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GENERAL COVARIANCE VIOLATION AND THE GRAVITATIONAL DARK MATTER. I. SCALAR GRAVITON

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Abstract

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The violation of the general covariance is proposed as a resource of the gravitational dark matter. The minimal violation of the covariance to the unimodular one is associated with the massive scalar graviton as the simplest representative of such a matter. The Lagrangian formalism for the continuous medium, the perfect fluid in particular, in the scalar graviton environment is developed. The implications for cosmology are shortly indicated.

Аннотация

Пирогов Ю.Ф. Нарушение общей ковариантности и гравитационная темная материя. I. Скалярный гравитон: Препринт ИФВЭ 2005–15. – Протвино, 2005. – 11 с., библиогр.: 11.

Нарушение общей ковариантности предлагается в качестве источника гравитационной темной материи. Минимальное нарушение этой ковариантности до унимодулярной ассоциируется с массивным скалярным гравитоном как простейшим представителем подобной материи. Развит лагранжев формализм для непрерывной среды и, в часности, идеальной жидкости в присутствии скалярных гравитонов. Кратко указаны следствия для космологии.

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1. Introduction

The General Relativity (GR) is the viable theory of gravity, perfectly consistent with observations. Nevertheless, it encounters a number of the conceptual problems among which one can mention that of the dark matter (DM). To solve the latter problem one adjusts usually the conventional or new matter particles, remaining still in the realm of the GR (for a recent review see, e.g., ref. [1]). The ultimate goal of the DM being in essence to participate only in the gravitational interactions, one can try and attribute to this purpose the gravity itself, going thus beyond the GR.

Namely, the GR is the field theory of the massless tensor graviton as a part of the metric. However, the metric contains also the extra degrees of freedom which could be associated with the massive scalar, vector and scalar-tensor gravitons. Nevertheless, the latter ones lack the explicit physics manifestations. This is due to the fact that the GR incorporates as the basic symmetry the general covariance (GC). The latter serves as the gauge symmetry to remove from the observables the degrees of freedom in excess of the massless tensor graviton. So, to associate the extra particles contained in the metric with (a part of) the DM the violation of the GC is obligatory.

Starting from the GR, an arbitrary variety of the GC violations is admissible. Contrary to this, a hierarchy of the GC violations is expected in the framework of the affine Goldstone approach to gravity [2]. In distinction with the GR, the latter approach is based on two symmetries: the global affine symmetry (AS) plus the GC. As a result, there are conceivable two types of the GC violations: those with and without the explicit AS violation. The first type of violation is to be strongly suppressed. But the second one is a priori arbitrary. In the reasonable assumptions, the GC violation can be parametrized in the given approach through the background metric [1].

Marginally, the latter description can depend only on the determinant of the background metric. In this case, there still resides the unimodular covariance (UC), i.e., that relative to the volume preserving diffeomorphisms. In comparison with the GC, the UC lacks only the local scale transformations. Being next-of-kin to the GC, the UC suffices to incorporate both the

[1] The basic ingredient of the approach is the spontaneous appearence of the metric in the affinely connected world. For the short exposition, see ref. [3]. For a related development, see ref. [4].

massive scalar graviton and the massless tensor one, but nothing redundant [2]. The violation of the GC to the residual UC is considered in the present paper to be the raison d'etre for the existence of the scalar graviton as a simplest representative of the gravitational DM. More elaborate dependencies on the background metric, with the residual covariance group being even more restricted, could correspond to the massive vector and scalar-tensor gravitons.

In the present paper, we confine ourselves by the scalar graviton, the vector and scalar-tensor ones being postponed to the future. In Sec. 2, the GC violation, in general, and, specifically, the scalar graviton are considered. The Lagrangian description of the continuous medium, the perfect fluid in particular, in the scalar graviton environment is developed in Sec. 3, with some remarks in Conclusion.

