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ON THE INTERNAL SOLUTION OF THE SCHWARZSCHILD TYPE IN THE FIELD THEORY OF GRAVITATION

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Abstract

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It is shown that the internal solution of the Schwarzschild type in the Relativistic Theory of Gravitation does not lead to an infinite pressure inside a body as it holds in the General Theory of Relativity. This happens due to the graviton rest mass, because of the stopping of the time slowing down.

Аннотация

Герштейн С.С., Логунов А.А., Мествиришвили М.А. О внутреннем решении типа Шварцшильда в полевой теории гравитации: Препринт ИФВЭ 2005–29. – Протвино, 2005. – 6 с., библиогр.: 6.

В статье показано, что внутреннее решение типа Шварцшильда в полевой теории гравитации не приводит к *бесконечному давлению* внутри тела, как это имеет место в общей теории относительности. Это происходит благодаря массе покоя гравитона из-за остановки замедления хода времени.

K. Schwarzschild found in papers [1, 2] a spherically symmetric static solution (internal and external) of the general relativity (GR) equations. The external solution is widely known and has the following form:

$$ds^2 = c^2\Big(1 - \frac{W_g}{W}\Big)dt^2 - \Big(1 - \frac{W_g}{W}\Big)^{-1} dW^2 + W^2(d\theta^2 + \sin^2\theta\, d\phi^2)\,, \quad (1)$$

where $W_g = (2GM)/c^2$ is the Schwarzschild radius.

The internal Schwarzschild solution for a **homogeneous ball** of the radius a is described by the interval:

$$ds^2 = c^2\Big(\frac{3}{2}\sqrt{1-qa^2} - \frac{1}{2}\sqrt{1-qW^2}\Big)^2 dt^2 - (1-qW^2)^{-1}dW^2 + W^2(d\theta^2 + \sin^2\theta\, d\phi^2)\,, \quad (2)$$

where $q = (1/3)\varkappa\rho = (2GM)/(c^2a^3)$, $\varkappa = (8\pi G)/c^2$, $\rho = (3M)/(4\pi a^3)$.

The general property of internal and external solutions is manifested in the fact that at a certain value of W the metric coefficients in front of dt^2 in intervals (1) and (2) vanish. Vanishing of this metric coefficient, which we will denote as U, means that the gravitational field acts in such a way that it not only slows down the run of time, but is able even to stop this run. For the external solution the vanishing of the metric coefficient U occurs at $W = W_g$.

To exclude such a possibility (not forbidden by the theory) one has to assume that the radius of the body obeys to the inequality

$$a > W_g\,. \quad (3)$$

For the internal solution this occurs at

$$W^2 = 9a^2 - 8(a^3/W_g)\,. \quad (4)$$

To exclude such a possibility of the vanishing of the metric coefficient U inside the body one has to assume that

$$a > (9/8)W_g\,. \quad (5)$$

We have to emphasize that inequalities (3) and (5) are not a consequence of GR.

The internal Schwarzschild solution is somewhat formal and is interesting first of all because it is an exact solution of the GR equations. In papers [3, 4] it was shown, taking as an example the external Schwarzschild solution that in the Relativistic Theory of Gravitation (RTG), as a field theory, inequality (3) arises due to an effective repulsive force, which is stipulated by the stop of the time slowing down, which, in its turn, is caused by the graviton rest mass. Below we consider, in the framework of the RTG, the internal solution of the Schwarzschild type. The internal Schwarzschild solution is the solution of the Hilbert–Einstein equations

$$1-\frac{d}{dW}\Big[\frac{W}{V}\Big]=\varkappa W^2\rho\,,$$

$$1-\frac{1}{V}-\frac{W}{UV}\frac{dU}{dW}=-\varkappa\frac{W^2}{c^2}p\,. \tag{6}$$

According to (2) the metric coefficients in front of dt^2 and dW^2 are, respectively,

$$U=\Big(\frac{3}{2}\sqrt{1-qa^2}-\frac{1}{2}\sqrt{1-qW^2}\Big)^2,\quad V=(1-qW^2)^{-1}. \tag{7}$$

We find there of

$$\frac{\acute{U}}{U}=\frac{qW}{\sqrt{1-qW^2}\Big(\frac{3}{2}\sqrt{1-qa^2}-\frac{1}{2}\sqrt{1-qW^2}\Big)}\,,\quad \acute{U}=\frac{dU}{dW}\,. \tag{8}$$

Substituting (7) and (8) into equation (6), we obtain the pressure

$$\frac{p}{c^2}=\frac{\rho}{2}\frac{(\sqrt{1-qW^2}-\sqrt{1-qa^2})}{\sqrt{U}}\,. \tag{9}$$

It is seen from this, in particular, that if equation (4) would not be excluded then the pressure inside the body would be infinite on the sphere defined by this equation. The singularity that arises due to the vanishing of the metric coefficient U cannot be eliminated by the choice of the coordinate system, because the scalar curvature R also possesses it:

$$R=-8\pi G\left[\frac{3\sqrt{1-qa^2}-2\sqrt{1-qW^2}}{\sqrt{U}}\right]. \tag{10}$$

Let us show now that in the RTG, when dealing with solutions of the Schwarzschild type, the situation is drastically different due to the repulsion force, which arises because of the stop in slowing down of the time.

