



STATE RESEARCH CENTER OF RUSSIA  
INSTITUTE FOR HIGH ENERGY PHYSICS

IHEP 2006–16

F.F. Tikhonin\*

## AN INSIGHT INTO THE ANATOMY OF ELECTRO-GRAVITATIONAL INTERACTIONS <sup>1</sup>

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<sup>1</sup> Talk given at the Seventh International Wigner Symposium, 24–29 August 2001.  
University of Maryland, College Park, Maryland, U.S.A.

\* E-mail: feodor.tikhonin@mx.ihep.ru

**Abstract**

Tikhonin F.F. An Insight into the Anatomy of Electro-Gravitational Interactions: IHEP Preprint 2006-16. – Protvino, 2006. – p. 7, figs. 3, refs.: 13.

An interplay between electromagnetic and gravitational interactions is studied with particular emphasis on the particles mass dependence of amplitudes. An interesting cancellations between diagrams are explicitly demonstrated.

**Аннотация**

Тихонин Ф.Ф. О структуре электрогравитационных взаимодействий: Препринт ИФВЭ 2006-16. – Протвино, 2006. – 7 с., 3 рис., библиогр.: 13.

Изучается взаимодействие между электромагнитными и гравитационными силами с акцентом на зависимость амплитуд от масс участвующих частиц. Демонстрируется взаимосокращение между диаграммами, описывающими процесс рождения пары фотон-гравитон.

*"No," replied Margarita, "what really puzzles me is where you have found the space for all this". With a wave of her hand Margarita emphasised the vastness of the hall they were in. Koroviev smiled sweetly, wrinkling his nose. "Easy!" he replied. "For anyone who knows how to handle the **fifth dimension** it's no problem to expand any place to whatever size you please. No, dear lady, I will say more - to the devil knows what size.*

M.A.Bulgakov, "The Master and Margarita" [1]

## 1. Introduction

The objective of this simple consideration is to attract slightly more careful attention to the interplay between electromagnetic and gravitational interactions, than usually is required for cross sections calculation. Total cross section for the typical elementary electrogravitational process like that considered in this note,  $\ell^+ + \ell^- \rightarrow \gamma + \text{graviton} \equiv \mathcal{G}$ , is constant proportional to  $\alpha \cdot G_N$  with  $\alpha \simeq 1/128$  and  $G_N \simeq 2.6 \cdot 10^{-20} \text{yb}$ , while typical elementary cross section for electroweak process (with gauge boson instead of  $\mathcal{G}$ ) is of the order  $\alpha^2/s$ , where  $s$  is the energy squared of collision. It follows, that both cross sections should equate at energies of the order  $\sqrt{s} \simeq 10^{18} \text{ GeV}$ . This huge gap is the reflection of the hierarchy problem, one of the ways how to circumvent this problem was proposed recently [2] and outlined in a cursory manner below.

First attempt of unified description of Electromagnetism and Gravity was undertaken by Theodor Kaluza [3]. He achieved his goal by adding an extra, fifth dimension to the visible four-dimensional space-time and interpreted the  $\mu 5$  ( $\mu = 1, 2, 3, 4$ ) components as the electromagnetic vector potential. Afterwards Oscar Klein [4] suggested that this fifth dimension has a peculiar, periodic topology, and, therefore, he "compactified" it to a circles of small radii, attached to each point of our visible world.

The fruitful Kaluza's idea revived in 1980's [5] and, more recently, with the advent of theories, known under the generic name "M theory" [6], which inevitably lead to consideration of ten or eleven (space) dimensions. At last, at the end of preceeding century, interest to the extra dimentions greatly arose, because, on the one hand these theories may provide a natural solution of hierarchy problem and on the other hand give rich phenomenology for the collider physics [2]. The idea is, that while all the usual particles live in our 3-dimensional world (brane), gravitons are ubiquitous and live in additional dimensions (bulk) also. This explains why the Newtonian constant is so small — gravitons waste their strength smearing it onto the extra dimensions. Potentially, gravitational interaction can be enhanced by Kaluza-Klein excitations

of gravitons up to order of electroweak one. How does this mechanism work see, for instance, in the recent reviews [7]. According to concept outlined above the gravitational and electromagnetic interactions can be considered on the equal foot and influence one of them to another might be perceptible. Therefore, we can now speak about electrogravitational processes hoping to observe them at the laboratory. Many such processes are considered already [8], but in what follows the particular accent will be done on the mass dependence of the corresponding cross sections. Two types of particles (both are yet to be discovered) are related closely to the masses: they are Higgs bosons and gravitons. So, it is not unexpected, that some similarity might be loomed between behaviours of these two particle types. Let us then first recall some features of the Higgs boson production amplitudes.

