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## A Simple Approach to Multiparticle Production

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**Abstract**

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A simple model which relates the energy dependence of mean charge multiplicity to the growth of the total cross-sections of strong interactions is presented. The model describes well the available data on  $e^+e^-$  annihilation and  $pp/\bar{p}p$  interactions.

**Аннотация**

Роинишвили В.Н. Простой подход к множественному рождению частиц: Препринт ИФВЭ 2004-31. – Протвино, 2004. – 6 с., 3 рис., библиогр.: 10.

Предлагается простая модель, связывающая энергетическую зависимость средней множественности заряженных частиц с ростом полных сечений сильных взаимодействий. Модель хорошо описывает существующие данные в  $e^+e^-$ -аннигиляциях и  $pp/\bar{p}p$ -взаимодействиях.

## 1. Introduction

The proposed model of multiparticle production is based on the assumption that the total time needed for all successive, in time, steps of an interaction to create  $n$  hadrons must be equal to the characteristic time of the strong interaction. It cannot be larger by definition and cannot be much less since in this case an additional step of the interaction producing hadrons has a chance to happen. Such approach leads to the condition

$$N_{\text{step}} \cdot \langle \tau \rangle = \tau_s, \quad (1)$$

where  $N_{\text{step}}$  is the number of the steps in the interaction,  $\langle \tau \rangle$  is a mean time needed for one step and  $\tau_s$  is the characteristic time of the strong interaction.

It follows from Eq.(1) that creation of large number of particles simultaneously is very improbable. Due to the conservation laws, each created particle has to interact somehow with all the others to share properly the available energy, momentum etc. before leaving the range of the strong interaction. In this case the number of sequential, in time, interactions  $N_{\text{step}}$  is proportional to the number of produced particles at least in the first power, and the condition (1) suppresses high multiplicity events very strongly.

More economical in time are consecutive ways of hadronization of the available energy, known as cascade models. A possible version of the cascade model which satisfies condition (1) is presented below.

## 2. The model of multiparticle production

In accordance with the cascade models (see e.g. Ref.[1]), we suppose that the condensed energy fragments into two objects (resonances, clusters, strings...) while conserving energy, charge, etc. via creating for example pairs of quarks. Each object then fragments again into two parts independently and so on, until the stable hadrons are formed. In the case of charged particle production  $N_{\text{step}}$  in Eq.(1) is the number of steps of the cascade process when new pairs of charged objects or particles are created. Then

$$n_{\text{ch}} = 2^{N_{\text{step}}} = 2^{\frac{\tau_s}{\langle \tau \rangle}} \quad (2)$$

where  $n_{ch}$  is the mean number of either directly produced charged hadrons or charged hadrons originating from the strong decays of objects with the lifetime not exceeding  $\tau_s$ .

What is known about  $\tau_s$  and  $\langle\tau\rangle$ ?

The value of  $\tau_s$  must be of the order of  $m_\pi^{-1}$  since pions are the lightest hadrons. However, the cross-sections of strong interactions grow with energy. According to the parameterisation used in Ref.[2], the rise of the total cross-sections at high energies, for all interactions in which secondary hadrons are produced, is described by the term:

$$\sigma_r(s) = B \cdot \log^2(s/s_0), \quad (3)$$

valid in the range of  $s > s_0$ . It is important to note, that the values of parameters  $B = 0.307$  mb and  $s_0 = 28.94$  GeV<sup>2</sup> are common for all interactions:  $pp$ ,  $p\bar{p}$ ,  $\pi p$ ,  $Kp$ ,  $\gamma p$ ,  $\gamma\gamma$ ... This means that the rise of the cross-sections with energy is not a property of the colliding particles but rather the property of the strong interactions. Since the characteristic time of interactions, as well as the radius of the range of interactions, is proportional to the square root of the cross-sections,  $\tau_s$  must depend on energy. We assume