2. Gravity

Gravitational DM Consider the classical theory of the metric $g_{\mu\nu}$ and the generic matter field ϕ with the action

$$I = \int \Big(L_g(g_{\mu\nu}) + \Delta L_g(g_{\mu\nu}, \bar{\kappa}_p) + L_m(\phi, \bar{\kappa}_p) \Big) \sqrt{-g}\, d^4x. \tag{1}$$

Here L_g and ΔL_g are, respectively, the generally covariant and GC violating contributions of the gravity, L_m being the matter Lagrangian. In general, the latter violates the GC, too. All the Lagrangians above are assumed to be the scalars. The functions $\bar{\kappa}_p = \bar{\kappa}_p(x^\mu)$, $p = 1, \ldots$, with $\bar{\kappa}'_p(x'^\mu) \neq \bar{\kappa}_p(x^\mu)$ parametrize the GC violation [3]. The number and the transformation properties of these background parameter-functions relative to the change of the coordinates $x^\mu \to x'^\mu$ determine ultimately the group of the residual covariance. The parameter-functions are the generalization of the constant parameters and are to be found through observations. The space-time dependence of $\bar{\kappa}_p$ tacitly implies the existence of the distinguished coordinates relative to some background. This means that the metric Universe, contrary to what is assumed in the GR, is not a self-contained system and can not exclusively be described in the internal dynamical terms. Conceivably, the background parameter-functions will also be put on the dynamical footing in the course of time.

It is well known, that any generally covariant Lagrangian L_g corresponds to the massless tensor graviton (see, e.g., ref. [5]). The reason is that the GC removes in the equations of motion all the degrees of freedom contained in the metric but those with the helicities $\lambda = \pm 2$. To make the rest of the metric degrees of freedom to be observable, the addition of the GC violating ΔL_g is obligatory.

Varying the action (1) with respect to $g^{\mu\nu}$ one arrives at the gravity equation of motion:

$$G_{\mu\nu} + \Delta G_{\mu\nu} = M_P^{-2}\, T^{(m)}_{\mu\nu}. \tag{2}$$

Here $G_{\mu\nu}$ is the gravity tensor defined as follows:

$$-M_P^2 G_{\mu\nu} = \frac{2}{\sqrt{-g}} \frac{\delta \mathcal{L}_g}{\delta g^{\mu\nu}} = 2\, \frac{\delta L_g}{\delta g^{\mu\nu}} - L_g g_{\mu\nu}\ , \tag{3}$$

[2] In fact, it is the UC, not the GC, which ensures the masslessness of the tensor graviton [5]. In other contexts, the UC is studied in refs. [6] – [9].

[3] We do not consider the space-time dependence of the scalar parameters, such as the Newton's constant, etc.

with $\mathcal{L}_g = \sqrt{-g}\, L_g$ being the Lagrangian density (and similarly for $\Delta G_{\mu\nu}$). In the above, one puts conventionally $M_P^{-2} = 8\pi G_N$, with M_P being the Planck mass and G_N being the Newton's constant. $T^{(m)}_{\mu\nu}$ is the conventional energy-momentum tensor of the matter defined through L_m by the r.h.s. of eq. (3).

Due to the general covariance of L_g, one arrives at the identity:

$$\nabla_\nu G^{\mu\nu} = 0, \tag{4}$$

with ∇_ν being the covariant derivative defined by the metric $g_{\mu\nu}$. Thus with account for eq. (2), the modified conservation condition

$$\nabla_\nu (T^{\mu\nu}_m - M_P^2 \Delta G^{\mu\nu}) = 0 \tag{5}$$

is to be fulfilled. Clearly, the energy-momentum of the ordinary matter alone ceases to conserve. To restore the conventional interpretation of the conserved matter, one should consider the extra gravity degrees of freedom as (a part of) the DM with the energy-momentum tensor

$$\Delta T^{\mu\nu}_g = -M_P^2 \Delta G^{\mu\nu}. \tag{6}$$

The respective particles have no specific quantum numbers and possess only the gravitational interactions, so that such an association is quite natural.

In other terms, eq. (5) can be written as

$$\nabla_\nu T^{\mu\nu}_m = Q^\mu, \tag{7}$$

with the vector

$$Q^\mu = M_P^2 \nabla_\nu \Delta G^{\mu\nu} \tag{8}$$

representing the extra gravity force acting on the ordinary matter. This force reflects the non-fulfilment of the equivalence principle due to the GC violation. In the case that L_m does not depend on $\bar{\kappa}_p$ and thus is generally covariant, one gets with account for the matter equation of motion:

$$\nabla_\nu T^{\mu\nu}_m = Q^\mu = 0. \tag{9}$$

Scalar graviton In what follows, we restrict ourselves by the minimal violation of the GC with one parameter-function, the scalar density, and respectively with the scalar graviton alone as the simplest representative of the gravitational DM. Conventionally, take as L_g the modified Einstein-Hilbert Lagrangian:

$$L_g = -M_P^2 \Big(\frac{1}{2} R(g_{\mu\nu}) - \Lambda \Big), \tag{10}$$

with R being the Ricci scalar and Λ being the cosmological constant (if any). Eq. (10) describes the massless tensor graviton.