## 2. Preliminaries

Consider the so called Björken process,  $\ell^+ + \ell^- \rightarrow Z + \text{higgs} \equiv H^0$ .

In calculation of the cross section for it one usually restricts to the consideration of the graph (a) from Fig.1 only.

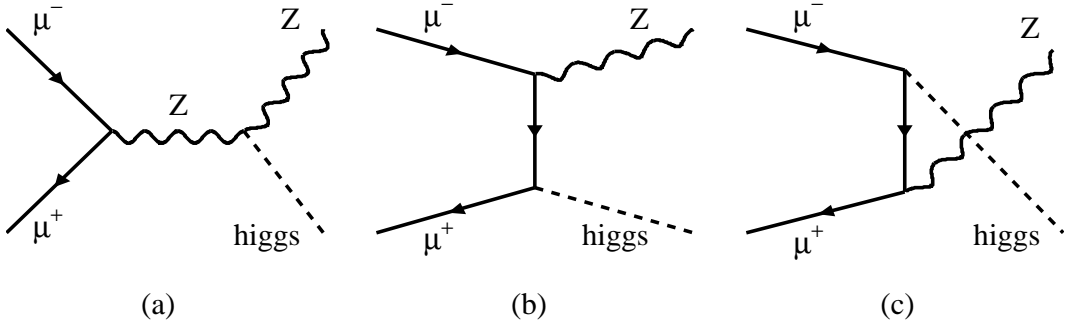


Figure 1. Diagrams contributing to the process  $\ell^+ + \ell^- \rightarrow Z + H^0$  are shown. Notations are selfevident.

So do we, but let us take into account masses of the initial fermions (muons, to be definite). Asymptotics of this process at  $\sqrt{s} \rightarrow \infty$  is as follows

$$\sigma_{as}^{(c)}(\mu^+ \mu^- \rightarrow ZH^0)|_{m_\mu \neq 0} = \frac{2\pi \alpha^2 g_A^2}{\sin^4(2\theta_{\mathcal{EW}})} \frac{m_\mu^2}{m_Z^4}, \quad (1)$$

where  $g_A^2$  is the constant of weak axial-vector coupling of the Z boson to muon,  $\theta_{\mathcal{EW}}$  is the angle of electroweak mixing, and all other notations are selfevident. It is easily to see that this result contradicts the unitarity condition. Indeed, the cross-section obtained is azimuthal angle independent. It means, that the scattering process occurs in the s-wave. But the s-wave amplitudes must satisfy the inequality

$$\sigma_{J=0} \leq 1/s,$$

while one sees from Eq.(1), that it is constant (equal to  $\cong 1.2 \cdot 10^{-2} \text{ fb}$ ). In the expression above  $J$  is the angular momentum of scattering.

In order to avoid the contradiction we include two more amplitudes in the calculation corresponding to the diagrams (b) and (c) of Fig.1. Cross section due to the sum of these two amplitudes is angle independent and is equal to  $\cong 1.2 \cdot 10^{-2} \text{ fb}$  again, but the interference between them and that of (a) results in  $\cong -2.4 \cdot 10^{-2} \text{ fb}$ , completely eliminating confusion [9]. This remarkable fine tuning reflects fundamental features of the underlying Standard Model [10]. Namely, it is related to the renormalizability of the latter, because the asymptotic behaviour of the tree amplitudes reflects the behaviour of the loop integrals with respect to their limits.

### 3. Graviton-photon annihilation of $\ell^+\ell^-$ pair

Now, let's turn to the "analogous" process  $\ell^+ + \ell^- \rightarrow \gamma + \mathcal{G}$ , to which diagrams on the Fig. 2 correspond.