$$\begin{aligned} \tau_s &= \frac{1}{m_\pi} + \frac{1}{\hbar c} \sqrt{\sigma_r(s)} \\ &= \tau_s^o \cdot \left[ 1 + \sqrt{\frac{\sigma_r(s)}{\sigma^0}} \right], \end{aligned} \quad (4)$$

where  $\tau_s^o = 1/m_\pi$ ,  $\sigma^0 = (\hbar c/m_\pi)^2$  and  $\sqrt{s}$  is the energy available for hadronization: the total energy in annihilation processes, the mass of a heavy particle decaying into hadrons, or the fraction of energy accessible for particle productions in  $pp$ ,  $p\bar{p}$ ,  $\pi p$ ... interactions.

The value of  $\langle\tau\rangle$ , which we expect to be of order of  $\Lambda_{QCD}^{-1}$ , is left as the only free parameter of the model.

In the following, Eq.(2) is applied to the experimental data on various processes.

### 3. Comparison of the model with the experimental data

#### 3.1. $e^+e^- \rightarrow h^\pm + X$

For  $e^+e^-$  annihilation, the data extracted from the COMPAS database system Ref.[3] were used with  $\sqrt{s} > 10$  GeV, in order to be far from the energy limit  $s_0$  for the validity of Eq.(4). Most of the data points include charged particles from  $K^0$  and  $\Lambda$  decays. Theories or models of strong interactions should predict the multiplicity of hadronically produced particles only, therefore the data were corrected for those decays using the measured yield of  $K^0$  and  $\Lambda$  at various energies [4]. This correction amounted to about 10%.

In fig.1 the experimental data for the mean charged multiplicity (hollow circles) are shown together with the results of the fit using Eq.(2) with one free parameter  $\langle\tau\rangle^{-1}$ . The quality of the fit is very good, with  $\chi^2/\text{n.d.f.} = 22.2/39$  for the obtained value of  $\langle\tau\rangle^{-1} = 0.344 \pm 0.001$  GeV.

It is worth mentioning that if in Eq.(4) we consider  $\tau_s^o$  (or equivalently  $m_\pi$ ) as an additional free parameter when fitting to the data, we get the same value for  $\langle\tau\rangle^{-1} = 0.344 \pm 0.003$  GeV, while for the other parameter we get  $m_\pi = 0.1385 \pm 0.0018$  GeV, which is very close to the table value of the pion mass.

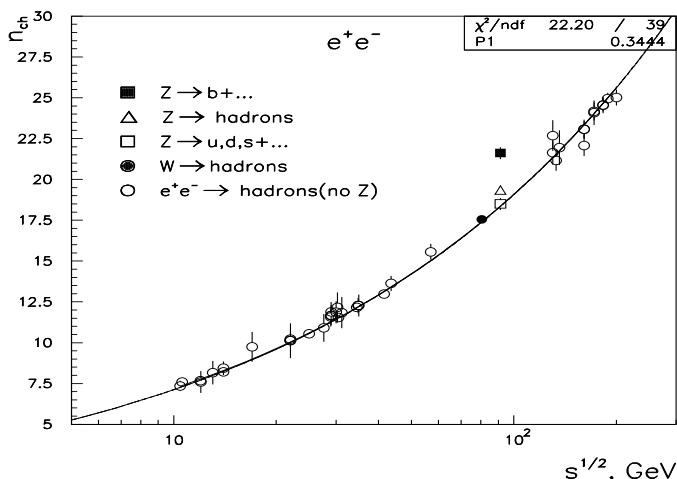


Figure 1. Energy dependence of the mean charge multiplicity for  $e^+e^-$  annihilation and hadronic decays of  $Z$  and  $W$ . The solid line represents the result of the fit to  $e^+e^- \rightarrow$  hadrons only (see text).