To incorporate additionally only the scalar graviton, the minimal GC violation with the residual UC is sufficient. To this end, use as the respective field variable the relative metric scale $e^\chi = \sqrt{-g}/\sqrt{-\bar{g}}$ or, otherwise,

$$\chi = \frac{1}{2} \ln \frac{g}{\bar{g}}. \tag{11}$$

Here one puts $g = \det g_{\mu\nu}$ and $\bar{g} = \det \bar{g}_{\mu\nu}$, with $g_{\mu\nu}$ and $\bar{g}_{\mu\nu}$ being the dynamical and background metrics, respectively. Being determined by the ratio of two scalar densities, χ itself is the scalar. In other terms, it can be expressed as follows:

$$\partial_\mu \chi = \Gamma^\lambda{}_{\lambda\mu} - \bar{\Gamma}^\lambda{}_{\lambda\mu}, \tag{12}$$

with $\Gamma^\lambda{}_{\nu\mu}$ and $\bar{\Gamma}^\lambda{}_{\nu\mu}$ being, respectively, the dynamical and background Christoffel connections. Clearly, the difference of the two transforms homogeneously under the change of the coordinates.

For the physics clarity, fix a coordinate system and put in this system by definition $g_{\mu\nu} \equiv (-\bar{g})^{1/4}(\eta_{\mu\nu} + h_{\mu\nu})$, with $\eta_{\mu\nu}$ being the Minkowski tensor. Then one gets in the linearized approximation $\chi \simeq 1/2\, \eta^{\mu\nu} h_{\mu\nu}$, in accord with treating χ as the scalar graviton. Note that knowing $\bar{g}$ in the observer's coordinates x^μ, one can always find the background-attached coordinates $\bar{x}^\mu$, where $\bar{g} = -1$.

Take the scalar graviton Lagrangian in the simplest form:

$$\Delta L_g = \frac{1}{2}\mu^2\, \partial\chi \cdot \partial\chi - V(\chi). \tag{13}$$

Here and in what follows, the notation $A \cdot B = A_\mu B^\mu$ is used for any two vectors. In the above, μ is a constant with the dimension of mass [4]. The variable $\mu\chi$ is the conventionally normalized field of the scalar graviton.

Due to dependence on $\bar{g}$, the GC is violated. Nevertheless, the Lagrangian possesses the residual UC, i.e., that relative to the variations of the coordinates $\delta x^\mu = \epsilon^\mu$ with the auxiliary condition $\partial \cdot \epsilon = 0$. This leaves $\bar{g}$ (in the line with g) unchanged. The derivative part of the Lagrangian above does not violate the AS [2], hence the constant μ is not a priori suppressed relative to M_P. The potential $V(\chi)$ ascribes mass to the scalar graviton. Violating the AS [2], the χ-dependent part of the potential is to be suppressed. As for the χ-independent part, one puts conveniently $V(0) = 0$ attributing the constant part to the Λ-term.

As for the matter field ϕ, the conventional generally covariant Lagrangian $L_m(D_\mu\phi, \phi)$, with $D_\mu\phi$ being the generally covariant derivative, should be substituted by the Lagrangian $L_m(D_\mu\phi, \phi, \partial_\mu\chi, \chi)$. Similarly to the gravity Lagrangian, the derivative χ-terms do not violate the AS, whereas those without derivative do violate it and thus should be suppressed.

Varying the total action with respect to $g^{\mu\nu}$ ($\bar{g}$ being unchanged under the variation) and taking into account the relations $\delta g_{\lambda\rho} = -g_{\lambda\mu} g_{\rho\nu}\, \delta g^{\mu\nu}$, $\delta\sqrt{-g} = -1/2\, \sqrt{-g} g_{\mu\nu}\delta g^{\mu\nu}$, so that $\delta\chi = -1/2\, g_{\mu\nu}\delta g^{\mu\nu}$, one arrives at the equation of motion for gravity (2), where

$$G_{\mu\nu} = R_{\mu\nu} - \frac{1}{2} R\, g_{\mu\nu} + \Lambda g_{\mu\nu} \tag{14}$$

and

$$\Delta G_{\mu\nu} = -M_P^{-2}\Big(\mu^2(\partial_\mu\chi\partial_\nu\chi - \frac{1}{2}\, \partial\chi \cdot \partial\chi\, g_{\mu\nu} + \Box\chi\, g_{\mu\nu}) + (V + \partial V/\partial\chi) g_{\mu\nu}\Big), \tag{15}$$

with $\Box$ being the covariant Dalambertian:

$$\Box \equiv \nabla \cdot \nabla = \frac{1}{\sqrt{-g}}\, \partial \cdot (\sqrt{-g}\, \partial). \tag{16}$$

