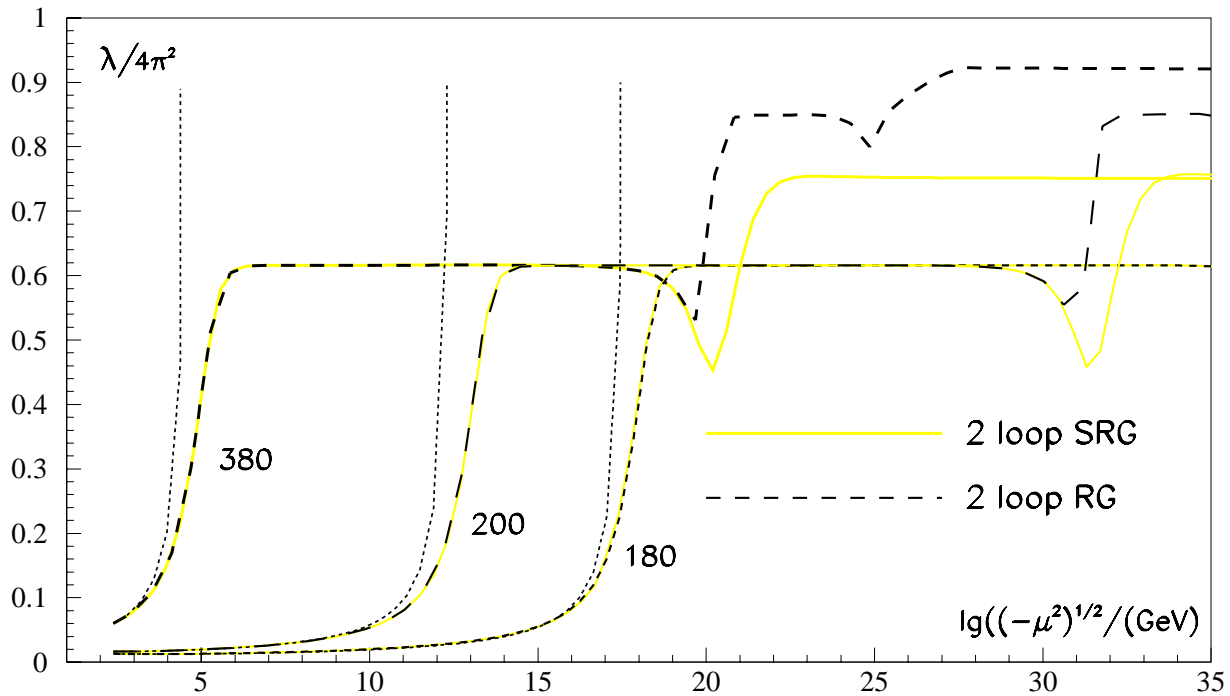
Fig. 2: The conventional (RG) and subtracted (SRG) two-loop running of the SM Higgs quartic coupling λ at M_H =180 GeV – 380 GeV. For comparison, the one-loop RG running of λ is shown by dots.

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[1]The upper and the lower bounds on the Higgs mass are also known in the literature as the triviality bound and the vacuum stability bound, respectively.