Also plotted are the data points for  $Z$  [5] and  $W$  [6] decays (corrected for  $K^0$  and  $\Lambda$  decays, as above):  $Z \rightarrow b\bar{b}$  (black square),  $Z \rightarrow h^\pm + X$  (triangle),  $Z \rightarrow u\bar{u}, d\bar{d}, s\bar{s}$  (hollow square) and  $W \rightarrow h^\pm + X$  (black circle). The mean multiplicities for the first two decays are higher than the values predicted by the model, because they include a large fraction of the particles from weak decays of  $b$  quarks (the branching ratio of  $Z \rightarrow b\bar{b}$  is about 15%). The measured values of the mean multiplicity for  $Z \rightarrow u\bar{u}, d\bar{d}, s\bar{s}$  and  $W$  decays are in very good agreement with the model. This indicates that the process of hadronization is the same for both  $e^+e^-$  annihilation and the decays of heavy particles.

### 3.2. $pp/p\bar{p} \rightarrow h^\pm + X$

For  $pp$  and  $p\bar{p}$  interactions the situation is more complicated because of the specific dynamics: there are quasi-elastic interactions and diffractive processes producing a small number of hadrons but carrying away the bulk of the total energy. The lump of energy that hadronises is not at rest in the center-of-mass system. Therefore only a fraction of energy

$$K \cdot \sqrt{s} \quad (5)$$

is available to create new particles where the so-called inelasticity  $K$  may itself depend on energy (see below).

#### 3.2.1. $pp$ and $p\bar{p}$ data

The highest energy data available for the mean multiplicity of charged particles is from  $p\bar{p}$  collisions at UA5 [7] which are for non-single-diffractive events (NSDE) and do not include particles from  $K^0$  and  $\Lambda$  decays. For lower energies, we used the data on NSDE in  $pp$  interactions at ISR [8], which also do not include particles from weak decays.

### 3.2.2. Inelasticity

Fig.2 shows the inelasticity  $K$  as a function of  $\sqrt{s}$  in  $pp/p\bar{p}$  interactions.  $K$  is defined as:

$$K = \frac{s_{e^+e^-}^{1/2}(n_{ch})}{s_{pp/p\bar{p}}^{1/2}(n_{ch})} \quad (6)$$

where  $s_{pp/p\bar{p}}^{1/2}(n_{ch})$  is the energy of the hadron colliders at which mean charge multiplicity is equal to  $n_{ch}$  and  $s_{e^+e^-}^{1/2}(n_{ch})$  is the energy of  $e^+e^-$  annihilation corresponding to the same multiplicity.

The black points refer to the multiplicity range where the data is available for both  $e^+e^-$  and  $pp/p\bar{p}$  collisions. For the hollow circles the energy of  $e^+e^-$  was calculated from Eq.(2). It's clear that for NSDE the inelasticity decreases with energy.

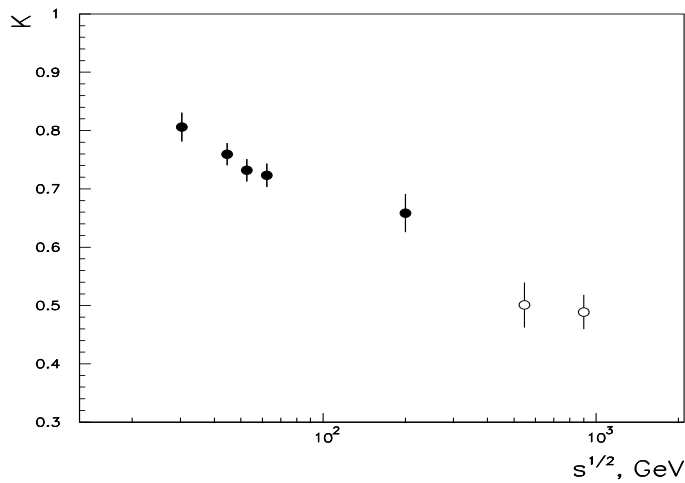


Figure 2. Inelasticity  $K$  as a function of CMS energy for non-single-diffractive  $pp/p\bar{p}$  interactions. The black points refer to the region of multiplicity where data for both  $e^+e^-$  and  $pp/p\bar{p}$  exist. For the hollow circles the equivalent energy of  $e^+e^-$  annihilation is defined using Eq.(2).

