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FIELD AND ITS ROLE IN THE UNIVERSE

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Abstract

Gershtein S.S., Logunov A.A., Mestvirishvili M.A. *Self-restriction* of Gravitational Field and its Role in the Universe: IHEP Preprint 2005-43. – Protvino, 2005. – p. 36, figs. 1, tables 1, refs.: 34.

It is shown in the article that according to the Relativistic Theory of Gravitation the gravitational field providing slowing down of the time rate nevertheless stops itself this slowing down in strong fields. So a physical tendency of this field to *self-restriction* of the gravitational potential is demonstrated. This property of the field leads to a stopping of the collapse of massive bodies and to the cyclic evolution of the homogeneous and isotropic Universe.

Аннотация

Герштейн С.С., Логунов А.А., Мествиришвили М.А. *Самоограничение* гравитационного поля и его роль во Вселенной: Препринт ИФВЭ 2005-43. – Протвино, 2005. – 36 с., 1 рис., 1 табл., библиогр.: 34.

В статье показано, что согласно релятивистской теории гравитации гравитационное поле, осуществляя своим действием замедление течения времени, само в сильном поле останавливает это замедление. Так проявляется физическое свойство поля к *самоограничению* гравитационного потенциала. Это свойство поля приводит к остановке коллапса массивных тел и циклическому развитию однородной и изотропной Вселенной.

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Introduction

Both in the Newton gravity theory and in the Einstein general theory of relativity (GTR) the gravitational forces are attractive forces. However the field approach to gravitation shows that it is not quite so in strong gravitational fields. We shall return to this point later.

The Relativistic Theory of Gravitation (RTG) was presented in detail on the pages of “Uspekhi” in paper [1]. Here we only briefly enlist its fundamental propositions.

In the basis of the RTG lies the special theory of relativity and this provides energy-momentum and angular momentum conservation laws for all physical processes including the gravitational ones. The RTG stems from a hypothesis that gravitation is universal and its source is the conserved energy-momentum tensor of all matter fields including that of the gravitational field.

That is why the gravitational field is a tensor field, $\phi^{\mu\nu}$. Such an approach corresponds to Einstein’s idea that he wrote about as early as in 1913 [2]: “...*the tensor of the gravitational field $\vartheta_{\mu\nu}$ is a source of the field equality with the material systems tensor $\Theta_{\mu\nu}$. Exeptional status of the energy of the gravitational field in comparison with other kinds of energy would leads to inadmissible consequences*”. This Einstein’s idea was put into the basis of the relativistic theory of gravitation. Einstein, when devising the general theory of relativity, did not manage to realize this idea because instead of the energy-momentum tensor a pseudo-tensor of the gravitational field arose in the GTR. This happened because Einstein did not consider the gravitational field as a physical field (of a Faraday–Maxwell type) in the Minkowski space. That is why the GTR equations do not contain the metric of the Minkowski space. The approach to gravitation adopted in the RTG implies *geometrization*, i.e. an effective Riemannian space results *but of a simple topology only*. This leads to the following picture: the test body motion in the Minkowski space under the action of the gravitational field is equivalent to the motion of this body in an effective Riemannian space created by the gravitational field. Namely this circumstance of the theory allows one to separate inertial forces from the gravitational ones.

In the field approach to gravitation an effective Riemannian space arises but of a simple topology only. This is the reason why the field approach cannot lead to the GTR in which the topology is not simple in general.

This idea described above leads to the following complete system of equations [1, 3, 4]:

$$\left(R^{\mu\nu} - \frac{1}{2}g^{\mu\nu}R\right) + \frac{m^2}{2}\left[g^{\mu\nu} + \left(g^{\mu\alpha}g^{\nu\beta} - \frac{1}{2}g^{\mu\nu}g^{\alpha\beta}\right)\gamma_{\alpha\beta}\right] = 8\pi GT^{\mu\nu}, \quad (1)$$

$$D_\nu \tilde{g}^{\nu\mu} = 0. \quad (2)$$

Here $T^{\mu\nu}$ is the substance energy-momentum tensor¹, D_ν stands for the covariant derivative in the Minkowski space, $\gamma_{\alpha\beta}$ is the metric of the Minkowski space; $g_{\alpha\beta}$ is the metric of the effective Riemannian space, $m = m_g c/\hbar$, m_g is the graviton mass, $\tilde{g}^{\nu\mu} = \sqrt{-g}g^{\nu\mu}$ is the density of the metric tensor $g^{\mu\nu}$.

Due to the non-zero mass at rest of the graviton in equation (83) equations (84) are a consequence of the gravitational field equations (83) and the substance equations.

The effective metric of the Riemannian space $g^{\mu\nu}$ is related to the gravitational field $\phi^{\mu\nu}$ by the relationship

$$\tilde{g}^{\mu\nu} = \tilde{\gamma}^{\mu\nu} + \tilde{\phi}^{\mu\nu},$$

where

$$\tilde{\gamma}^{\mu\nu} = \sqrt{-\gamma}\gamma^{\mu\nu}, \quad \tilde{\phi}^{\mu\nu} = \sqrt{-\gamma}\phi^{\mu\nu}, \quad \gamma = \det \gamma_{\mu\nu}.$$

The system of equations (1), (2) is generally covariant under arbitrary coordinate transformations and form-invariant under the Lorentz transformations. It follows directly from the least action principle with a Langrangean density

$$L = L_g(\gamma_{\mu\nu}, \tilde{g}^{\mu\nu}) + L_M(\tilde{g}^{\mu\nu}, \phi_A),$$

here

$$L_g = \frac{1}{16\pi}\tilde{g}^{\mu\nu}(G_{\mu\nu}^\lambda G_{\lambda\sigma}^\sigma - G_{\mu\sigma}^\lambda G_{\nu\lambda}^\sigma) - \frac{m^2}{16\pi}\left(\frac{1}{2}\gamma_{\mu\nu}\tilde{g}^{\mu\nu} - \sqrt{-g} - \sqrt{-\gamma}\right),$$

$$G_{\mu\nu}^\lambda = \frac{1}{2}g^{\lambda\sigma}(D_\mu g_{\sigma\nu} + D_\nu g_{\sigma\mu} - D_\sigma g_{\mu\nu}), \quad \phi_A \text{ are the substance fields.}$$

In order that timelike and isotropic intervals in the effective Riemannian space could not get out of the light-cone of the underlying Minkowski space the causality condition has to hold

$$\gamma_{\mu\nu}v^\mu v^\nu = 0, \quad g_{\mu\nu}v^\mu v^\nu \leq 0, \quad (3)$$

Here v^ν is the velocity four-vector.

Thus the motion of test bodies under the action of the gravitational field proceeds always *inside* both the Riemannian cone and the cone of the Minkowski space. This provides the geodesic completeness.

The rest mass of the graviton arises unavoidably in the theory because only in this way one can consider the gravitational field as a physical field in the Minkowski space whose source is the total conserved energy-momentum tensor of all matter. And this is a non-zero mass of the graviton that changes completely the picture both of the collapse process and the evolution of the Universe.

When A. Einstein in 1913 related the gravitational field to the metric tensor of the Riemannian space it appeared that such a field caused a slowing down of the lapse of a physical process.

¹Under the "substance" we mean all physical fields except the gravitational one.

This slow down can be illustrated, in particular, in the case of the Schwarzschild solution, if to compare the lapse of time in the presence of the gravitational field with the lapse of time for a distant observer. However, generally, only the metric tensor of the Riemannian space takes place in the GTR and therefore any trace of inertial time of the Minkowski space is absent from the Hilbert–Einstein equations. Due to this reason the universal property of the gravitational field to slow down the lapse of time in comparison with the inertial time could not get a further development in the framework of the GTR.

The rise of the effective Riemannian space in the field theory of gravitation with preservation of the Minkowski space as a basic space gives a special importance to the property of the gravitational field to exert a slowing down influence on the lapse of time. Only in this case can one argue truly about the *slowing down of the lapse of time when making the comparison of the lapse of time in the gravitation field with that in an inertial frame of the Minkowski space in the absence of gravitation*.

All this is realized in the RTG because the metric tensor $\gamma_{\mu\nu}$ of the Minkowski space enters explicitly the full system of its equations.

To demonstrate that the change of the lapse of time implies the appearance of a force we turn to the Newton equation

$$m \frac{d^2 x}{dt^2} = F.$$

If one passes formally from the inertial time to a time τ with

$$d\tau = U(t)dt,$$

then it is easy to obtain

$$m \frac{d^2 x}{d\tau^2} = \frac{1}{U^2} \left\{ F - \frac{dx}{dt} \frac{d}{dt} \ln U \right\}.$$

One can see from this that the change of the lapse of time defined by the function U results in the appearance of an effective force. All this bears a formal character here as in this case there is no physical reason which would change the lapse of time. But this formal example shows that if a process of slowing down of the lapse of time occurs is *Nature* then it unavoidably generates effective field forces, and so it is necessary to take them into account as something absolutely new and surprising. The physical gravitational force changes both the lapse of time and parametersclowing of the space quantities in comparison with the same quantities is an inertial system of the Minkowski space without gravitation.

The field approach to gravitation excludes the concept of black holes and explains the evolution both of massive bodies and the Universe on the basis of more profound insight into the physical properties of the very gravitational field.

This confirms the deep intuition of A. S. Eddington who said at the session of the Royal Astronomical Society 11 January 1935: *“The star has to go on radiating and radiating and contracting and contracting until, I suppose, it gets down to a few km. radius, when gravity becomes strong enough to hold in the radiation, and the star can at last find peace. . . . I felt driven to the conclusion that this was almost **a reductio ad absurdum** of the relativistic degeneracy formula. Various accidents may intervene to save the star, but I want more protection than that. I think there should be a law of Nature to prevent a star from behaving in this absurd way !”*²

²The Observatory. 1935. Vol. 58. P. 373.

It appears that in the framework of the field formulation of gravitation such a *Law of Nature* is contained in the physical property of the gravitational field to stop the *process* of the slowing down of the lapse of time and hence to limit its potential. This stops the *process* of compression.

Below, taking as examples the collapse and the evolution of the homogeneous and isotropic Universe, we will see in what way the *self-restriction* of the gravitational field potential arises which stops both the process of the slowing down of time and the process of the substance compression.

1. Equations of the spherically symmetric static gravitational field

The interval in the Minkowski space has the following form in spherical coordinates

$$d\sigma^2 = (dx^0)^2 - (dr)^2 - r^2(d\theta^2 + \sin^2\theta d\phi^2), \quad (4)$$

here $x^0 = ct$. The interval in the effective Riemannian space for a spherically symmetric static field can be written in the form

$$ds^2 = U(r)(dx^0)^2 - V(r)dr^2 - W^2(r)(d\theta^2 + \sin^2\theta d\phi^2). \quad (5)$$

Equations (1), (2) of the RTG we represent in the form

$$R^\mu_\nu - \frac{1}{2}\delta^\mu_\nu R + \frac{m^2}{2}\left(\delta^\mu_\nu + g^{\mu\alpha}\gamma_{\alpha\nu} - \frac{1}{2}\delta^\mu_\nu g^{\alpha\beta}\gamma_{\alpha\beta}\right) = \varkappa T^\mu_\nu, \quad (6)$$

$$D_\mu \tilde{g}^{\mu\nu} = 0. \quad (7)$$

Equation (7) reads in more detail

$$\partial_\mu \tilde{g}^{\mu\nu} + \gamma^\nu_{\lambda\sigma} \tilde{g}^{\lambda\sigma} = 0. \quad (8)$$

Here $\gamma^\nu_{\lambda\sigma}$ are Christoffel's symbols of the Minkowski space. For a spherically symmetric static source the components of the tensor T^μ_ν are:

$$T^0_0 = \rho(r), \quad T^1_1 = T^2_2 = T^3_3 = -\frac{p(r)}{c^2}, \quad (9)$$

here ρ is the mass density, p stands for the isotopic pressure.