[4] Were it not for $\mu \neq 0$, χ would become the auxiliary nonpropagating field. A priori, the mass squared of the scalar graviton may be negative, with χ corresponding to the tachion instead of the physical particle. One could envisage also $\mu^2 < 0$, with χ corresponding to the ghost.

Thereof one gets for the scalar gravity force, eq. (8),

$$Q_\mu = -\mu^2(\Box\chi\,\partial_\mu\chi + \partial_\mu\Box\chi) - \partial_\mu(V + \partial V/\partial\chi). \tag{17}$$

>From the gravity equation of motion (2), there follows for the scalar curvature:

$$R = 4\Lambda + M_P^{-2}\Big(\mu^2(\partial\chi\cdot\partial\chi - 4\Box\chi) - 4(V + \partial V/\partial\chi) - T_{m\mu}^{\ \ \mu}\Big). \tag{18}$$

Contrary to the GR, R can be nonzero even in the absence of the Λ-term and matter. Conceivably, the gravitational DM alone, the scalar graviton in particular, suffices to form the (quasi)-stable gravitational configurations.

3. Continuous medium

Distorted medium Having in mind the application to cosmology, choose as the matter the continuous medium. Such a macroscopic approach is to be applicable at the sufficiently long length scales compared to the mean free path of the matter particles, with the quantum coherence effects being averaged. Conventionally, one restricts himself by the description of the continuous medium at the level of the equations of motion directly in terms of the energy-momentum tensor. This, in principle, suffices under the validity of the GC. To comprehend the new effects produced the scalar graviton one has to adhere to the Lagrangian framework.

The mathematically strict Lagrangian description of the continuous medium proceeds in terms of the comoving coordinates z^a and $z^a_\mu \equiv \partial_\mu z^a$, $a = 0, \ldots, 3$, as the Lagrangian variables (see, e.g., ref. [10]). z^0 is closely related with the proper time for a medium element, so that $\partial/\partial z^0 \sim U\cdot\partial$, with U^μ being the medium 4-velocity. z^a, $a = 1, 2, 3$, are the fields which label the streamlines. Using these fields, one can automatically account for the continuity condition, the latter being the generic property of the continuous media. Inasmuch as such a consistent description is possible, we develop in what follows the more straightforward formalism directly in terms of the observer's x^μ coordinates.

To describe the medium at the Lagrangian level, we choose, in addition to U^μ, $U\cdot U = 1$, the proper (i.e., measured in the comoving coordinates) concentration n of the medium particles and the specific entropy σ (the entropy per particle). Equivalently, one could chose as the pair of the independent dynamical variables the entropy $s = n\sigma$ per unit proper volume and σ. Besides, the medium is characterized by the nondynamical parameters such as the particle mass m, etc. The medium has to satisfy the continuity condition for the particle number current $N^\mu = nU^\mu$

$$\nabla\cdot N = \frac{1}{\sqrt{-g}}\,\partial\cdot(\sqrt{-g}N) = 0, \tag{19}$$

This constraint has to be valid identically, independently of the equations of motion.

Take the Lagrangian for the continuous medium generically as

$$L_m(U^\mu, n, \sigma) = -E(|N|, \sigma), \tag{20}$$

with $E(|N|, \sigma)$ being the scalar energy function. Here one puts $|N| = (N\cdot N)^{1/2}$. One can also add the derivative interactions of the scalar graviton with the medium as follows:

$$L'_m(U^\mu, n, \sigma) = -F(|N|, \sigma)\frac{1}{|N|}\,N\cdot\partial\chi, \tag{21}$$

with $F(|N|,\sigma)$ being the scalar formfactor. Violating the AS, the derivativeless χ-dependence is supposed to be absent [5].