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To illustrate, consider the two-loop RG evolution of the SM with the maximally heavy Higgs, $M_H = 200$ GeV, nearly allowed by the electroweak precision data [2]. Varying the modification radius $\tilde{\Lambda}$ in the range 10^{19} GeV $< \tilde{\Lambda} < 10^{42}$ GeV^2, we find numerically the scale of the two-loop hidden singularity to be $\Lambda_s \simeq 10^{31}$ GeV. This can be seen from Fig. 2 showing the conventional (RG) and subtracted (SRG) two-loop running of the Higgs quartic coupling λ. Note that λ gets in fact rather large decrement, of 10% or so, after the integration contour crosses over the implicit singularities. For the lighter Higgs (not shown), λ stays actually unmodified[3]. Fig. 3 shows the conventional and modified two-loop evolution of the SM gauge couplings. In these figures, the modification radius is $\tilde{\Lambda} = 10^{42}$ GeV and $|\mu_0|$ is equal to the Higgs VEV, $v = 246.2$ GeV. The extension of $\tilde{\Lambda}$ even beyond the position of the Landau singularity results in the relative variation of the modified running couplings at the level of 10^{-3}. The case $M_H = 380$ GeV corresponds to $\Lambda_s = M_{Pl} = 10^{19}$ GeV. Also shown in Fig. 3 is the evolution of $\alpha_1(\mu^2)$ at $M_H \simeq 1.2$ TeV which corresponds to $\Lambda_s = M_{GUT} = 10^{14}$ GeV.
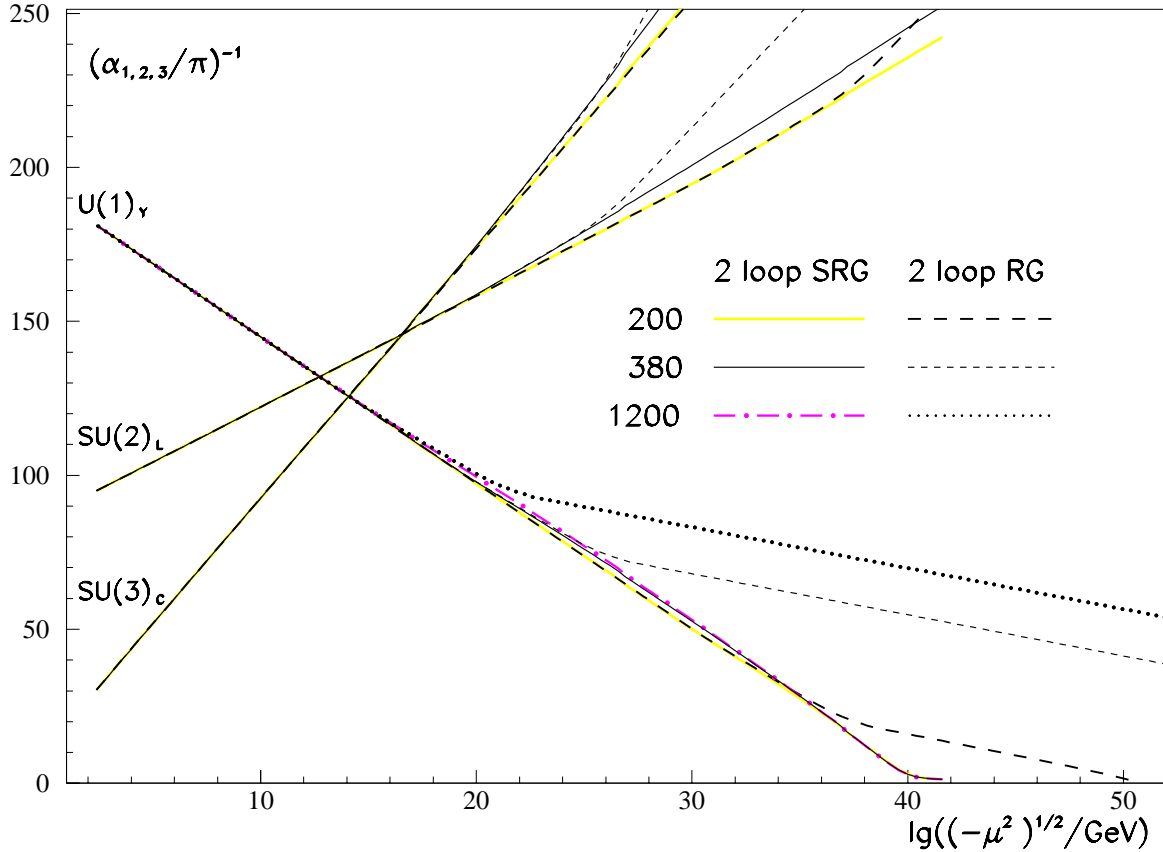
Fig. 3: The conventional (RG) and subtracted (SRG) two-loop running of the SM gauge couplings at $M_H = 200$ GeV and 380 GeV. $M_H = 380$ GeV corresponds to $\Lambda_s = M_{Pl}$. The running of the $U(1)_Y$ gauge coupling at $M_H \simeq 1.2$ TeV corresponding to $\Lambda_s = M_{GUT}$ is also shown.