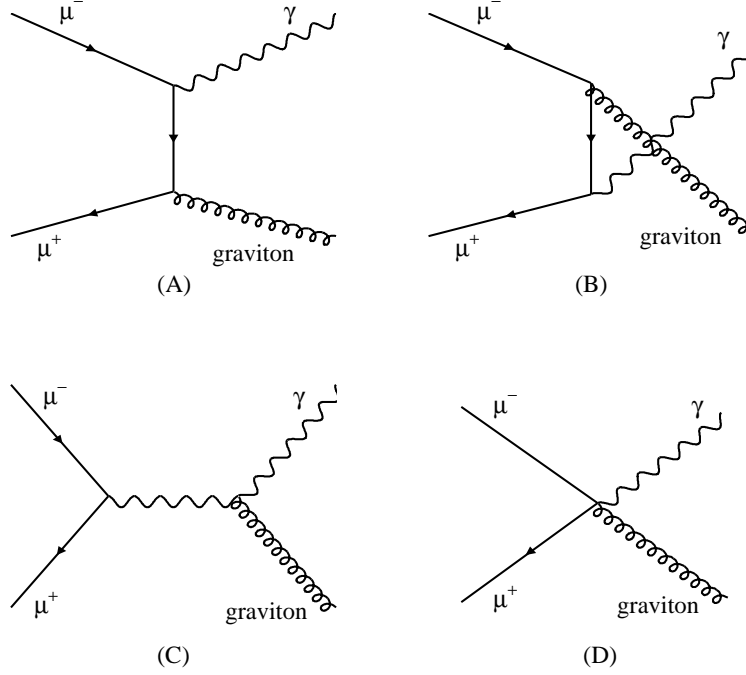


Figure 2. Diagrams contributing to the process  $\ell^+ + \ell^- \rightarrow \gamma + \mathcal{G}$  are shown. Notations are selfevident.

They are labeled by capital letters: (A) + (B) are **Compton like**, (C) is the **photon exchange** and (D) is the **contact** one, for future references.

In what follows we consider several cases with respect to the mass values of participating particles.

#### 3.1. Massless graviton, the lepton mass values are neglected

The cross section is given in the differential form as

$$\frac{d\sigma}{d\cos\vartheta_\gamma} = \frac{\pi}{2} \alpha G_N^{enh} (1 + \cos^2\vartheta_\gamma), \quad (2)$$

where  $\vartheta_\gamma$  is the emitting angle of the photon with respect to the negative lepton beam and  $G_N^{enh}$  is now the constant of graviton coupling to the matter fields enhanced by the virtue of the Kaluza–Klein mechanism.

If, however, we calculate the cross section, corresponding to the diagram (C) of Fig. 2 only, we obtain the same result as that of Eq.(2). This is a consequence of the Ward identity. For the illustration, let us write down only the contributions of the Compton like diagrams (A) and (B). Squaring the sum of (A) and (B) amplitudes, we obtain the following expression

$$\frac{''d\sigma''^{Compton\ like}}{d\cos\vartheta_\gamma} = -\frac{5}{2}\pi\alpha G_N^{enh} . \quad (3)$$

Note, that this expression is constant and scattering angle independent. Therefore, in the full expression for the cross section the interference term appears:

$$2\Re(\mathcal{M}_A + \mathcal{M}_B)(\mathcal{M}_C + \mathcal{M}_D)^* + 2\Re\mathcal{M}_C\mathcal{M}_D^* . \quad (4)$$

This term completely compensates the contribution of the Compton like diagrams (A) and (B). Note that after summing over the lepton polarizations the squared (D) diagram vanishes:

$$\sum_{spins} |\mathcal{M}_D|^2 = 0 . \quad (5)$$

It is expedient to mention the paper [11], where the fact of mutual cancellation between contributions of diagrams, describing the gravity in the strong electromagnetic field, was also noted.