### 3.2.3. Fit to $pp$ and $p\bar{p}$ data

In fig.3 the experimental data on mean charge multiplicity in  $pp$  and  $p\bar{p}$  are plotted (hollow circles). The solid line corresponds to the result of the fit using Eq.(2), with  $\langle\tau\rangle$  fixed to the value  $\langle\tau\rangle^{-1} = 0.344$  GeV obtained from  $e^+e^-$  data, and  $s$  replaced by  $K^2 \cdot s$ . The inelasticity  $K$  was parameterised as proposed in Ref.[9]:

$$K = K_0 \times s^{-\epsilon}. \quad (7)$$

The fit for two free parameters  $K_0$  and  $\epsilon$  gives

$$K_0 = 1.34 \pm 0.09 \quad \text{and} \quad \epsilon = 0.074 \pm 0.008 \quad (8)$$

with  $\chi^2/\text{n.d.f.} = 2.8/5$ . The obtained value for  $\epsilon$  is not far from the range  $0 \leq \epsilon \leq 0.06$  mentioned in Ref.[9].

The black points in fig.3, corresponding to the Fermilab energies, are taken from Ref.[10]. They were not used during the fit because in Ref.[10] there is no mention of the associated errors. However, one can see that these data points are in a very good agreement with the predictions of our model, even if the errors are very small.

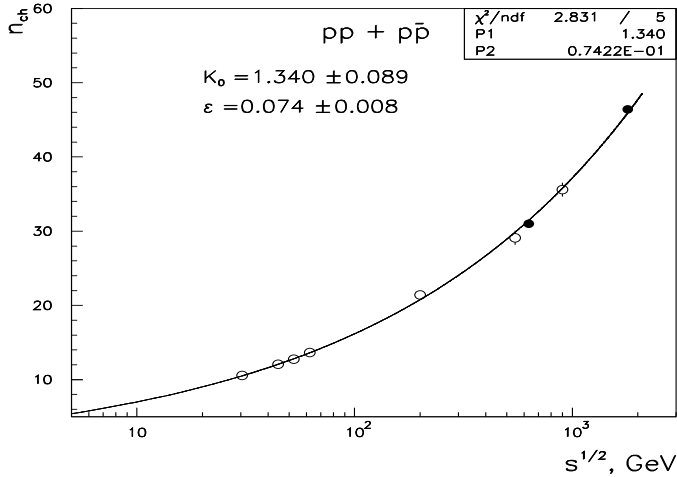


Figure 3. Energy dependence of the mean charge multiplicity for  $pp/pp\bar{p}$  collisions. Hollow circles represent ISR and UA5 data, dark points — Fermilab data from Ref.[10]. The solid line is the result of the fit to ISR and UA5 data only (see text).

## 4. Conclusions

Unless the good agreement of the presented model with the experimental data is purely accidental, one can conclude that

- the rise of the multiplicity of hadrons with energy is restricted by the condition that the total time needed to produce particles must be equal to the characteristic time of the strong interactions, Eq.(1);
- the rise of the multiplicity and the growth of the total cross-sections with energy may have the same origin;
- the mean time needed for a condensed energy to fragment into two parts (one step in the cascade model) is determined to be  $(0.344 \text{ GeV})^{-1}$ ;
- the process of hadronization of energy in  $e^+e^-$  annihilation and in the decays of heavy particles is the same. If the process of hadronization in  $e^+e^-$  annihilation and in  $pp$  and  $pp\bar{p}$  interactions is similar, then the inelasticity  $K$  for non-single-diffractive events in  $pp$  and  $pp\bar{p}$  collisions decreases with energy as  $s^{-0.07}$ .

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