The continuity condition (19) is clearly equivalent to the ordinary conservation of the vector density $\mathcal{N}^\mu = \sqrt{-g}\, N^\mu$, $\partial\cdot\mathcal{N} = 0$, which does not include the metric. Not to vary the continuity condition under the change of the metric, it is $\mathcal{N}^\mu$, not N^μ, to be taken as the independent variable. Introducing the scalar density $|\mathcal{N}| = (\mathcal{N}\cdot\mathcal{N})^{1/2} = \sqrt{-g}\,|N|$, wright finally the total effective Lagrangian density as follows:

$$\mathcal{L}_m^{(tot)} = -\sqrt{-g}\Big(E\Big(\frac{|\mathcal{N}|}{\sqrt{-g}}\,,\sigma\Big) + F\Big(\frac{|\mathcal{N}|}{\sqrt{-g}}\,,\sigma\Big)\frac{1}{|\mathcal{N}|}\,\mathcal{N}\cdot\partial\chi\Big) + \lambda\,\partial\cdot\mathcal{N}. \tag{22}$$

where λ is the Lagrange's multiplier.

Varying eq. (22) with respect to λ one reproduces the constraint eq. (19). Further, varying $\mathcal{L}_m^{(tot)}$ relative to $g^{\mu\nu}$ one gets the energy-momentum tensor:

$$T_{\mu\nu}^{(m)} = (\rho+p)U_\mu U_\nu - p\,g_{\mu\nu} - f(U_\mu\partial_\nu\chi + U_\nu\partial_\mu\chi), \tag{23}$$

with

$$\begin{aligned} \rho &= e + fU\cdot\partial\chi + \nabla\cdot(fU), \\ p &= ne' - e + (nf' - f)U\cdot\partial\chi - \nabla\cdot(fU), \end{aligned} \tag{24}$$

where $e(n,\sigma) \equiv E(|N|,\sigma)|_{|N|=n}$, $e' = \partial e(n,\sigma)/\partial n$ and similarly for f. For the trace of the energy-momentum tensor one has

$$T_m{}_\mu^\mu = \rho - 3p - 2fU\cdot\partial\chi. \tag{25}$$

In the above, ρ is the scalar coinciding with the energy per unit proper volume, p is the scalar coinciding with the (isotropic) pressure, while f is the new scalar state function. The terms proportional to f distort the medium. Being of the odd degree in velocity these terms reflect the dissipation (pumping) caused by the environment of the scalar gravitons. As a result, there appears in $T_{\mu\nu}^{(m)}$ one more independent tensor structure

$$\theta_{\mu\nu} = U_\mu\partial_\nu\chi + U_\nu\partial_\mu\chi \tag{26}$$

accounting, in particular, for the anisotropy of the medium in the preferred direction $\partial_\mu\chi$.

The dynamical variables U^μ, n and σ have to satisfy the modified conservation condition eq. (5) and the continuity condition eq. (19). This gives five equations for the same number of the independent variables. Supplemented by the ten gravity equations of motion (2) for the metric $g_{\mu\nu}$ and thus χ, this presents the complete system of the dynamical equations.

Projecting eq. (5) on the streamlines by multiplying on U_μ and accounting for $U\cdot\nabla_\nu U = 0$, one gets the energy equation in the external field:

$$\nabla\cdot\Big((\rho+p)U\Big) - U\cdot\partial p - U_\mu\nabla_\nu(f\theta^{\mu\nu}) = U\cdot Q. \tag{27}$$

[5] The terms quadratic in $\partial_\mu\chi$, being treated as the corrections to the Lagrangian ΔL_g, are neglected in the first approximation.

The scalar $q = U \cdot Q$ on the r.h.s. of the equation above is nothing but the power Q_0 dissipating (depositing) in the medium per unit proper volume. Likewise, restricting eq. (7) by the projector $P^{\mu\nu} = g^{\mu\nu} - U^\mu U^\nu$, $P^{\mu\nu} U_\nu = 0$, on the hypersurface orthogonal to the streamlines, one gets the modified Euler equation:

$$(\rho + p) U \cdot \nabla U_\mu + U \cdot \partial p \, U_\mu - \partial_\mu p - P_{\mu\nu} \nabla_\lambda (f \theta^{\nu\lambda}) = Q_\mu - U \cdot Q \, U_\mu. \tag{28}$$

When all the terms but the one proportional to ρ are missing, eq. (28) is nothing but the geodesic condition: $U \cdot \nabla U_\mu = 0$. Otherwise, it describes the deviation of the flow from the geodesics due to the pressure and the influence of the scalar gravitons.