For the definition of the metric coefficients U , V and W one can make use of Eqs. (6) for $\mu = 0, \nu = 0; \mu = 1, \nu = 1$.

$$\begin{aligned} & \frac{1}{W^2} - \frac{1}{VW^2} \left(\frac{dW}{dr} \right)^2 - \frac{2}{VW} \frac{d^2W}{dr^2} - \frac{1}{W} \frac{dW}{dr} \frac{d}{dr} \left(\frac{1}{V} \right) + \\ & + \frac{1}{2} m^2 \left[1 + \frac{1}{2} \left(\frac{1}{U} - \frac{1}{V} \right) - \frac{r^2}{W^2} \right] = \varkappa \rho, \end{aligned} \quad (10)$$

$$\begin{aligned} & \frac{1}{W^2} - \frac{1}{VW^2} \left(\frac{dW}{dr} \right)^2 - \frac{1}{UVW} \frac{dW}{dr} \frac{dU}{dr} + \\ & + \frac{1}{2} m^2 \left[1 - \frac{1}{2} \left(\frac{1}{U} - \frac{1}{V} \right) - \frac{r^2}{W^2} \right] = -\varkappa \frac{p}{c^2}. \end{aligned} \quad (11)$$

Equation (8) takes the form

$$\frac{d}{dr} \left(\sqrt{U/V} W^2 \right) = 2r \sqrt{UV}. \quad (12)$$

With account of the identity

$$\frac{dr}{dW} \frac{1}{W^2} \frac{d}{dr} \left[\frac{W}{V} \left(\frac{dW}{dr} \right)^2 \right] = \frac{1}{VW^2} \left(\frac{dW}{dr} \right)^2 + \frac{2}{VW} \frac{d^2W}{dr^2} + \frac{1}{W} \frac{dW}{dr} \frac{d}{dr} \left(\frac{1}{V} \right),$$

and after passing from the derivatives in r to the derivatives in W equations (10), (11) and (12) take the form

$$1 - \frac{d}{dW} \left[\frac{W}{V(dr/dW)^2} \right] + \frac{1}{2} m^2 \left[W^2 - r^2 + \frac{W^2}{2} \left(\frac{1}{U} - \frac{1}{V} \right) \right] = \varkappa W^2 \rho, \quad (13)$$

$$\begin{aligned} & 1 - \frac{W}{V(dr/dW)^2} \frac{d}{dW} [\ln(UW)] + \\ & + \frac{1}{2} m^2 \left[W^2 - r^2 - \frac{W^2}{2} \left(\frac{1}{U} - \frac{1}{V} \right) \right] = -\varkappa W^2 \frac{p}{c^2}, \end{aligned} \quad (14)$$

$$\frac{d}{dW} [\sqrt{U/V} W^2] = 2r \sqrt{UV} \frac{dr}{dW}. \quad (15)$$

Further on we will use these equations for various state equations of the substance. Namely on the basis of these equations it will be demonstrated in Parts 2, 3 and 4 that the gravitational field possesses a property of *self-restriction* which imposes the limit for the slowing down of the lapse of time by the gravitational field.

2. External solution for the spherically symmetric static body

In this Part it will be shown that presence of the rest mass of the graviton changes qualitatively the character of the solution in the region near to the Schwarzschild sphere. Below we consider this in detail.

Subtracting Eq. (14) from (13) and introducing new variables

$$Z = \frac{UW^2}{V\dot{r}^2}, \quad \dot{r} = \frac{dr}{dt}, \quad t = \frac{W - W_0}{W_0}, \quad (16)$$

we obtain

$$\frac{dZ}{dW} - \frac{2Z}{U} \frac{dU}{dW} - 2 \frac{Z}{W} - \frac{m^2 W^3}{2W_0^2} \left(1 - \frac{U}{V} \right) = -\varkappa \frac{W^3}{W_0^2} \left(\rho + \frac{p}{c^2} \right) U. \quad (17)$$

Adding Eqs. (13) and (14), we find

$$1 - \frac{1}{2} \frac{W_0^2}{W} \frac{1}{U} \frac{dZ}{dW} + \frac{m^2}{2} (W^2 - r^2) = \frac{1}{2} \varkappa W^2 \left(\rho - \frac{p}{c^2} \right). \quad (18)$$

Let us consider (17) and (18) outside the substance in the region defined by the inequalities

$$\frac{U}{V} \ll 1, \quad \frac{1}{2}m^2(W^2 - r^2) \ll 1. \quad (19)$$

In this region Eq. (18) has the form

$$U = \frac{1}{2} \frac{W_0^2}{W} \frac{dZ}{dW} = \frac{1}{2} \frac{W_0}{W} \frac{dZ}{dt}. \quad (20)$$

Taking into account (20) we bring Eq. (17) to the form

$$Z \frac{d^2 Z}{dW^2} - \frac{1}{2} \left(\frac{dZ}{dW} \right)^2 + \frac{1}{4} m^2 \frac{W^3}{W_0^2} \frac{dZ}{dW} = 0. \quad (21)$$

Let us introduce, according (16), variable t . Then Eq. (21) assumes the form

$$Z \ddot{Z} - \frac{1}{2} (\dot{Z})^2 + \alpha (1+t)^3 \dot{Z} = 0, \quad (22)$$

Here $\alpha = m^2 W_0^2 / 4$, $\dot{Z} = dZ/dt$. For t defined by the inequality

$$0 \leq t \ll 1/3, \quad (23)$$

Eq. (22) simplifies

$$Z \ddot{Z} - \frac{1}{2} (\dot{Z})^2 + \alpha \dot{Z} = 0. \quad (24)$$

It has a solution

$$\lambda \sqrt{Z} = 2\alpha \ln \left(1 + \frac{\lambda \sqrt{Z}}{2\alpha} \right) + \frac{\lambda^2}{2} t. \quad (25)$$

Here λ is an arbitrary constant.

On the basis of (20) and (16) we have

$$U = \frac{1}{2} \frac{W_0}{W} \dot{Z}, \quad V \dot{r}^2 = \frac{1}{2} W_0 W \frac{\dot{Z}}{Z}. \quad (26)$$

Making use of (25) we find

$$\dot{Z} = 2\alpha + \lambda \sqrt{Z}. \quad (27)$$

Substituting (27) into (26), we obtain

$$U = \frac{W_0}{W} \left(\alpha + \frac{\lambda}{2} \sqrt{Z} \right), \quad V \dot{r}^2 = W_0 W \frac{\alpha + \lambda \sqrt{Z}/2}{Z}. \quad (28)$$

At $\alpha = 0$ we have from (25)

$$\sqrt{Z} = \frac{\lambda}{2} t. \quad (29)$$

Substituting this expression into (28), we find

$$U = \left(\frac{\lambda}{2} \right)^2 \frac{W - W_0}{W}. \quad (30)$$

But this expression for U has to coincide exactly with the Schwarzschild solution

$$U = \frac{W - W_g}{W}, \quad W_g = \frac{2GM}{c^2}. \quad (31)$$

Comparing (30) and (31), we obtain

$$\lambda = 2, \quad W_0 = W_g. \quad (32)$$

Thus we find:

$$U = \frac{W_g}{W}(\alpha + \sqrt{Z}), \quad V\dot{r}^2 = W_g W \frac{\alpha + \sqrt{Z}}{Z}. \quad (33)$$

We need now to define how r depends on W with help of (15) Substituting (33) into Eqs. (15) and passing to the variable

$$\ell = r/W_g, \quad (34)$$

we obtain

$$\frac{d}{d\sqrt{Z}} \left[(1+t) \frac{dZ}{dt} \frac{d\ell}{d\sqrt{Z}} \right] = 4\ell. \quad (35)$$

Taking into account (27) and making differentiation in \sqrt{Z} in (35), we find

$$(1+t)(\alpha + \sqrt{Z}) \frac{d^2\ell}{(d\sqrt{Z})^2} + (1+t+\sqrt{Z}) \frac{d\ell}{d\sqrt{Z}} - 2\ell = 0. \quad (36)$$

As we are interested in the region of t defined by inequality (23), Eq. (36) is simplified and has the form

$$(\alpha + \sqrt{Z}) \frac{d^2\ell}{(d\sqrt{Z})^2} + (1 + \sqrt{Z}) \frac{d\ell}{d\sqrt{Z}} - 2\ell = 0. \quad (37)$$

The general solution of Eqs. (37) will be

$$\ell = A\ell_1 + B\ell_2, \quad (38)$$

where

$$\ell_1 = F[-2, 1 - \alpha, -(\alpha + \sqrt{Z})], \quad \ell_2 = (\alpha + \sqrt{Z})^\alpha F[-2 + \alpha, 1 + \alpha, -(\alpha + \sqrt{Z})].$$

Here A and B are arbitrary constants, F is a degenerated hypergeometric function.

The analysis of solution (38) in the region defined by inequalities (19) and (23) leads to an equality

$$\dot{r} = W_g. \quad (39)$$

Let us consider the following limiting case

$$\sqrt{Z} \gg \alpha. \quad (40)$$

In this case we have from expression (25) with account of (32)

$$\sqrt{Z} = t. \quad (41)$$

Substituting this expression into (28) and taking into consideration (32), (39), we obtain the Schwarzschild solution

$$U = \frac{W - W_g}{W}, \quad V = \frac{W}{W - W_g}. \quad (42)$$

Now we come to another limiting case when the influence of the graviton mass is essential, Let the following inequality takes place

$$\sqrt{Z} \ll \alpha. \quad (43)$$

In this approximation we find from expression (25) with account of (32)

$$Z = 2\alpha t. \quad (44)$$

Substituting this expression into (28) and taking into consideration (32) and (39), we obtain

$$U = \alpha \frac{W_g}{W}, \quad V = \frac{1}{2} \frac{W}{W - W_g}. \quad (45)$$

This solution, on the basis of (43) and (44), holds in the region

$$t \ll \frac{\alpha}{2} \quad \text{i.e.} \quad W - W_g \ll \frac{1}{2} W_g \left(\frac{m_g c}{\hbar} \frac{W_g}{2} \right)^2.$$

We see from (45), that the graviton mass m_g does not allow for vanishing of the quantity U . *The rest mass of the graviton imposes for any body its limit for slowing-down of the lapse of time.* This limit is defined by a linear function of the Schwarzschild radius, i.e. of the mass of the body

$$\frac{1}{2} \left(\frac{m_g c}{\hbar} \right) W_g.$$

There is no such a limit in the GTR. Such a property of the gravitational field leads to a cardinal change in the motion of a test body in the gravitational field.