The same mechanism of the "selflimitation of the field", which led, in the RTG [3–4], to inequality (3) for the external Schwarzschild solution, leads to inequality of the type (5) for the internal Schwarzschild solution. RTG equations for the metric defined by the interval

$$ds^2=c^2U(W)dt^2-V(W)\acute{r}^2dW^2-W^2(d\theta^2+\sin^2\theta\,d\phi^2)\,, \tag{11}$$

(here $\acute{r}=dr/dW$) assume the form [5, 6]:

$$1-\frac{d}{daw}\Big[\frac{W}{V\acute{r}^2}\Big]+\frac{1}{2}\Big(\frac{m_gc}{\hbar}\Big)^2\Big[W^2-r^2+\frac{W^2}{2}\Big(\frac{1}{U}-\frac{1}{V}\Big)\Big]=\varkappa W^2\rho\,, \tag{12}$$

$$1-\frac{1}{V\acute{r}^2}-\frac{W}{UV\acute{r}^2}\acute{U}+\frac{1}{2}\Big(\frac{m_gc}{\hbar}\Big)^2\Big[W^2-r^2-\frac{W^2}{2}\Big(\frac{1}{U}-\frac{1}{V}\Big)\Big]=-\varkappa W^2\frac{p}{c^2}\,, \tag{13}$$

$$\frac{d}{dW}\left[\sqrt{\frac{U}{V}}\,W^2\right] = 2r\sqrt{UV}\,\acute{r}\,.$$

Introducing a new variable $Z = (UW^2)/(V\acute{r}^2)$ and adding Eqs. (12) and (13)we obtain:

$$1 - \frac{1}{2UW}\overset{\prime}{Z} + \frac{m^2}{2}(W^2 - r^2) = \frac{1}{2}\varkappa W^2\Big(\rho - \frac{p}{c^2}\Big)\,. \tag{14}$$

Subtracting (13) from Eq. (12)we find

$$\overset{\prime}{Z} - 2Z\frac{\overset{\prime}{U}}{U} - 2\frac{Z}{W} - \frac{m^2}{2}W^3\Big(1 - \frac{U}{V}\Big) = -\varkappa W^3\Big(\rho + \frac{p}{c^2}\Big)U\,, \tag{15}$$

where $m = (m_g c)/\hbar$.

In our problem the components of the energy-momentum tensor of matter are

$$T_0^0 = \rho, \quad T_1^1 = T_2^2 = T_3^3 = -\frac{p(W)}{c^2}\,.$$

The equation of matter

$$\nabla_\nu(\sqrt{-g}\,T_\mu^\nu) = \partial_\nu(\sqrt{-g}\,T_\mu^\nu) + \frac{1}{2}\sqrt{-g}\,T_{\sigma\nu}\partial_\mu g^{\sigma\nu} = 0$$

for the given problem reduces to the following form

$$\frac{1}{c^2}\frac{dp}{dW} = -\Big(\rho + \frac{p}{c^2}\Big)\frac{1}{2U}\frac{dU}{dW}\,. \tag{16}$$

As the pressure grows towards the center of the ball, this leads to the inequality

$$\frac{dU}{dW} > 0\,, \tag{17}$$

which means that the function U decreases towards the center of the ball, and hence the run of time slows down in compare with that of an inertial system.

Due to the *constancy* of the pressure, ρ, (16) is readily solved:

$$\rho + \frac{p}{c^2} = \frac{\alpha}{\sqrt{U}}\,. \tag{18}$$

Comparing (9) and (18) one finds the constant α

$$\alpha = \rho\sqrt{1 - qa^2}\,. \tag{19}$$

If we assume that

$$m^2(W^2 - r^2) \ll 1, \quad (U/V) \ll 1\,,$$

and introduce a new variable $y = W^2$, then (14) and (15) take the form:

$$\overset{\prime}{Z} = U(1 - 3qy) + \frac{\alpha\varkappa}{2}y\sqrt{U}\,, \tag{20}$$

$$\sqrt{U}\overset{\prime}{Z} - \frac{1}{y}Z\sqrt{U} - 4Z(\sqrt{U})' + \frac{\alpha\varkappa}{2}yU - \frac{m^2}{4}y\sqrt{U} = 0\,. \tag{21}$$

Here and below $\overset{\prime}{Z} = dZ/dy$.

When analysing the external spherically symmetric Schwarzschild solution [3, 4] we have found that due to an effective repulsive force the metric coefficient U that defined the time run slowing down in compare with an inertial run, did not vanish even in a strong gravitational field.

That is why in what follows we will investigate the behaviour of the solution to this equations at small values of y. If the graviton mass is zero then it follows from (7) for small y that

$$\sqrt{U} \simeq \frac{1}{2}(3\sqrt{1-qa^2} - 1) + \frac{qy}{4} + \frac{1}{16}q^2y^2\,. \tag{22}$$

From this one can see, that the function $\sqrt{U}$ for the internal Schwarzschild solution can vanish if

$$3\sqrt{1-qa^2} = 1\,, \tag{23}$$

what leads to the infinite value both of the pressure p and the scalar curvature R at the center of the ball.