Undistorted medium Restrict ourselves in what follows by the undistorted medium, $F = 0$. In this case due to the GC of L_m, there should be fulfilled eq. (9). The medium is described as usual by the two state functions: ρ and p. According to the second thermodynamic law one has now

$$kT d(sv) = d(\rho v) + p \, dv, \tag{29}$$

the equality sign referring to the reversible processes. In the above, v is the proper volume element, T is the temperature and k is the Boltzmann constant. In particular, applied to the one-particle volume, $1/n$, eq. (29) reads

$$kT d\sigma = d\frac{\rho}{n} + p \, d\frac{1}{n}, \tag{30}$$

which means

$$\begin{aligned} p &= n \frac{\partial \rho}{\partial n} - \rho, \\ kT &= \frac{1}{n} \frac{\partial \rho}{\partial \sigma}. \end{aligned} \tag{31}$$

The first part of the equation above coincides with eq. (24) at $f = 0$, the second part being nothing but the thermodynamic definition of the temperature.

>From eq. (29), one gets otherwise

$$kT \frac{d(sv)}{d\tau} = \frac{d((\rho + p)v)}{d\tau} - v \, \frac{dp}{d\tau} \, , \tag{32}$$

with τ being the proper time. Thereof, with account for the relations (see, e.g., ref. [11])

$$\begin{aligned} \frac{dv}{d\tau} &= v \nabla \cdot U, \\ \frac{du}{d\tau} &= U \cdot \partial u, \end{aligned} \tag{33}$$

valid for the proper volume v and an arbitrary scalar u, one presents the energy equation (27) as that for the entropy conservation:

$$\nabla \cdot S = 0 \ , \tag{34}$$

with $S^\mu = \sigma N^\mu$ being the entropy current. This reflects the adiabaticity of the flow in the absence of the external force Q^μ. With account for the continuity condition, eq. (34) expresses the isentropy along the streamlines:

$$U \cdot \partial\sigma = 0. \tag{35}$$

Eqs. (19), (28) and (35), augmented by the equations of state $\rho = \rho(n, \sigma)$ and $p = p(n, \sigma)$, present the five equations of motion for the five dynamical variables U^μ, n and σ.

The preceding formalism can be extended to the multicomponent medium consisting of the fractions $i = 1, \ldots, I$. In this case, the medium depends generally on the $(2I+3)$, $I \geq 1$, dynamical variables U^μ, n_i and σ_i. Each one of the medium components has to satisfy the continuity condition eq. (19) for the partial number currents $N_i^\mu = n_i U^\mu$. For the reversible processes, one can impose additionally the $(I-1)$ thermodynamic equilibrium conditions expressing the equality of all the partial temperatures T_i to the common temperature T:

$$T_i = T. \tag{36}$$

The temperatures T_i are to be defined by the counterpart of eq. (31). The metric $g_{\mu\nu}$ as well as χ being given, this, together with the four conservation equations (9), constitutes $(2I+3)$, $I \geq 1$ equations for the same number of the independent variables.

Perfect fluid Consider one of the medium components, index i being omitted. The function $e(s, \sigma)$ being given, one derives by eq. (24) the equations of state $\rho = \rho(n, \sigma)$ and $p = p(n, \sigma)$. Thereof, one can get the equation of state $p = p(n, \rho)$ to be used in cosmology. And v.v., imposing on the physical ground the last equation and some thermodynamic relations one can, generally, restore the respective function $e(s, \sigma)$.

To this end, impose the equation of state in the conventional form of the proportionality between the pressure and the excitation energy per the unit proper volume:

$$p = w(\rho - mn), \tag{37}$$

with w being a constant. Put without loss of generality

$$e = (m + \epsilon)n, \tag{38}$$

with $\epsilon(n, \sigma)$ being a function to be found. With account for eq. (31), this means

$$\begin{aligned} p &= n^2 \frac{\partial \epsilon}{\partial n}, \\ kT &= \frac{\partial \epsilon}{\partial \sigma}. \end{aligned} \tag{39}$$

As the general solution to the differential equation ensuing from eq. (37), one gets

$$\epsilon = \varepsilon n^w, \tag{40}$$

with $\varepsilon = \varepsilon(\sigma)$ being an integration constant.