The motion of a test body proceeds along a geodesic line of the Riemannian space

$$\frac{dv^\mu}{ds} + \Gamma_{\alpha\beta}^\mu \frac{dx^\alpha}{ds} \cdot \frac{dx^\beta}{ds} = 0. \quad (46)$$

Here $v^\mu = dx^\mu/ds$ is the velocity four-vector, and v^μ satisfies the condition

$$g_{\mu\nu} v^\mu v^\nu = 1. \quad (47)$$

Let us consider the radial motion

$$v^\theta = v^\phi = 0, \quad v^r = dr/ds. \quad (48)$$

Taking into consideration that the Christoffel symbol Γ_{01}^0 is

$$\Gamma_{01}^0 = \frac{1}{2U} \frac{dU}{dr}, \quad (49)$$

we find from Eq. (46)

$$\frac{dv^0}{ds} + \frac{1}{U} \frac{dU}{dr} v^0 v^r = 0. \quad (50)$$

Resolving equation (50), we obtain

$$\frac{d}{dr} \ln(v^0 U) = 0. \quad (51)$$

We have thereof:

$$v_0 = \frac{dx^0}{ds} = \frac{U_0}{U}, \quad (52)$$

U_0 being an integration constant. If to assume the velocity of the falling body equal to zero at infinity we obtain $U_0 = 1$. From relation (47) we find

$$\frac{dr}{ds} = -\sqrt{\frac{1-U}{UV}}. \quad (53)$$

Substituting this into expression (45) and taking into consideration (39), we obtain

$$\frac{dW}{ds} = -\left(\frac{\hbar}{m_g c}\right) \frac{2}{W_g} \sqrt{2 \frac{W}{W_g} \left(1 - \frac{W_g}{W}\right)}. \quad (54)$$

It is seen thereof that a turning point appears. Differentiating (54) with respect to s , we find

$$\frac{d^2 W}{ds^2} = 4 \left(\frac{\hbar}{m_g c}\right)^2 \frac{1}{W_g^3}. \quad (55)$$

We see that in the turning point the acceleration is positive, i.e. a repulsion takes place, and it is significant. Integrating (54) we obtain

$$W = W_g + 2 \left(\frac{\hbar}{m_g c}\right)^2 \frac{(s - s_0)^2}{W_g^3}. \quad (56)$$

It is clear from expression (56) that *a test body cannot cross the Schwarzschild sphere*.

According to expressions (45) the scalar quantity g/γ , where $g = \det g_{\mu\nu}$, $\gamma = \det \gamma_{\mu\nu}$, has a singularity at $W = W_g$ that cannot be eliminated by the choice of the coordinate system. Therefore the presence of such a singularity in vacuum is inadmissible, otherwise one cannot sew up external solution with the solution inside the body. The conclusion follows from this that the body radius has to be larger than the Schwarzschild radius. In such a way a *self-restriction* of the field strength arises in the RTG and so the very reason of the appearance of the ‘‘Schwarzschild singularity’’ dissapears. This corresponds completely to A. Einstein opinion that he expressed as early as in 1939 in paper [5]: *‘‘The main result of the conducted study is a clear understanding that ‘‘Schwarzschild singularities’’ are absent from the real world (emphasized by us. Authors)’’* And further on: *‘‘The Schwarzschild singularity is absent because one cannot concentrate the substance in an arbitrary way, otherwise the particles forming the clusters will achieve the velocity of light (emphasized by us. Authors)’’*.

As an example we consider gravitational field in a shrinking (synchronous) coordinate system. The passage to this coordinate system from an inertial one is being made by means of transformations

$$dt = \frac{1}{U} [d\tau - dR(1 - U)], \quad dW = \sqrt{\frac{1-U}{UV}} (dR - d\tau).$$

In the synchronous coordinate system the interval of the Riemannian and pseudo-Euclidean space-times has the form

$$ds^2 = d\tau^2 - [1 - U(X)]dR^2 - W^2(X)(d\theta^2 + \sin^2 \theta d\phi^2),$$

$$d\sigma^2 = d\tau^2 \frac{1 - \dot{r}^2 U^2}{U^2} + 2 dR d\tau \frac{\dot{r}^2 U^2 - (1 - U)}{U^2} - dR^2 \frac{\dot{r}^2 U^2 - (1 - U)^2}{U^2} - r^2(d\theta^2 + \sin^2 \theta d\phi^2),$$

where $X = R - \tau$, $\dot{r} = dr/dX$.

The RTG equations

$$R_{\mu\nu} = 8\pi G \left(T_{\mu\nu} - \frac{1}{2} g_{\mu\nu} T \right) + \frac{m^2}{2} (g_{\mu\nu} - \gamma_{\mu\nu}), \quad (a)$$

$$D_\nu \tilde{g}^{\mu\nu} = 0,$$

for the problem defined by the intervals ds^2 and $d\sigma^2$ lead (outside the substance) to the equations of the form

$$R_{01} = \frac{2\ddot{W}}{W} + \frac{1}{(1-U)W} \dot{U}\dot{W} = \frac{m^2}{2} \left(\frac{1-U}{U^2} - \dot{r}^2 \right), \quad (b)$$

$$R_{00} + R_{01} = \frac{1}{1-U} \left[\frac{1}{2} \ddot{U} + \frac{\dot{U}^2}{4(1-U)} + \frac{1}{W} \dot{U}\dot{W} \right] = -\frac{m^2}{2} \frac{1-U}{U}. \quad (c)$$

In the region of the variable X where one can neglect the graviton mass due to its smallness we find from these equations

$$W = W_g^{1/3} \left[\frac{3}{2} X \right]^{2/3}, \quad 1 - U = \left[\frac{2}{3} W_g \right]^{2/3} X^{-2/3}, \quad (d)$$

From expression (d) for the function U it follows that it decreases with decreasing of X , and its derivative \dot{U} is positive.

In approximation (19) we find from equation (a) outside the substance

$$R_{22} = -\frac{UW}{1-U} \ddot{W} - \frac{U}{1-U} \dot{W}^2 - \frac{W(2-U)}{2(1-U)^2} \dot{U}\dot{W} + 1 = 0.$$

In the region of small values $0 < U \ll 1$ the equation is somewhat simplified and assumes the form

$$UW\ddot{W} + U\dot{W}^2 + W\dot{U}\dot{W} - 1 = 0.$$

This equation has a solution

$$\dot{W} = \frac{X}{UW}.$$

At the stopping point

$$\dot{W} = 0,$$

the second derivative \ddot{W} at small values of U is positive according to equations (b) and (c) and this is an evidence of the presence of a repulsive force.

It is this point where the process of expansion starts from. This expansion stops is the region of X where equalities (d) hold. In this region \ddot{W} is negative

$$\ddot{W} = -\frac{1}{2}W_g^{1/3}\left[\frac{3}{2}X\right]^{-4/3},$$

and consequently attraction takes place. So if the stopping point were outside the substance then compression would start after expansion, then again stop and again expansion and so on. However the real gravitational field excludes such a regime.

While in the GTR for the given problem an equation takes place

$$W = \left[\frac{3}{2}(R - c\tau)\right]^{2/3}W_g^{1/3},$$

in our case we obtain the expression

$$W = W_g + 2\left(\frac{\hbar}{m_g c}\right)^2 \frac{(R - c\tau)^2}{W_g^3},$$

that excludes achieving the point $W = 0$. This means that

$$\frac{d^2W}{d\tau^2} = \frac{4c^2}{W_g^3}\left(\frac{\hbar}{m_g c}\right)^2.$$

All this occurs because of the passage from the inertial time t to the physical time τ .

As the gravitational field is created by the substance and the very gravitational field restricts its potential, it follows from the example above that for obtaining a physical solution one needs to sew up the solution inside the substance with the external solution but it is necessary that the gravitational field potential would be bounded on the surface of the body by the inequality

$$\frac{|\phi|}{c^2} < 1.$$

It is such a solution, which corresponds to the real gravitational field, that implies that the stopping point cannot lie in vacuum. That is why the world lines of particles which are at rest in the shrinking coordinate system will collide with the substance of the source of the field, and besides these collisions will occur during a finite time for any observer. All this excludes the regime of motion about which we wrote above. At the same time this excludes the appearance of “black holes”.

Let us turn now to analysis of the internal solution.

3. Internal solution of the Schwarzschild type

In paper [6] Schwarzschild has found a spherically symmetric static internal solution of the equations of the general theory of relativity. For a *homogeneous ball* of radius a it is described by the interval

$$ds^2 = c^2\left(\frac{3}{2}\sqrt{1 - qa^2} - \frac{1}{2}\sqrt{1 - qW^2}\right)^2 dt^2 - (1 - qW^2)^{-1}dW^2 + W^2(d\theta^2 + \sin^2\theta d\phi^2); \quad (57)$$

here $q = (1/3)\kappa\rho = (2GM)/(c^2a^3)$, $\kappa = (8\pi G)/c^2$, $\rho = (3M)/(4\pi a^3)$.

The general property of external and internal solutions is manifested in the fact that at a definite value of W the metric coefficients in front of the differential dt^2 in the intervals vanishes.

Vanishing of the metric coefficient U of dt^2 means that the gravitational field can, by its action, not only slows down the lapse of time but can even *stop the flow of time*. For the external solution the vanishing of the metric coefficient U occurs at $W = W_g$. To exclude such a possibility (which is not forbidden by the theory) one must assume that the body radius satisfies the inequality

$$a > W_g. \quad (58)$$

For the internal solution this happens at

$$W^2 = 9a^2 - 8(a^3/W_g). \quad (59)$$

To exclude such a possibility of the vanishing of the metric coefficient U inside the body one must assume that

$$a > (9/8)W_g. \quad (60)$$

One has to emphasize that inequalities (58) and (60) are not a consequence of the GTR.

The internal Schwarzschild solution is somewhat formal but it is interesting first all because it is an exact solution of the equations of the GTR. In Part 2 it is shown taking as an example the external Schwarzschild solution that in the relativistic theory of gravitation, as in a field theory, inequality (58) results exactly due to the stop of the process of slowing down of the lapse of time. Below we consider an internal solution of the Schwarzschild type in the framework of the RTG.

The internal Schwarzschild solution arose on the basis of the Hilbert–Einstein equations

$$\begin{aligned} 1 - \frac{d}{dW} \left[\frac{W}{V} \right] &= \kappa W^2 \rho, \\ 1 - \frac{1}{V} - \frac{W}{UV} \frac{dU}{dW} &= -\kappa \frac{W^2}{c^2} p. \end{aligned} \quad (61)$$

Since, according to (57), the metric coefficients are

$$U = \left(\frac{3}{2} \sqrt{1 - qa^2} - \frac{1}{2} \sqrt{1 - qW^2} \right)^2, \quad V = (1 - qW^2)^{-1}, \quad (62)$$

we find hence

$$\frac{\dot{U}}{U} = \frac{qW}{\sqrt{1 - qW^2} \left(\frac{3}{2} \sqrt{1 - qa^2} - \frac{1}{2} \sqrt{1 - qW^2} \right)}, \quad \dot{U} = \frac{dU}{dW}. \quad (63)$$

Substituting (62) and (63) into (61), we obtain expression for the pressure

$$\frac{p}{c^2} = \frac{\rho}{2} \frac{(\sqrt{1 - qW^2} - \sqrt{1 - qa^2})}{\sqrt{U}}. \quad (64)$$

It is seen, in particular, from here that if equality (59) were not excluded then the pressure inside the body on the circumference, defined by this equality, would become infinite. The singularity

that arises due to the vanishing of the metric coefficient U cannot be eliminated by the choice of the coordinate system because the scalar curvature also has it:

$$R = -8\pi G \left[\frac{3\sqrt{1 - qa^2} - 2\sqrt{1 - qW^2}}{\sqrt{U}} \right]. \quad (65)$$

Let us show now, taking as an example an internal solution of the Schwarzschild type, that the situation in the RTG changes drastically due to the stop of the of the time lapse clowing-down. The same mechanism of self-restriction which led, in the RTG, to inequality (58) in the external Schwarzschild solution leads to an inequality of the type (60) for the internal Schwarzschild solution.