### 3.2. Massless graviton, the lepton mass values are not neglected

First, let us calculate that piece of cross section, which corresponds to the **photon exchange** diagram only in order to see if the case of preceeding subsection is repeated. The result is as follows

$$\frac{d\sigma^{(C)}}{d\cos\theta_\gamma} = \frac{\pi}{2}\alpha G_N^{enh} \left(2 - \beta^2 \sin^2\theta_\gamma\right), \quad (6)$$

while the cross section corresponding to the full set of diagrams is given in the differential form as

$$\frac{d\sigma}{d\cos\vartheta_\gamma} = \frac{1}{2}\pi\alpha G_N^{enh} \beta \frac{1 + 2\beta^2 \sin^2\vartheta_\gamma - \beta^4(1 + \sin^4\vartheta_\gamma)}{(1 - \beta^2 \cos^2\vartheta_\gamma)^2} \sin^2\vartheta_\gamma , \quad (7)$$

where  $\beta = \sqrt{1 - \frac{4m_\ell^2}{s}}$ ,  $m_\ell$  is the mass of initial lepton.

Evidently, the expression of Eq.(6) and that of Eq.(7) look utterly unlike — the marvellous cancellation of the preceeding subsection disappeared due to presence of lepton masses. Explicitly its behaviour with respect to the scattering angle and to the center of mass energy of collision is depicted on the Fig. 3. Being zero at the threshold it growths with energy and quickly goes to plateau. On this picture both variable change in the following limits:  $-1 \leq \cos\vartheta_\gamma \leq +1$ , and  $2m_\mu \simeq 0.212\text{ GeV} \leq \sqrt{s} \leq 3\text{ GeV}$ . Units for the cross section values are arbitrary.

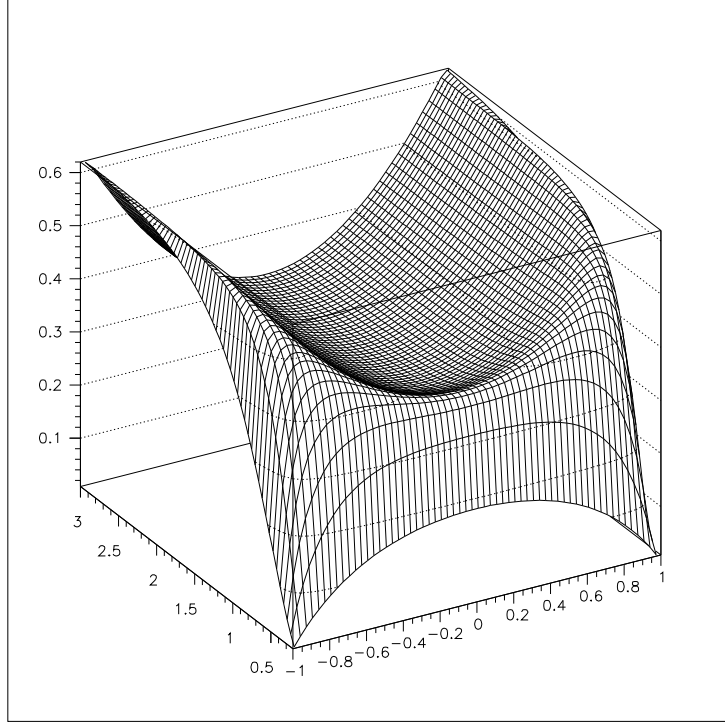


Figure 3. Angular distribution versus energy dependence of the process under study is shown.

Integrating the expression (7) over  $\cos\vartheta_\gamma$  in the limits above we obtain

$$\sigma_{tot} = \frac{4}{3}\pi\alpha G_N^{enh} \left\{ \left(1 + 8\frac{m_\ell^2}{s}\right)(1 - 3\mathcal{L}) + 12\frac{m_\ell^4}{s^2}(1 + 2\mathcal{L}) \right\} , \quad (8)$$

where  $\mathcal{L} = \frac{m_\ell^2}{s} \frac{1}{\beta} \ln \frac{1+\beta}{1-\beta}$ .