Identifying conventionally for the perfect fluid

$$\begin{aligned} \rho &= (m + c_v kT)n, \\ p &= kTn, \end{aligned} \tag{41}$$

with c_v being the specific heat, one gets

$$kT = w\varepsilon n^w, \;\; w = c_v^{-1}. \tag{42}$$

On the other hand, from the second part of eq. (39) there follows

$$kT = \frac{\partial \varepsilon}{\partial \sigma} n^w. \tag{43}$$

Combining the last two expression one gets

$$\varepsilon = Ce^{w\sigma}, \tag{44}$$

with C being an integration constant. To find the latter constant, one should match, at a critical temperature T_c, the relations obtained for the fluid with those for the crystal phase setting in the vicinity of $T = 0$. This should ensure $T(n, \sigma)|_{\sigma=0} = 0$ in accord with the third thermodynamic law: $s(n, T)|_{T=0} = 0$.

Altogether, one arrives at the sought energy function:

$$e = mn + Ce^{w\sigma} n^\gamma, \tag{45}$$

with

$$\gamma = 1 + w = \frac{c_p}{c_v} \tag{46}$$

and $c_p = 1 + c_v$. Remind, that the specific heat c_v is related with the number ν of the internal degrees of freedom of the fluid as $c_v = \nu/2$. In this, one has $\nu = 3$ for the monatomic fluid at any temperature, as well as for the cold nonrelativistic two- and multiatomic fluids. For the hot ultrarelativistic fluids, one has in the last two cases, respectively, $\nu = 5$ and $\nu = 6$. Thus for the real fluids, one has $1/3 \leq w \leq 2/3$. For the multiatomic fluids, the equations above are be substituted by the picewise-smooth expressions due to the step-like dependence $c_v(T)$.

Radiation The radiation ($m = 0$) can not be described consistently by the series eq. (45). The reason is that the conserved current N^μ and thus the continuity condition are missing in this case. To balance between the number of the equations of motion and the number of the independent dynamical variables, there should be left just one scalar variable. For consistency, one chooses $s = n\sigma$, the Lagrangian being in this case independent of n and σ apart. Now the argument about the choice of the Lagrangian set for eq. (22) does not apply. Nevertheless, one can retain the same procedure treating the radiation as the marginal case with $E = E(|\mathcal{S}|/\sqrt{-g}\,)$, $\mathcal{S}^\mu$ being the entropy current density. From the requirement $T_{m\mu}^{\mu} = 0$, there follows immediately the equation of state $p = w\rho$, $w = 1/3$. With account for eq. (24) at $f = 0$ and the relation $n(\partial e/\partial n)_\sigma = s(\partial e/\partial s)_\sigma$ (prior to cast σ aside), one gets the energy function

$$e = Cs^\gamma, \tag{47}$$

with C being an integration constant and $\gamma = 1 + w = 4/3$ ($\nu = 6$, $c_v = 3$). The energy function e and thus ρ for the radiation having been found, one can choose according to eq. (29) the temperature T as the independent variable conjugate to s:

$$kT = \frac{\partial \rho}{\partial s} = (1 + w)Cs^w, \tag{48}$$

so that $s \sim (kT)^3$, with $s(T)|_{T=0} = 0$. Thus, one gets in accord with the Stefan-Boltzman law: $\rho = 3p \sim T^4$.

Vacuum In the same vein, considering the vacuum formally as the perfect fluid with the equation of state $p = -\rho$ ($m = 0$, $\gamma = 0$), one gets $E = E_0$, the constant independent of n and σ. Note that there corresponds to the vacuum the specific heat $c_v = -1$ and thus, formally, the number of the internal degrees of freedom $\nu = -2$.

4. Conclusion

In conclusion, the classical theory of gravity with the residual UC is developed. In excess of the massless tensor graviton, the theory describes the massive scalar one as a part of the metric. Violating the GC, the theory presents a viable alternative to the GR, as well as to the generally covariant extensions thereof. The scalar graviton is to be associated naturally with (a part of) the DM. The Lagrangian formalism for the continuos medium interacting with the scalar gravitons is developed. The effects due to the scalar gravitons of possible interest in cosmology are mainly twofold: the modification of the gravity and medium equations of motion, the distortion of the medium. These effects are typical for any gravitational DM. The case study for the massive vector and scalar-tensor gravitons is expected to be given in the future.

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