We will get the equations for this problem from (13) and (14). Introducing a new variable

$$Z = \frac{UW^2}{V\dot{r}^2}, \quad \dot{r} = \frac{dr}{dW}$$

and adding equations (13) and (14), we obtain

$$1 - \frac{1}{2UW} \dot{Z} + \frac{m^2}{2}(W^2 - r^2) = \frac{1}{2} \varkappa W^2 \left(\rho - \frac{p}{c^2} \right). \quad (66)$$

Subtracting equation (14) from equation (13), we find

$$\dot{Z} - 2Z \frac{\dot{U}}{U} - 2 \frac{Z}{W} - \frac{m^2}{2} W^3 \left(1 - \frac{U}{V} \right) = -\varkappa W^3 \left(\rho + \frac{p}{c^2} \right) U. \quad (67)$$

In our problem the component of the energy-momentum tensor of the substance are

$$T_0^0 = \rho, \quad T_1^1 = T_2^2 = T_3^3 = -\frac{p(W)}{c^2}.$$

The substance equation

$$\nabla_\nu (\sqrt{-g} T_\mu^\nu) = \partial_\nu (\sqrt{-g} T_\mu^\nu) + \frac{1}{2} \sqrt{-g} T_{\sigma\nu} \partial_\mu g^{\sigma\nu} = 0$$

is reduced, for this problem, to the following form

$$\frac{1}{c^2} \frac{dp}{dW} = - \left(\rho + \frac{p}{c^2} \right) \frac{1}{2U} \frac{dU}{dW}. \quad (68)$$

Since the pressure increases towards the center of the ball this leads to an inequality

$$\frac{dU}{dW} > 0, \quad (69)$$

which indicates that with the approach to the center of the ball function U decreases and hence a slowing down of the lapse of time occurs in comparison with the inertial one. Since in the internal Schwarzschild problem the density ρ is assumed *constant*, equation (68) is solved easily:

$$\rho + \frac{p}{c^2} = \frac{\alpha}{\sqrt{U}}. \quad (70)$$

Comparing (64) and (70), we find the constant α

$$\alpha = \rho\sqrt{1 - qa^2}. \quad (71)$$

Equations (66) and (67), on the assumption that

$$m^2(W^2 - r^2) \ll 1, \quad (U/V) \ll 1,$$

and after introduction of a new variable $y = W^2$, take the form

$$\dot{Z} = U(1 - 3qy) + \frac{\alpha\kappa}{2}y\sqrt{U}, \quad (72)$$

$$\sqrt{U}\dot{Z} - \frac{1}{y}Z\sqrt{U} - 4Z(\sqrt{U})' + \frac{\alpha\kappa}{2}yU - \frac{m^2}{4}y\sqrt{U} = 0. \quad (73)$$

Here and further we use the notation $\dot{Z} = dZ/dy$.

In Part 2 in the analysis of the spherically symmetric Schwarzschild solution we have seen that due to an effective gravitational force of repulsion the metric coefficient U defining the slowing down of the lapse of time, as compared with the inertial one, does not vanish even in a strong gravitational field.

That is why below we will investigate the behaviour of the solution to these equations in the region of small y . For a zero-mass graviton we get from expression (62) for small y

$$\sqrt{U} \simeq \frac{1}{2}(3\sqrt{1 - qa^2} - 1) + \frac{qy}{4} + \frac{1}{16}q^2y^2. \quad (74)$$

It is also seen from this expression that the function \sqrt{U} for the internal Schwarzschild solution can be equal to zero if

$$3\sqrt{1 - qa^2} = 1, \quad (75)$$

and this leads to an infinite value both of the pressure ρ and the scalar density R . Since with the rest mass of the graviton equations (72), (73) stop the processes of slowing down of the lapse of time, it is naturally to expect that equality (75) cannot take place in the physical (real) range of the function \sqrt{U} . On the basis of (74) we will search for a solution to equations (72), (73) for the function \sqrt{U} in the form

$$\sqrt{U} = \beta + \frac{qy}{4} + \frac{1}{16}q^2y^2, \quad (76)$$

where β is an unknown constant which has to be determined, making use of Eqs. (72), (73).

Substituting expression (76) into equation (72) and integrating, we find

$$Z = \beta^2y + \frac{y^2}{2}\left(\frac{\beta q}{2} - 3\beta^2q + \frac{\alpha\kappa\beta}{2}\right) + \frac{y^3}{3}\left[\frac{q^2}{8}\left(\beta + \frac{1}{2}\right) - \frac{3\beta}{2}q^2 + \frac{\alpha\kappa q}{8}\right]. \quad (77)$$

Taking into account expressions (76) and (77) in equation (73) and neglecting small terms of order $(my)^2$, we obtain for the determination of the constant β the equation

$$2\beta^2q + \beta(q - \alpha\kappa) + m^2/3 = 0. \quad (78)$$

To clarify the inference let us note that the member containing y^2 has the following form:

$$-\frac{qy^2}{48}\{7[2\beta^2q + \beta(q - \alpha\kappa)] + 3m^2\}.$$

With account of equation (78) it can be reduced to

$$-\frac{q}{72}m^2y^2.$$

Taking into consideration that by definition

$$\alpha\kappa - q = \frac{\kappa\rho}{3}(3\sqrt{1 - qa^2} - 1),$$

we find from equation (78)

$$\beta = \frac{3\sqrt{1 - qa^2} - 1 + \left[(3\sqrt{1 - qa^2} - 1)^2 - (8m^2)/\kappa\rho\right]^{1/2}}{4}. \quad (79)$$

Thus the metric coefficient U defining the process of slowing down of the lapse of time as compared to the inertial on хода времени по сравнению с инерциальным *is not zero*.

If to put the graviton rest mass zero, expression (79), as one could expect, coincides exactly with the constant terms of expression (74). One can the minimum value of the quantity β from formula (65):

$$\beta_{\min} = \left(\frac{m^2}{2\kappa\rho}\right)^{1/2}. \quad (80)$$

The quantity β in the function \sqrt{U} defines a bound for the process of the lapse of time slowing down by the gravitational field of the ball. It means that further *slowing down* of the lapse of time by the gravitational field is *impossible*. That is why the scalar cutvature defined by expression (65) will be, in contrast to the GTR, finite everywhere. Thus the very gravitational field stops, due to the rest mass of the graviton, the process of the lapse of time slowing down.

According to (79) equality (75) is, due to the rest mass of the graviton, *impossible* because the inequality takes place

$$3\sqrt{1 - qa^2} - 1 \geq 2\sqrt{2}\left(\frac{m^2}{\kappa\rho}\right)^{1/2}. \quad (81)$$

Since by definition the equality holds

$$qa^2 = W_g/a,$$

we find on the basis of inequality (81) for $\kappa\rho \gg m^2$

$$a \geq \frac{9}{8}W_g\left(1 + \sqrt{\frac{m^2}{2\kappa\rho}}\right). \quad (82)$$

This *limit for the radius of the body* arising from the study of the internal solution is *stronger* than the limit (58) obtained in Part 2 from the analysis of the external solution. Inequality (82), as we see, follows directly from the theory, while in the GTR one has to specially introduce

inequality (60) in order to avoid an infinite pressure inside the body. On the basis of (70) and (71) we find for the pressure:

$$\frac{p}{c^2} = \frac{-\rho\sqrt{U} + \rho\sqrt{1 - qa^2}}{\sqrt{U}}.$$

Taking into account equality (80) we obtain the maximum pressure in the center of the ball

$$\frac{p}{c^2} \simeq \rho \left[\frac{2\kappa\rho}{m^2} (1 - qa^2) \right]^{1/2}.$$

The pressure in the center of the ball is finite, while in the GTR, according to (57), it is infinite.

The self-restriction of the magnitude of the gravitational field arising in the relativistic theory of gravitation distinguishes it as a matter of principle from the GTR and Newtonian theory of gravity in which only *attractive forces* are present. In the field theory of gravitation the presence of the rest-mass of the graviton and the fundamental property of the gravitational field to stop the process of the lapse of time slowing down imply that the *gravitational force* can be not only an *attractive force* but at certain conditions (strong fields) it manifests itself as an *effective decelerating force*. It is this force which stops the processes of a slowing down of the lapse of time by the gravitational field. Thus the gravitational field cannot, in principle, stop the lapse of time of a physical process because it possesses a fundamental property of “*self-restriction*”.

In Parts 2, 3 we have seen that in the GTR the metric coefficient U defining the slowing down of the lapse of time by the gravitational field can become zero. R. Feynmann mentioned this circumstance and wrote on this occasion [7]: “... *if our formula for the time dilation were correct then the physical processes would stop at the center of the Universe because the time would not lapse there at all. This is not only physically unacceptable prediction: since we could expect that the matter near the edge of the Universe would interact faster, the light from the distant galaxies would have a violet shift. In fact it is well known that it is shifted to lower, more red frequencies. Thus, our formula for the time dilation is evidently needed to be discussed in the following in connection with analysis of possible models of the Universe. The following discussion is purely qualitative and intended only to stimulate more wise thoughts on this subject*”.

4. On impossibility of the utmostly hard state equation of substance

The self-restriction of the potential, as we have seen, is an important property of the gravitational field. It is this property that provides the presence of the bound of time dilation.

Such a bound must exist because otherwise we come to a physically inadmissible conclusion. Thus any metric field theory of the gravitational field has to include this general statement as a *physical principle*.

The RTG combined with this general physical requirement allows to ascertain that *the utmostly hard state equation of the substance is not realistic*.

For the first time the question of the utmostly hard state equation of the substance was discussed in the Ya.B. Zeldovich paper [8]. This state equation of the substance has the form:

$$p/c^2 = \rho - a, \tag{83}$$

here p is the pressure, ρ is the substance density, a is some constant. With such a state equation of the substance the sound velocity is equal to the velocity of light.

Let us consider, in the RTG, a spherically symmetric problem defined by the interval

$$ds^2 = c^2 U(W) dt^2 - V(W) dW^2 - W^2 (d\theta^2 + \sin^2 \theta d\phi^2) \quad (84)$$

and the state equation of the substance (83). The state equation of the substance is written in the form

$$\frac{1}{c^2} \frac{dp}{dW} = - \left(\rho + \frac{p}{c^2} \right) \frac{1}{2U} \frac{dU}{dW}. \quad (85)$$

Taking into account (83) we find from equation (85)

$$\left(\frac{p}{c^2} + \frac{a}{2} \right) U = \alpha, \quad (86)$$

here α is an integration constant.

The system of the RTG equations for interval (84) in approximation (19) has the form:

$$\dot{Z} - \frac{2Z}{U} \dot{U} - 2 \frac{Z}{W} = -2 \left(\kappa \alpha - \frac{m^2}{4} \right) W^3, \quad (87)$$

$$2UW - \dot{Z} = \kappa a W^3. \quad (88)$$

Here the function Z is defined by the expression $Z = (UW^2)/V$.

This system of equations has an exact solution

$$U = 2 \left(\kappa \alpha - \frac{m^2}{4} \right) W^2, \quad Z = W^4 \left(\kappa \alpha - \frac{m^2}{4} \right) \left(1 - \frac{1}{3} \kappa a W^2 \right). \quad (89)$$

From the definition of Z we have

$$V = 2 \left(1 - \frac{1}{3} \kappa a W^2 \right)^{-1}. \quad (90)$$

At small values of W Eq. (15) is easily solved and one comes to the expression

$$r = \text{const } W^{\sqrt{5}-1}. \quad (91)$$

It is not difficult to see, making use of (89), (90) and (91), that at small W inequalities (19) hold rigorously. From (88) and (89) we find for the pressure, as a *scalar* quantity, the expression

$$\frac{p}{c^2} = -\frac{a}{2} + \frac{\alpha}{2(\kappa \alpha - m^2/4)W^2}. \quad (92)$$

That is why the singularity at $W = 0$ cannot be eliminated by the choice of coordinate system. At $m^2 = 0$ solution (11) becomes the solution to the RTG, found in ref. [9].

From expression for U it is evident that no restriction for the time slowing down by the gravitational field arises from the state equation of the substance (83), and therefore the pressure at the center, $W = 0$, according to (92), gets infinite, which is physically inadmissible.

Thus, *the utmostly hard state equation of the substance (83) does not realize* because it leads to the time stop at the center, $W = 0$, and so *violates* the above mentioned principle of the bound for the time slowing down.

5. Is the Minkowski space observable?

Now we ask a question: if the Minkowski space observable, at least in principle?