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[2]I.e. well below the Landau singularity of the $U(1)_Y$ gauge coupling at $\Lambda_s \simeq 5 \cdot 10^{50}$ GeV for this M_H.

[3]For the 380 GeV Higgs, the modification of the t, b, and τ Yukawa couplings (not shown) cancels the unification of the latter ones [3] above the singularity scale Λ_s.

The upper Higgs bound. An important conclusion follows hereof. For the 200 GeV Higgs, all the SM couplings demonstrate very close conventional and modified two-loop running up to the two-loop singularity scale Λ_s. The Higgs mass $M_H = 200$ GeV spoils the analytic properties of the SM two-loop running couplings only at the scale $\Lambda_s \simeq 10^{31}$ GeV, i.e. well above the Planck scale. This can imply that to improve the analytic properties of the SM two-loop couplings, the contributions of the third and higher loops are needed only at scales $\mu > M_{Pl}$. To break down the perturbativity of the SM prior to the Planck scale $M_{Pl} = 10^{19}$ GeV the Higgs mass $M_H > 380$ GeV is required. This lifts up the commonly accepted upper bound on the Higgs mass $M_H \leq 180$ GeV derived in the conventional manner from the same requirement. Moreover, to guarantee the SM perturbativity up to the GUT scale, $M_{GUT} = 10^{14}$ GeV, it is not actually necessary to impose any upper bound on M_H. Thus the Higgs is light probably for reasons other than the absence of the strong coupling in the SM. These reasons might lie beyond the SM. E.g., the Higgs could be the composite pseudo-Goldstone boson having the natural mass $\sim M_Z$ [9].

To resolve the uncertainty of the Higgs upper bound the third and fourth loops in the SM are urgently needed. Two extreme possibilities can be envisaged. First, the higher loops are large and do not compensate each other. In this case, the conservative conventional upper bound $M_H < 180$ GeV at $\Lambda = M_{Pl}$ would follow. Second, the higher loops are either small, or large but mutually compensated. In this case, the more liberal modified upper bound is appropriate, and M_H up to 380 GeV would be allowed at the same Λ. More realistically, an intermediate case may realize so that the upper bound on M_H should lie somewhere in between 180 GeV and 380 GeV.

The lower Higgs bound. The low Higgs masses, $M_H \leq 138.1$ GeV[4], give rise to the electroweak vacuum instability prior to the Planck scale. However at the vacuum instability scale, the SM running couplings develop no singularities and hence require no subtractions. Thus the analytic modification method taken as it is cannot clarify the electroweak vacuum instability problem.

4. Conclusion

The subtracted RG is applied to study the two-loop self-consistency of the SM. It is found that at the Higgs mass $M_H < 380$ GeV, the two-loop singularity scale is $\Lambda_s > M_{Pl}$. This implies that $M_H < 380$ GeV does not necessarily threaten with the strong coupling prior to the Planck scale. Even allowing Λ_s as low as M_{GUT}, the SM self-consistency may actually impose no upper bound on M_H. In other words, the light Higgs might be preferred for reasons other than the SM perturbativity, i.e. for reasons beyond the SM. To clarify the issue the third and fourth loops in the SM RG are needed. On the other hand the method cannot resolve the SM vacuum instability problem arising, in two loops, at $M_H < 138.1$ GeV. Thus, out of the entire experimentally allowed range for the Higgs mass 114.1 GeV $< M_H < 194$ GeV, only the lowest Higgs masses 114.1 GeV $< M_H < 138.1$ GeV could definitely give rise to the SM inconsistency prior to the Planck scale and would require new physics.

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[4]This corresponds to the recalculated result of Ref. [3] for the central value 174.3 GeV [10] of the top mass.

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