As for the non-zero graviton rest mass Eqs. (20)–(21) stop the time-slowing-down process, one can naturally expect that inequality (23) cannot take place in the physical (real) domain of values of the function $\sqrt{U}$. We will search, on the basis of (22), for a solution to equations (20)–(21) $\sqrt{U}$ in the form

$$\sqrt{U} = \beta + \frac{qy}{4} + \frac{1}{16}q^2y^2\,, \tag{24}$$

where β is an unknown constant, which has to be defined making use of Eqs.(20)–(21). Substituting expression (24) and (25), and integrating, we find

$$Z = \beta^2 y + \frac{y^2}{2}\Big(\frac{\beta q}{2} - 3\beta^2 q + \frac{\alpha\varkappa\beta}{2}\Big) + \frac{y^3}{3}\Big[\frac{q^2}{8}\Big(\beta + \frac{1}{2}\Big) - \frac{3\beta}{2}q^2 + \frac{\alpha\varkappa q}{8}\Big]\,. \tag{25}$$

Taking into expressions (24) and (25), and neglecting small terms of order $(my)^2$, we obtain the following equation for β

$$2\beta^2 q + \beta(q - \alpha\varkappa) + m^2/3 = 0\,. \tag{26}$$

It is instructive to note that the term containing y^2 has the following form:

$$-\frac{qy^2}{48}\big\{7[2\beta^2 q + \beta\,(q - \alpha\varkappa)] + 3m^2\big\}\,.$$

One can, with the use of Eq.(26), reduce it to

$$-\frac{q}{72}m^2y^2\,.$$

Taking into consideration that, by definition,

$$\alpha\varkappa - q = \frac{\varkappa\rho}{3}\big(3\sqrt{1-qa^2} - 1\big)\,,$$

we find from Eq.(26)

$$\beta = \frac{3\sqrt{1-qa^2} - 1 + \Big[\big(3\sqrt{1-qa^2}-1\big)^2 - (8m^2)/\varkappa\rho\Big]^{1/2}}{4}\,. \tag{27}$$

Thus, the metric coefficient U defining the time-slowing-down processes *is not zero*.

If to put the graviton rest mass to zero, expression (27), as one should expect, coincides exactly with the last term of expression (22). From formula (27) one can obtain the minimum value of β

$$\beta_{\min} = \Big(\frac{m^2}{2\varkappa\rho}\Big)^{1/2}. \tag{28}$$

The quantity β in the function $\sqrt{U}$ defines the limit for the time-slowing-down process by the gravitational field of the ball. This means that further slowing down of the time run by the gravitational field is *impossible*. That is why the scalar curvature defined by expression (10) will be, as distinct from the General Theory of Relativity (GTR), finite everywhere. So, the very gravitational field stops the time-slowing-down process due to the non-zero graviton mass.

According to (27) equality (23) is *impossible* due to the non-zero graviton mass, because the following inequality takes place:

$$3\sqrt{1-qa^2} - 1 \geq 2\sqrt{2}\Big(\frac{m^2}{\varkappa\rho}\Big)^{1/2}. \tag{29}$$

Taking into account the definition

$$qa^2 = W_g/a\,,$$

we find on the basis of inequality (29) for $\varkappa\rho \gg m^2$

$$a \geq \frac{9}{8}W_g\left(1+\sqrt{\frac{m^2}{2\varkappa\rho}}\right). \tag{30}$$

This bound for the body radius, which arises when studying the internal solutions, is stronger than bound (29), obtained in [3, 4], in the course of the analysis of the external solution. Inequality (30), as we see, follows directly from theory, while in the GTR inequality (5) is specially introduced to avoid an infinite pressure inside the body.

From (18) and (19) we find the pressure:

$$\frac{p}{c^2} = \frac{-\rho\sqrt{U} + \rho\sqrt{1-qa^2}}{\sqrt{U}}\,.$$

With account of equality (28) we obtain the maximum pressure at the center of the ball

$$\frac{p}{c^2} \simeq \rho\Big[\frac{2\varkappa\rho}{m^2}(1-qa^2)\Big]^{1/2}.$$

The pressure at the center of the ball is finite, while in the GTR, due to (2), it is infinite.

The presence, in the Relativistic Theory of Gravitation, of the *effective repulsive force*, which arises in strong gravitational fields, differs it the essence from Einstein's GTR and Newton's theory of gravitation in which only *attractive forces* rule. In the field theory of gravitation the presence of the non-zero graviton mass and the fundamental property to stop the time run slowing down process lead to the fact that the *gravitational force* may be not only an *attractive force*, but at some circumstances (in strong fields) even an *effectively repulsive* one. The effective repulsive force stops the time run slowing down process by the gravitational field. The gravitational field, in the main, cannot stop the time run of a physical process because it possesses a fundamental property of *self-restriction*.

Namely this property of the gravitational field, excluding a possibility of the black holes formation as non-physical objects, drastically changes the picture of the matter evolution as compared with the GTR.

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