To answer it we write down equations (83) in the form

$$\frac{m^2}{2}\gamma_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) - R_{\mu\nu} + \frac{m^2}{2}g_{\mu\nu}.$$

It is seen from here that in the r.h.s. there are only geometric characteristics of the effective Riemannian space and quantities which define the substance distribution in this space.

Now let us make use of the Weyl–Lorentz–Petrov theorem [10], according to which: “*If one knows . . . the equations of all timelike and all isotropic geodesic lines it is possible to determine the metric tensor up to a constant multiplier*”. Hence it follows that with help of the experimental study of the particles and the photon in the Riemann space one can, *in principle*, determine the *metric tensor* $g_{\mu\nu}$ of the effective Riemannian space. Substituting further $g_{\mu\nu}$ into the equation one can determine the Minkowski space *metric tensor*. After that one can, with help of coordinate transformations, provide a passage to an *inertial* Galilean coordinate system. Thus the Minkowski space is, in principle, *observable*.

It is proper to quote here the words by V.A.Fock [11]: “*How has one to define the straight line: as a light ray or as a straight line in the Euclidean space in which harmonic coordinates x_1, x_2, x_3 are used as Cartesian coordinates? We believe that the second definition is the only correct one. Actually we used them when we said that the ray of light near the Sun has a form of a hyperbola*”, and further apropos of this: “*. . . consideration that the straight line, as a ray of light, is more directly observable, it has no significance: what is decisive in definitions is not their direct observability but rather a correspondence to Nature, though this correspondence is established by an indirect deduction*”.

The *inertial* coordinate system, as we see, is related to the substance distribution in the Universe. Thus, the RTG gives us, *in principle*, an opportunity to determine the *inertial* coordinate system.

6. The evolution of homogeneous and isotropic Universe

6.1. The equations of the evolution of the scale factor

In a homogeneous and isotropic Universe the interval in the effective Riemannian space can be presented in the Friedmann–Robertson–Walker metric:

$$ds^2 = c^2 U(t) dt^2 - V(t) \left[\frac{dr^2}{1 - kr^2} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2) \right], \quad (93)$$

whereas the interval in the Minkowski space takes the form

$$d\sigma^2 = c^2 dt^2 - dr^2 - r^2 (d\theta^2 + \sin^2 \theta d\phi^2). \quad (94)$$

Let us write down equations (1), (2) of the RTG in the form

$$\frac{m^2}{2}\gamma_{\mu\nu} = 8\pi G\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) - R_{\mu\nu} + \frac{m^2}{2}g_{\mu\nu}, \quad (95)$$

$$\partial_\mu \tilde{g}^{\mu\nu} + \gamma_{\lambda\sigma}^\nu \tilde{g}^{\lambda\sigma} = 0. \quad (96)$$

With account of equations

$$\begin{aligned} \gamma_{22}^1 &= -r, \quad \gamma_{33}^1 = -r \sin^2 \theta, \quad \gamma_{12}^2 = \gamma_{13}^3 = 1/r, \\ \gamma_{33}^2 &= -\sin \theta \cos \theta, \quad \gamma_{23}^3 = \cot \theta, \\ \tilde{g}^{00} &= V^{3/2} U^{-1/2} (1 - kr^2)^{-1/2} r^2 \sin \theta, \\ \tilde{g}^{11} &= -V^{1/2} U^{1/2} (1 - kr^2)^{1/2} r^2 \sin \theta, \\ \tilde{g}^{22} &= -V^{1/2} U^{1/2} (1 - kr^2)^{-1/2} \sin \theta, \\ \tilde{g}^{33} &= -V^{1/2} U^{1/2} (1 - kr^2)^{-1/2} (\sin \theta)^{-1}, \end{aligned} \quad (97)$$

equations (96) for $\nu = 0$ and $\nu = 1$ take the form

$$\frac{d}{dt} \left(\frac{V}{U^{1/3}} \right) = 0, \quad (98)$$

$$-\frac{d}{dr} \left[(1 - kr^2)^{1/2} r^2 \right] + 2(1 - kr^2)^{-1/2} r = 0. \quad (99)$$

For components $\nu = 2$ and $\nu = 3$ equations (96) hold identically. It follows from equations (98) and (99) that

$$V/U^{1/3} = \text{const} = \beta^4 \neq 0, \quad k = 0. \quad (100)$$

Thus, since the system of equations of the RTG is complete, it *leads unambiguously, in contrast to the GTR, to a unique solution, that is the flat spatial (Euclidean) geometry of the Universe.*

Assuming that

$$a^2 = U^{1/3}, \quad (101)$$

we obtain

$$ds^2 = \beta^6 \left[c^2 d\tau_g^2 - \left(\frac{a}{\beta} \right)^2 (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) \right]. \quad (102)$$

Here the quantity

$$d\tau_g = \left(\frac{a}{\beta} \right)^3 dt \quad (103)$$

determines the rate of dilating of the lapse of time in the presence of the gravitational field in comparison with the inertial time t .

Common constant numerical factor β^6 in the interval ds^2 equally increases both the time and the spatial variables. It does not reflect the dynamics of the Universe development but determines the time of the Universe and its spatial scale. The time of the Universe is determined by the quantity $d\tau$ as a timelike part of the interval ds^2

$$d\tau = \beta^3 d\tau_g = a^3 dt, \quad (104)$$

$$ds^2 = c^2 d\tau^2 - \beta^4 a^2(\tau) (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2). \quad (105)$$

The energy-momentum tensor of the substance in the effective Riemannian space takes the form

$$T_{\mu\nu} = (\rho + p) U_\mu U_\nu - g_{\mu\nu} p, \quad (106)$$

where ρ and p are the density and the pressure, respectively, of the substance in its rest frame, while U_μ is its velocity. Since g_{0i} and R_{0i} are zero for interval (105), it follows from equation (95) that

$$T_{0i} = 0 \text{ и } U_i = 0. \quad (107)$$

This means that in the *inertial frame* defined by the interval (94) the *substance* during the Universe evolution remains in the state of rest. Immobility of the substance in the homogeneous and isotropic Universe (leaving aside peculiar velocities of Galaxies) corresponds, in some sense, to early (pre-Friedmann) A. Einstein's concepts of the Universe.

So-called "expansion of the Universe", observed with the red shift, is caused *not by the substance motion* but rather by *the change* of the *gravitational field* with time. One has to keep in mind this observation when using the established term "expansion of the Universe".

With the description of interval (105) in the proper time τ the interval of the primordial Minkowski space (94) assumes the form

$$d\sigma^2 = \frac{c^2}{a^6} d\tau^2 - dr^2 - r^2(d\theta^2 + \sin^2 \theta d\phi^2). \quad (108)$$

On the basis of (105) and (108) and taking into account that

$$R_{00} = -3\frac{\ddot{a}}{a}, \quad R_{11} = \beta^4(a\ddot{a} + 2\dot{a}^2), \quad (109)$$

$$T_{00} - \frac{1}{2}g_{00}T = \frac{1}{2}(\rho + 3p), \quad T_{11} - \frac{1}{2}g_{11}T = \frac{1}{2}\beta^4 a^2(\rho - p), \quad (110)$$

we obtain from equations (95) the equations for the scale factor

$$\frac{1}{a} \frac{d^2 a}{d\tau^2} = -\frac{4\pi G}{3} \left(\rho + \frac{3p}{c^2} \right) - \frac{1}{6} (mc)^2 \left(1 - \frac{1}{a^6} \right), \quad (111)$$

$$\left(\frac{1}{a} \frac{da}{d\tau} \right)^2 = \frac{8\pi G}{3} \rho(\tau) - \frac{1}{12} (mc)^2 \left(2 - \frac{3}{a^2 \beta^4} + \frac{1}{a^6} \right). \quad (112)$$

In the absence of the substance and gravitational waves equations (111), (112) have a trivial solution $a = \beta = 1$, i.e. the evolution of the empty Universe does not occur and the effective Riemannian space coincides with the Minkowski space. *Let us note that in the developed theory the absolute meaning of the scale factor a acquires the physical sense.* At $m = 0$ equations (111) and (112) coincide with the Friedmann equation for the evolution of the flat Universe. However, the presence of terms with $m \neq 0$ essentially changes the character of evolution at small and large values of the scale factor.

The appearance of additional terms in equations (111) and (112) at $m^2 \neq 0$ (in particular, of the terms $\sim m^2/a^6$) is related to the passage from the inertial system time t to the time τ (104). Since the gravitation influences the lapse of time, these terms appear to be large enough to influence the character of evolution in the strong gravitational fields (in spite of the smallness of the graviton mass).

Specifically, because of this change of the lapse of time in the gravitational field the forces arise that are manifested as repulsive forces during the shrinking of the Universe or as attractive forces in the final stage of the expansion.

The proportionality of the r.h.s. terms in (111) and (112) to the square of the rest mass of the graviton is a manifestation of the fact that only at $m^2 \neq 0$ the effective Riemannian space preserves the connection with the basic Minkowski space.

6.2. Absence of the cosmological singularity

From the covariant conservation law of the energy-momentum tensor $\tilde{T}^{\mu\nu} = \sqrt{-g} T^{\mu\nu}$

$$\nabla_\mu \tilde{T}^{\mu\nu} = \partial_\mu \tilde{T}^{\mu\nu} + \Gamma_{\alpha\beta}^\nu \tilde{T}^{\alpha\beta} = 0 ,$$

where ∇_μ is the covariant derivative and $\Gamma_{\alpha\beta}^\nu$ are the Christoffel symbols in the Riemannian space, which follows from (1), (2) and from expression (106), the expression results

$$-\frac{1}{a} \frac{da}{d\tau} = \frac{1}{3\left(\rho + \frac{p}{c^2}\right)} \frac{d\rho}{d\tau} . \quad (113)$$

For the state equation of the substance $p = f(\rho)$ equation (113) determines the dependence of the substance density on the scale factor. In the case when the state equation has the form

$$\frac{p}{c^2} = \omega\rho ,$$

this dependence is given by the expression

$$\rho = \frac{\text{const}}{a^{3(\omega+1)}} .$$

For the cold substance, including the dark and baryon masses, $\omega_{CDM} = 0$; for the radiation density $\omega_r = 1/3$, and for the quintessence $\omega_q = -1 + \nu$. Thus the total substance density in Eqs. (111) and (112) has the form:

$$\rho = \frac{A_{CDM}}{a^3} + \frac{A_r}{a^4} + \frac{A_q}{a^{3\nu}} , \quad (114)$$

where A_{CDM} , A_r and A_q are constant quantities. According to (114) the radiation dominated stage of the Universe evolution takes place at small values of the scale parameter ($a \ll 1$):

$$\rho \approx \rho_r = \frac{A_r}{a^4} .$$

Turning to equation (112), one can notice that at $a \ll 1$ the negative term in the r.h.s. of the equation grows with decreasing of the scale factor as $1/a^6$ in modulus. Since the l.h.s. of the equation is positive definite there must exist a minimum value of the scale factor

$$a_{\min} = \frac{mc}{(32\pi G A_r)^{1/2}} = \left(\frac{m^2 c^2}{32\pi G \rho_{\max}} \right)^{1/6} . \quad (115)$$

The existence of the minimal value of the factor (115) means that the process of slowing down of the lapse of time by the gravitational field during the compression of the Universe stops. Therefore, the gravitational field cannot stop the lapse of time.

Thus, *due to the graviton mass and, hence, to the presence of the gravitational forces related with the change of the lapse of time the cosmological singularity is eliminated*, and the expansion of the Universe starts from a finite value of the scale factor (115). Specifically, here a surprising property of the gravitational field is manifested: an ability to create in the strong fields the

repulsive forces which stop the process of the compression of the Universe and then provide its accelerated expansion.

On the basis of (111) and (115) we determine the initial acceleration which was a “push” to the expansion of the Universe.

It is

$$\left. \frac{1}{a} \frac{d^2 a}{d\tau^2} \right|_{\tau=0} = \frac{8\pi G}{3} \rho_{\max},$$

and hence, in the RTG, in the radiation dominated stage of the Universe *in the period of the accelerated expansion*, which *precedes* the Friedmann expansion stage, the scalar curvature is not zero and at $\tau = 0$

$$R = -\frac{16\pi G}{c^2} \rho_{\max},$$

while it is equal to zero in the GTR. When the scale factor $a(\tau)$ is

$$a^2(\tau) = \frac{3}{2} a_{\min}^2,$$

the Hubble constant gets maximum

$$H_{\max} = 3^{-2} (32\pi G \rho_{\max})^{1/2},$$

the scalar curvature R is

$$R = -\left(\frac{2}{3}\right)^3 \frac{16\pi G \rho_{\max}}{c^2},$$

and

$$R_{\rho\lambda\mu\nu} R^{\rho\lambda\mu\nu} = 8 \cdot 3^{-7} \left(\frac{32\pi G}{c^2} \rho_{\max} \right)^2.$$

Since the scalar curvature R and the invariant $R_{\rho\lambda\mu\nu} R^{\rho\lambda\mu\nu}$ depend of ρ_{\max} one can expect an intensive production of the gravitons in the radiation dominated stage.

In such a way a relativistic relic gravitational background of non-thermal origin can arise.

6.3. Impossibility of the unlimited “expansion of the Universe”

Considering the gravitational field $\phi^{\mu\nu}$ as a physical field in the Minkowski space, one has to require the fulfilment of the causality principle. This means that the light cone in the effective Riemannian space has to lie inside the light cone of the Minkowski space, i.e. for $ds^2 = 0$ the requirement $d\sigma^2 \geq 0$ holds. Writing down $d\sigma^2$ in the spherical coordinate system

$$d\sigma^2 = c^2 dt^2 - (dr^2 + r^2 d\theta^2 + r^2 \sin^2 \theta d\phi^2) \quad (116)$$

and determining the spatial part of the interval from the condition $ds^2 = 0$, we have

$$d\sigma^2 = c^2 dt^2 \left(1 - \frac{a^4}{\beta^4} \right) \geq 0,$$

i. e.

$$(a^4 - \beta^4) \leq 0. \quad (117)$$

Thus the scale factor (a) is bounded by the condition $a \leq \beta$ and so it would be natural to assume its maximum value as

$$a_{\max} = \beta.$$

With such a choice of a_{\max} the rate of the lapse of time $d\tau_g$ in the moment of the stop of the Universe expansion becomes equal to the rate of the lapse of the inertial time t in the Minkowski space, though the second derivative \ddot{a} and, hence, the scalar curvature R are non-zero. This is this point from which the slowing down of the rate of the lapse of time under the action of the attractive forces will proceed up to the point of the stop of compression, when under the action now already of repulsive forces the opposite process of the acceleration of the rate of the lapse of time up to the rate of the inertial time t of the Minkowski space starts. *Exactly all these physical consequences require necessarily the condition $a_{\max} = \beta$ to be held.* As we will see further (see Part. 6.7), the value of the quantity β is determined by the integral of motion.

Condition (117) does not admit an unlimited growth of the scale factor with time τ , i.e. an unlimited “expansion” of the Universe (in the above-indicated sense) which is provided by the dynamical evolution equation of the scale factor a . Let us note, besides, that the very Universe is infinite because the radial coordinate is defined in the range $0 < r \leq \infty$.

6.4. Evolution of the early Universe

In the radiation dominated stage of the Universe ($\rho = \rho_r$) at $a \ll 1$ equations (111), (112) assume the form

$$\left(\frac{1}{\xi} \frac{d\xi}{d\tau} \right)^2 = \frac{1}{\tau_r^2} \left(1 - \frac{1}{\xi^2} \right) \frac{1}{\xi^4}, \quad (118)$$

$$\frac{1}{\xi} \frac{d^2\xi}{d\tau^2} = \frac{1}{\tau_r^2} \left(\frac{2}{\xi^2} - 1 \right) \frac{1}{\xi^4}, \quad (119)$$

where

$$\xi = \frac{a(\tau)}{a_{\min}}; \quad \tau_r = \left(\frac{3}{8\pi G \rho_{\max}} \right)^{1/2}.$$

The solution to equation (118) is

$$\frac{\tau}{\tau_r} = \frac{1}{2} \left\{ \xi(\xi^2 - 1)^{1/2} + \ln[\xi + (\xi^2 - 1)^{1/2}] \right\}. \quad (120)$$

It $\xi - 1 \ll 1$ ($\tau \ll \tau_r$):

$$a \simeq a_{\min} \left\{ 1 + \frac{1}{2} \left(\frac{\tau}{\tau_r} \right)^2 - \frac{7}{24} \left(\frac{\tau}{\tau_r} \right)^4 \right\}.$$

Adding equations (118) and (119), we obtain

$$\ddot{a}/a + (\dot{a}/a)^2 = (mc)^2/12a^6, \quad \text{где} \quad \dot{a} = da/d\tau.$$

In the GRT the l.h.s. of this equation in the radiation dominated domain is equal to zero exactly and hence the Friedmann stage takes place when the scale factor $a(\tau)$ changes with time as

$\tau^{1/2}$. In the RTG, according to this equation, there exists, in the radiation dominated phase, “pre-Friedmann” stage of the Universe evolution where the scalar curvature R is

$$R = -\frac{1}{2}(mc)^2 \frac{1}{a^6}.$$

Here the particle horizon is

$$R_{\text{part}}(\tau) = a(\tau) \int_0^\tau \frac{c d\tau'}{a(\tau')} \simeq c\tau \left(1 + \frac{1}{3} \frac{\tau^2}{\tau_r^2}\right).$$

The accelerated expansion proceeds, according to (119), till the values $\xi = \sqrt{2}$ (i. e. $a = \sqrt{2} a_{\min}$) during the time

$$\tau_{in} = \tau_r \frac{1}{2}(\sqrt{2} + \ln(1 + \sqrt{2})) \simeq 1.15\tau_r.$$

The quantity \dot{a}/a achieves its maximum value $[\dot{a}/a]_{\max} = 2/3\sqrt{3}\tau_r$ somewhat earlier, at $a/a_{\min} = \sqrt{3/2}$ and at $\tau \sim 0,762\tau_r$. *The large acceleration during the growth of the scale factor starting from its minimum value $(\ddot{a}/a)_0 = 1/\tau_r^2$ is related with effective forces arising because of the difference in the lapse of time t and τ (see equation (104)), caused by the action of gravitation.* Exactly these forces lead to the terms m^2/a^6 in equations (111), (112). At $\tau > \tau_{in}$ the acceleration changes to the deceleration. At $\xi \gg 1$ the expansion (120) passes to the Friedmann regime corresponding to the radiation dominated stage

$$a(\tau) = a_{\min} \xi \simeq a_{\min} \left(\frac{2\tau}{\tau_r}\right)^{1/2},$$

at to the dependence known for this regime

$$\rho \simeq \rho_r(\tau) = \frac{3}{32\pi G\tau^2}; \quad \tau \gg \tau_r. \quad (121)$$

In order that in the first seconds since the beginning of the expansion the laws of primordial nucleosynthesis to be held it is necessary that $\tau_r \lesssim 10^{-2}$ [s]. The bound for ρ_{\max} corresponding to this requirement is rather weak:

$$\rho_{\max} > 2 \cdot 10^{10} \text{ [g} \cdot \text{cm}^{-3}\text{]}.$$

The value ρ_{\max} at energies $kT \simeq 1 \text{ TeV}$ corresponding to the electroweak scale and with account of all degrees of freedom of leptons, quarks etc. is

$$\rho_{\max} \simeq 10^{31} \text{ [g} \cdot \text{cm}^{-3}\text{]},$$

while at the Grand Unification scale $kT \simeq 10^{15} \text{ GeV}$

$$\rho_{\max} \simeq 10^{79} \text{ [g} \cdot \text{cm}^{-3}\text{]}.$$

Thus, since the scale factor cannot become zero, this means that according to the RTG no *Big Bang* could occur in the Universe. In the past, everywhere in the Universe, *the substance* was in the gravitational field in the state of high density and high temperature, and then it evolved as was described above.

6.5. Total relative density of the substance and the graviton mass

Let a_0 is the present value of the scale factor, while ρ_c^0 is the critical density related to the present value of the Hubble constant $H = \left(\frac{1}{a} \frac{da}{dt}\right)_0$ by the relation

$$H^2 = \frac{8\pi G}{3} \rho_c^0 .$$

Introducing the variable

$$x = \frac{a}{a_0} ,$$

and the ratios of the densities

$$\Omega_r^0 = \frac{\rho_r^0}{\rho_c^0}; \quad \Omega_m^0 = \frac{\rho_m^0}{\rho_c^0}; \quad \Omega_q^0 = \frac{\rho_q^0}{\rho_c^0} ,$$

one can, with account of relation (114), write down equations (111), (112) in the form

$$\left(\frac{1}{x} \frac{dx}{d\tau}\right)^2 = H^2 \left\{ \frac{\Omega_r^0}{x^4} + \frac{\Omega_m^0}{x^3} + \frac{\Omega_q^0}{x^{3\nu}} - \frac{f^2}{6} \left(1 - \frac{3}{2\beta^4 a^2} + \frac{1}{2a^6}\right) \right\}; \quad (122)$$

$$\left(\frac{1}{x} \frac{d^2 x}{d\tau^2}\right) = -\frac{H^2}{2} \left\{ \frac{2\Omega_r^0}{x^4} + \frac{\Omega_m^0}{x^3} - 2 \left(1 - \frac{3\nu}{2}\right) \frac{\Omega_q^0}{x^{3\nu}} + \frac{f^2}{3} \left(1 - \frac{1}{a^6}\right) \right\}, \quad (123)$$

where

$$f = \frac{mc}{H} = \frac{m_g c^2}{\hbar H} . \quad (124)$$

For the present value of the quantities at $a_0 \gg 1$ equation (122) gives the relationship

$$1 = \Omega_{\text{tot}}^0 - \frac{f^2}{6} ,$$

i.e. the total relative density is equal to

$$\Omega_{\text{tot}}^0 = \frac{\rho_{\text{tot}}^0}{\rho_c^0} = \Omega_r^0 + \Omega_m^0 + \Omega_q^0 = 1 + \frac{f^2}{6} . \quad (125)$$

Hence the Universe possessing (according to the RTG) the Euclidean spatial geometry has to have $\Omega_{\text{tot}}^0 > 1$, while in the theories with a primordial inflationary expansion leading to the flat geometry the condition $\Omega_{\text{tot}}^0 = 1$ has to be satisfied with a great accuracy ($\sim 10^{-3} \div 10^{-5}$). Equation (125) gives a possibility to evaluate the graviton mass from the recent experimental measurements of Ω_{tot}^0 and H .

6.6. Upper limit for the graviton mass

Determining the cosmological parameters from the observation of the angular asymmetry of the micro-wave background radiation (or CMB) [12] leads systematically to an average value $\Omega_{\text{tot}}^0 > 1$. This concerns both the first quantitative experimental data from COBE [13], Maxima-1 [14] и Boomerang-98 [15] joint data processing of which [16] gives the value $\Omega_{\text{tot}}^0 = 1,11 \pm 0,07$, and excellent data of the experiment WMAP [17] which alone (without taking into account the data on observation of the supernovae SNIa [12, 18] and the galaxy catalogue (2dFGRS [19] and SDSS [20])) give, dependent on the choice of parameters, the values $\Omega_{\text{tot}}^0 = 1,095^{+0,094}_{-0,144}$ and $\Omega_{\text{tot}}^0 = 1,086^{+0,057}_{-0,128}$ [17].

Within the error bars these values certainly do not contradict the value $\Omega_{\text{tot}}^0 = 1$ following from the inflationary model, but they can indicate also the existence of the non-zero graviton mass, according to relations (124), (125). At any rate if to take the value $\Omega_{\text{tot}}^0 = 1,3$ which exceeds more than 2σ the average value of Ω_{tot}^0 then we obtain from (124), (125) with probability 95% the upper bound for the graviton mass. It is convenient to represent the quantity f from (124) in the form of the ratio of the graviton mass to the quantity

$$m_H = \frac{\hbar H}{c^2} = 3,80 \cdot 10^{-66} h ,$$

which could be called the ‘‘Hubble mass’’. At $f^2/6 = 0,3$ the upper limit for the graviton mass is

$$m_g \leq 1,34 m_H \approx 5,1 \cdot 10^{-66} h \text{ [g]} ,$$

or, at $h = 0,70$,

$$m_g < 3,6 \cdot 10^{-66} \text{ [g]} . \quad (126)$$

The Compton wavelength of the graviton appears to be compatible with the Hubble radius of the Universe, c/H

$$\frac{\hbar}{m_g c} \lesssim 0,75 \frac{c}{H} .$$

The estimates of the upper limit for the graviton mass obtained earlier were based on the fact that the gravitational potential with non-zero graviton mass has to have the Yahawa form.

On the basis of analysis of the dynamics of the galaxy clusters and conservative estimates of the distances (~ 580 kps) at which the gravitational connection between the galaxies in the cluster still holds there was obtained in works [21, 22] an upper limit for the graviton mass

$$m_g < 2 \cdot 10^{-62} \text{ [g]} .$$

Our estimate (126) improves this bound more than 5000 times. This is related to the fact that a consistent consideration of the gravitational field in the Minkowski space includes not only the equation, according to which the potential of the weak gravitational field has the Yukawa form, but also general equations of gravitation (1), (2), which are in agreement with all gravitational phenomena in the Sun system and are applicable to the whole Universe, i.e. at distances of the order $c/H \simeq 10^{28}$ [cm], more than 5000 times longer than the distances among gravitationally binded galaxies in the clusters.

6.7. Integral of the Universe evolution and present value of the scale factor

Making use of relation (113) one can exclude the pressure from equation (111), bring it to the form

$$\frac{1}{a} \frac{d^2 a}{d\tau^2} = \frac{4\pi G}{3} \left(a \frac{d\rho}{da} + 2\rho \right) - \frac{1}{6} (mc)^2 \left(1 - \frac{1}{a^6} \right),$$

and to write it further in the form

$$\frac{d^2 a}{d\tau^2} + \frac{dV}{da} = 0, \quad (127)$$

where

$$V = -\frac{4\pi G}{3} a^2 \rho + \frac{(mc)^2}{12} \left(a^2 + \frac{1}{2a^4} \right). \quad (128)$$

Multiplying both sides of equation (127) by $\frac{da}{d\tau}$, we obtain

$$\frac{d}{d\tau} \left[\frac{1}{2} \left(\frac{da}{d\tau} \right)^2 + V \right] = 0,$$

or

$$\frac{1}{2} \left(\frac{da}{d\tau} \right)^2 + V = E = \text{const.} \quad (129)$$

Expression (129) reminds the energy of the unit mass. If the quantity a had the dimension of length then the first term in (129) would correspond to the kinetic energy and the second to the potential one. The quantity $\left(-\frac{4\pi G}{3} \rho a^2 \right)$ in (128) corresponds to the gravitational potential on the boundary of the ball of radius a filled with a substance of constant density ρ while the extra terms in (128) proportional to m^2 correspond to effective forces arising, as was mentioned above, because of the influence of gravitation on the lapse of time.

The quantity E is the integral of the Universe evolution. It is extremely small but at $m \neq 0$ is not zero. Having substituted $(da/d\tau)^2$ from equation (112) into equality (129), we get

$$E = \frac{(mc)^2}{8\beta^4}. \quad (130)$$

In such a way the constant β (see (117)) entering interval (105) and, according to (117), limiting the growth of the scale factor a is expressed via the integral of motion E .

In what follows we shall need the present value of the scale factor, a_0 . One can obtain an estimate of this quantity from the following considerations. Assuming that the Universe evolution begins in the radiation dominated epoch, we have for the ratio a_0/a_{\min}

$$\frac{a_0}{a_{\min}} = \left(\frac{\rho_{\max}}{\rho_r^0} \right)^{1/4},$$

where ρ_r^0 is the present density of the radiation energy. In its turn ρ_r^0 can be expressed in terms of the relative density Ω_r^0 and critical density ρ_c^0

$$\rho_r^0 = \Omega_r^0 \rho_c^0 = \Omega_r^0 \left(\frac{3H^2}{8\pi G} \right).$$

Thus

$$\frac{a_0}{a_{\min}} = \left(\frac{8\pi}{3} \frac{G\rho_{\max}}{H^2 \Omega_r^0} \right)^{1/4} \approx 1,34 \cdot 10^{10} (G\rho_{\max})^{1/4},$$

where $G\rho_{\max}$ is taken in sec^{-2} . (In the course of the calculation of the numerical factor in the above-said expression we used the standard value $H = h/3,0857 \cdot 10^{17} c$ and $\Omega_r^0 = \Omega_\gamma^0 = 2,471 \cdot 10^{-5}/h^2$.)

Then, making use of definition (124), one can represent the value a_{\min} from (115) in the form

$$a_{\min} = \left(\frac{f^2}{6} \right)^{1/6} \left(\frac{3}{16\pi} \frac{H^2}{G\rho_{\max}} \right)^{1/6} = 8,21 \cdot 10^{-7} \left(\frac{f^2}{6} \right)^{1/6} \frac{1}{(G\rho_{\max})^{1/6}},$$

where, according (125)

$$\frac{f^2}{6} = \Omega_{\text{tot}}^0 - 1,$$

a_{\min} at the electroweak scale is

$$a_{\min} \simeq 5 \cdot 10^{-11},$$

and at the scale of the Grand Unification

$$a_{\min} \simeq 5 \cdot 10^{-19}.$$

For the quantity a_0 we have from the ratio a_0/a_{\min} ³

$$a_0 = \left(\frac{f^2}{6} \right)^{1/6} \left(\frac{2\pi}{3} \frac{G\rho_{\max}}{H^2} \right)^{1/12} \frac{1}{(\Omega_r^0)^{1/4}} \simeq 1,1 \cdot 10^4 \left(\frac{f^2}{6} \right)^{1/6} (G\rho_{\max})^{1/12}, \quad (131)$$

where a_0 at ρ_{\max} chosen at the electroweak scale is

$$a_0 \simeq 5 \cdot 10^5,$$

and at the scale of the Grand Unification

$$a_0 \simeq 5,5 \cdot 10^9.$$

As was already mentioned (see Part 6.1.) in the RTG an absolute value of the scale factor acquires a sense. At the average value $\Omega_{\text{tot}} = 1,02$ (i.e. $f^2/6 = 0,02$) and $\rho_{\max} \gtrsim 10^{10} [\text{g}\cdot\text{cm}^{-3}]$ the quantity $a_0 \gg 1$. This justifies the approximations made during the inference of equality (125).

³In the course of the numerical estimate one takes as a relative density of the relativistic particles Ω_r^0 the relative density of the microwave relativistic radiation Ω_γ^0 because it follows from the data on neutrino oscillations that at least two types of neutrino are nonrelativistic at present time. When extrapolating to the early Universe one should certainly take into account that the temperature of the relic radiation in the course of the evolution raised due to e^+e^- -annihilation; before the onset of the annihilation it was equal to the temperature of the neutrino gaz which at that time also consisted of relativistic neutrinos and contributed into the total density of the relativistic particles. In the same way, when extrapolating to the early Universe, the density of the relativistic gaz raises due to relativisation of other particles that are produced. However, due to the fact that the quantity Ω_r^0 enters (131) in the form of $(\Omega_r^0)^{1/4}$, the numerical estimate (131) changes no more than three times (even if to assume that the number of degrees of freedom in the relativistic gaz is about 100).

6.8. Incompatibility of the RTG with the existence of a constant cosmological term (Λ CDM theory). Necessity of quintessence with $\nu > 0$

As was already mentioned, when considering the gravitational field as a physical field in the Minkowski space, it is necessary to require the fulfilment of the causality principle. This requirement, applied to the Universe evolution, leads to inequality (117) according to which the scale factor is bounded by the inequality $a \leq a_{\max} = \beta$. In other words, according to the RTG, the unlimited expansion of the Universe is impossible. The mathematical apparatus of the RTG automatically provides the fulfilment of this condition in the case *when the matter density decreases with increase of the scale factor*. In fact, the structure of the term proportional to m_g^2 in equation (112) is such that due to positive definiteness of the l.h.s. of the equation the third term in the bracket provides the absence of the cosmological singularity at $a \ll 1$ while the first term limits the minimum value of the matter density (and by this limits from above the value of the scale factor) at $a \gg 1$. The condition $\frac{8\pi G}{3}\rho - \frac{(mc)^2}{6} = 0$ written in the form $\frac{H^2}{\rho_c^0}\rho - \frac{(mc)^2}{6} = 0$ (where H is the present value of the Hubble constant) leads to the equality $\rho_{\min} = \frac{(mc)^2}{6H^2}\rho_c^0$, or in another form

$$\frac{\rho_{\min}}{\rho_c^0} = \frac{f^2}{6} = \Omega_{\text{tot}}^0 - 1. \quad (132)$$

The field theory of gravitation appears incompatible with the existence of the constant cosmological term leading to an unlimited expansion of the Universe. In fact, at $a \gg 1$ it follows from equation (122):

$$\Omega_{\Lambda}^0 < \frac{f^2}{6}.$$

However, this inequality is incompatible with the condition

$$\Omega_{\Lambda}^0 > \frac{f^2}{6},$$

which is needed in order that in the present time there existed, according to equation (123), an accelerated expansion.

Thus the only possibility to explain, in the framework of the RTG, the accelerated expansion of the Universe observed at the present time is the existence of a quintessence with $\nu > 0$ or some other substance the density of which decreases with increase of the scale factor (but not faster than const/a^2). The RTG excludes a possibility of the existence both of the constant cosmological term ($\nu = 0$) and the “phantom” expansion ($\nu < 0$) [23].

6.9. Start and finish of the present accelerated expansion

The most strict bounds, $\Omega_{\text{tot}}^0 = 1.018_{-0.022}^{+0.013}$, obtained in the experiment WMAP [17] in the framework of Λ CDM-model with use of the data from the galaxy catalogue SDSS and the data on

supernovae SNIa admit within 1σ the value $\Omega_{\text{tot}}^0 = 1,03$. This difference in the RTG, according to relations (124), (125), *determines the graviton mass*

$$m_g = 0,424 m_H = 1,6 \cdot 10^{-66} h .$$

Further we shall use for definiteness namely this value of the graviton mass. Since by the beginning of the epoch of the present acceleration $\Omega_r \ll \Omega_m$ and $a \gg 1$, then the start and finish of the accelerated expansion are determined, according to (123), by the roots $x_1 < 1 < x_2$ of the equation $F(x) = 0$, where the function $F(x)$ is

$$F(x) = \frac{\Omega_m^0}{x^3} - 2 \left(1 - \frac{3\nu}{2} \right) \frac{\Omega_q^0}{x^{3\nu}} + \frac{f^2}{3} .$$

Here the value of the first root x_1 is related to the red shift Z_1 , corresponding to the beginning of the acceleration epoch

$$\frac{1}{x_1} = \frac{a_0}{a_1} = Z_1 + 1 . \quad (133)$$

The time lapsed from the beginning of the Universe expansion till the beginning of the present acceleration can be established from equation (122). Neglecting the duration of the radiation dominated epoch and the value of the scale factor a by its end, we have

$$\tau_1 \approx \frac{1}{H} \int_0^{x_1} \frac{dx}{x[\Phi(x)]^{1/2}} = \frac{1}{H} \int_{Z_1+1}^{\infty} \frac{dy}{y (\Omega_m^0 y^3 + \Omega_q^0 y^{3\nu} - f^2/6)^{1/2}} ,$$

where

$$\Phi(x) = \frac{\Omega_m^0}{x^3} + \frac{\Omega_q^0}{x^{3\nu}} - \frac{f^2}{6} .$$

Here the values $\Omega_m^0 = 0,27$, $\Omega_q^0 = 0,73$ are assumed according to [25].

Correspondingly, the time of the termination of the epoch of accelerated expansion and of the passage to deceleration is

$$\tau_2 = \frac{1}{H} \int_0^{x_2} \frac{dx}{x[\Phi(x)]^{1/2}} ,$$

and the present age of the Universe, τ_0 , is:

$$\tau_0 = \frac{1}{H} \int_0^1 \frac{dx}{x[\Phi(x)]^{1/2}} .$$

The physical distance passed by the light (particle horizon) by the present moment of time is determined by the following expansion

$$\begin{aligned} D_{\text{part}}(\tau_0) &= \frac{c}{H} \int_1^{a_0/a_{\min}} dy \frac{1}{[\Omega_r^0 y^4 + \Omega_m^0 y^3 + \Omega_q^0 y^{3\nu} - f^2/6 \times (1 + y^6/2a_0^6)]^{1/2}} \simeq \\ &\simeq \frac{2}{\sqrt{\Omega_m^0}} \frac{c}{H} . \end{aligned}$$

This quantity determines the size of the observed Universe by the present time. Qualitatively (without exact scales) the temporal dependence of the scale factor, its velocity \dot{a} and \ddot{a} is given in the Figure.

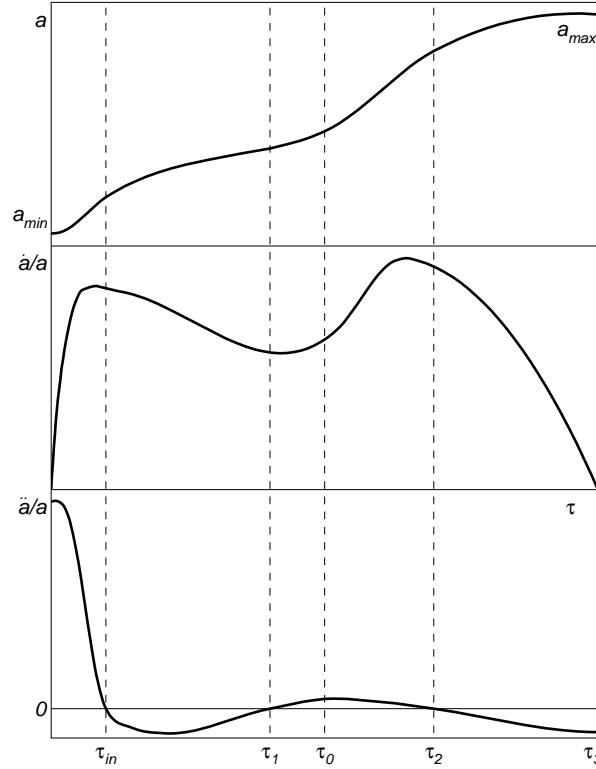


Figure. Qualitative curves of the dependence of the scale factor (upper part) velocity and acceleration (lower part) dependent on time τ . Here $\tau_{in} = 1,15\tau_r$. The present moment of time is designated τ_0 . In the beginning the scale factor grows from its minimum value a_{min} with a very high acceleration which in a short time enough, τ_{in} , becomes zero. The velocity in this period of time increases from the zero value up to the maximum one. The scale factor during this period of time changes insignificantly: $a(\tau_{in}) = \sqrt{2}a_{min}$. Further on the expansion occurs with negative acceleration which becomes zero at some moment of time τ_1 . The value of the velocity drops and somewhat later than τ_1 it achieves its minimum value. The scale factor in this period of time continues to rise (expansion continues). The motion with positive acceleration continues till the moment τ_2 . The velocity and the scale factor increase. At $\tau > \tau_2$ the expansion occurs again with negative acceleration until the time when at the momentum τ_3 the expansion stops. The scale factor achieves its maximum value. On this the half-cycle is completed and everything repeats in the opposite order: the expansion epoch is changed by the compression epoch. For the quantity \dot{a}/a the first maximum is situated at $a = \sqrt{3/2} a_{min}$ ($\tau \sim 0,76 \tau_r$) somewhat earlier than τ_{in} , in the same way as the second maximum happens prior to τ_2 . The minimum of \dot{a}/a , contrary to this, is situated later than τ_1 . This follows from the fact that the quantity $(d/d\tau)(\dot{a}/a) = (\ddot{a}/a) - (\dot{a}^2/a^2)$ at $\ddot{a} = 0$ is negative.

6.10. Maximum value of the scale factor and the integral of the Universe evolution

The time corresponding to the end of the accelerated expansion and the beginning of deceleration leading to the stop of expansion strongly depends on parameter ν (see the Table).

Table. The time of the beginning of the accelerated expansion of the Universe, τ_1 , that of its termination, τ_2 , and the time of the maximum expansion (oscillation half-period)

| τ_{\max} [bill. years]. | | | |
|------------------------------|-----------|------------|---------------|
| ν | τ_1 | τ_2 | τ_{\max} |
| $\nu = 0,05$ | 7,0 - 8,2 | 980 - 1080 | 1220 - 1360 |
| $\nu = 0,10$ | 7,0 - 8,2 | 440 - 485 | 620 - 685 |
| $\nu = 0,15$ | 7,1 - 8,3 | 275 - 295 | 430 - 460 |
| $\nu = 0,20$ | 7,1 - 8,3 | 190 - 205 | 325 - 347 |
| $\nu = 0,25$ | 7,2 - 8,5 | 142 - 149 | 263 - 280 |
| $\nu = 0,30$ | 7,5 - 8,7 | 109 - 113 | 227 - 235 |

The scale-factor, corresponding to the stop of the expansion, x_{\max} , is determined by the root of equation (122) and at small ν is, with a good accuracy,

$$x_{\max} \simeq \left(\frac{6\Omega_q^0}{f^2} \right)^{1/3\nu} = \left(\frac{\Omega_q^0}{\Omega_{\text{tot}}^0 - 1} \right)^{1/3\nu}. \quad (134)$$

Substituting the value of a_0 from equation (131) into this expression, we find

$$a_{\max}^4 = \frac{1}{\Omega_r^0} \left(\frac{f^2}{6} \right)^{2/3} \left(\frac{2\pi}{3} \frac{G\rho_{\max}}{H^2} \right)^{1/3} \left(\frac{\Omega_q^0}{\Omega_{\text{tot}}^0 - 1} \right)^{4/3\nu}.$$

Taking into account this equality and that the integral of motion is

$$E = \frac{(mc)^2}{8a_{\max}^4},$$

we obtain

$$E = \frac{(mc)^2}{8} \Omega_r^0 \left(\frac{6}{f^2} \right)^{2/3} \left(\frac{3}{2\pi} \frac{H^2}{G\rho_{\max}} \right)^{1/3} \left(\frac{\Omega_{\text{tot}}^0 - 1}{\Omega_q^0} \right)^{4/3\nu}.$$

It is evident from here that the integral of motion of the Universe evolution is a very small quantity. Making use of the expression for x_{\max} , it is easy to determine the relative acceleration of the attraction in the moment of the stop of expansion:

$$\frac{\ddot{a}}{a} \sim -\frac{\nu}{4} \left(\frac{m_g c^2}{\hbar} \right)^2,$$

and therefore the scalar curvature R is

$$R = \frac{3\nu}{2c^2} \left(\frac{m_g c^2}{\hbar} \right)^2.$$

It is essential that the relative minimum value of the density (ρ_{\min}/ρ_c^0), corresponding to the maximum of expansion depends on the value of ($\Omega_{tot}^0 - 1$) only, i.e. on the graviton mass (see (125), (126)). At ($\Omega_{tot}^0 = 1,02$) the value of ρ_{\min} is quite large and even exceeds very much the *present density of radiation*. In paper [24] the authors proceeded from the present age of the Universe, $(13,7 \pm 0,2) \cdot 10^9$ years given in [25, 26]. This quantity is calculated in [25, 26] basically in the Λ CDM-model. It is very important that the recent observation of SN1a [27, 28] in the range $Z \gtrsim 1$ can give a *straightforward information* about the beginning of the present acceleration. Such data were obtained in the excellent paper by A. Ries et al [28] according to which the deceleration was changed by the present acceleration at the following values of the red shift

$$Z = 0,46 \pm 0,13 .$$

This result conforms with the presented picture of evolution. It enables to directly determine the value x_1 (see (133)) and to precise the admissible range of the cosmological parameters ⁴.

The expansion up to the maximum value of the scale parameter and its consequent compression lead to the oscillatory character of the Universe evolution. The idea of the oscillatory character of the Universe evolution was repeatedly advanced earlier proceeding mainly from the philosophical considerations (see, e.g., [29] – [31]). Such a regime could, in principle, be expected in the closed Friedmann model with $\Omega_{tot} > 1$. However, firstly, the insurmountable difficulty related to the passage through the cosmological singularity and, secondly, considerations related to the growth of entropy from cycle to cycle [31] do not allow this.

It is necessary to emphasize that in the framework of the Hilbert–Einstein equations the flat Universe cannot be oscillatory ⁵. These difficulties for the infinite Universe are eliminated in the RTG. Since singularities are absent from the RTG the Universe could exist an infinite time during which the interaction occurred among its domains and this led to homogeneity and isotropy of the Universe with some structure of inhomogeneity which we did not take into account for the sake of simplicity.

In the above-mentioned approximation x_{max} is related to the scale factor (x_2) corresponding to the termination of the accelerated expansion by the relation

$$x_2 = \left(1 - \frac{3}{2}\nu\right)^{1/3\nu} \cdot x_{max} \approx \frac{1}{\sqrt{e}} x_{max} .$$

The time corresponding to the stop of the expansion (half-period of oscillation) with chosen in [24] value of the graviton mass $m_g = 0,49 m_H$ is about $1300 \cdot 10^9$ years for $\nu = 0,05$, near $650 \cdot 10^9$ years for $\nu = 0,10$ and $270 \cdot 10^9$ years for $\nu = 0,25$.

⁴Let us notice that the distance to supernovae (D_L), determined from the relation $F = L/4\pi D_L^2$ (where L and F are the luminosity of the standard SN1a and observed flow, respectively), is expressed via cosmological parameters of the RTG by the relation

$$D_L = \frac{c}{H}(Z+1) \int_1^{1+Z} \left[\Omega_m^0 y^3 + \Omega_q^0 y^{3\nu} - \frac{f^2}{6} \right]^{-1/2} dy .$$

⁵Paper [32] on the cyclic evolution of the Universe is erroneous because the “solution” given in it is not in fact a solution to the basic system of the Hilbert–Einstein equations, which can be checked by the direct substitution. Paper [33] is also erroneous because the system of equations (3), (17) and (18) of this article is internally contradictory.

The attractiveness of the oscillatory evolution of the Universe is mentioned in the recent paper [34]. The oscillatory regime is realized by the price of introducing a scalar field interacting with the substance and use of the extra dimensions. Some important consideration were advanced that the phase of the accelerated expansion promotes the entropy conservation in the repeating cycles of the evolution. In the RTG the oscillatory character of the Universe evolution is achieved as a result of introducing of the only massive gravitational field as a physical field generated by the total energy-momentum tensor in the Minkowski space.

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