### 3.3. Massive graviton, the lepton mass values are neglected

When the lepton mass is negligible and the graviton mass  $m_G$  is kept finite, the expression for the cross section looks as

$$\frac{d\sigma}{d\cos\vartheta_\gamma} = \frac{\pi}{2} \frac{\alpha G_N^{enh}}{\left(1 - \frac{m_G^2}{s}\right)^2} \left\{ 1 - \left(\frac{m_G^2}{s}\right)^5 - \left(1 - \frac{m_G^2}{s}\right)^3 \cos^2\vartheta_\gamma \left[ 3\frac{m_G^2}{s} + \left(1 - \frac{m_G^2}{s}\right)^2 \cos^2\vartheta_\gamma \right] \right\} \frac{1}{\sin^2\vartheta_\gamma}. \quad (9)$$

The remarkable feature of this expression is the fact, that it has smooth limit at  $m_G \rightarrow 0$ . Namely, in this limit we obtain the expression of the Eq.(2). Thus, the so called van Dam–Veltman–Zakharov discontinuity [12] is absent in the physical quantity — cross section, although in deriving expressions of Eq.(2) and that of Eq.(9) we have used projection operators for the massless and massive gravitons, for which such a discontinuity presents explicitly.

Now, the cross-section of the Eq.(9) is evidently singular with respect to the scattering angle. This is consequence of neglecting the initial state masses. The analytical expression for the cross section with the initial muon masses taken into account is too cumbersome, so we will treat this case numerically in the next subsection.

### 3.4. Massive graviton, the lepton mass values are not neglected

In this subsection we present the results of the numerical study of the process under investigation with masses of participating particles kept finite (apart from the zero mass value for the photon, obviously).

Results of this analytical computation of the matrix element squared with the aid of HECAS program [13] and Monte Carlo phase space integration are very different from that of the massless case and are as follows.

We consider only the one particular case, when  $m_{\mathcal{G}}^2/s \rightarrow 1$ , i.e.  $E_{\gamma} \rightarrow 0$ . It appears, that in this case the cross section behaviour is completely determined by the **Compton like** diagrams only,

$$\sigma^{all\ diagrams} \equiv \sigma^{tot} \cong \sigma^{Compton\ like}$$

and there is no infrared cut-off by gravitation, as in the case of massless graviton, i.e.

$$\sigma^{tot}|_{m_{\mathcal{G}}^2 \rightarrow s} \rightarrow \infty. \quad (10)$$

At the same time contributions from the **photon exchange** diagram and the **contact** one tend to go to the zero in this limit,  $\sigma^{(C+D)}|_{m_{\mathcal{G}}^2 \rightarrow s} \rightarrow 0$ , while the interference between two subsets, namely, between **Compton like** and the sum of **photon exchange** and **contact** one remains constant and negative,  $\sigma^{(A+B)(C+D)}|_{m_{\mathcal{G}}^2 \rightarrow s} \simeq const$ , which, however, has no strong influence on the whole cross section behaviour in view of Eq.(10). So, we were not able to draw any definite conclusion from the case under consideration as opposite to the case of massless graviton. Nevertheless, questions raised in this talk, deserve future study, because not all possibilities are yet considered. To this end we need the fully analytical expression for the cross section in the most general case, i.e. when all masses of the participating particles except photon, are taken into account. We hope to do this job in the nearest future.

## 4. Conclusion

By a simple example of the process  $\ell^+ + \ell^- \rightarrow \gamma + \mathcal{G}$  the interplay between electromagnetic and gravitational interactions is considered. Some striking features of the amplitudes behaviour are revealed:

- in the case of massless graviton and in the limit of neglecting the initial state mass values the cross section is completely determined by the **photon exchange** diagram only (diagram (C) of Fig 2);
- the phenomenon disappears, when masses of initial states are taken into account;
- in the case of massive graviton the behaviour of amplitudes is very different from that considered above.



## Acknowledgments

I would like to thank Prof. Y.S.Kim for invitation me to participate in the work of Symposium and Mrs. Mary Ridgell for excellent job. Also, I am indebted to Drs. O.S.Pavlova and K.V.Stepanyantz for useful information they delivered to me and to Prof. D.V.Gal'tsov for making me aware of Ref. [11].

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*Received August 18, 2006.*

Препринт отпечатан с оригинала-макета, подготовленного автором.

Ф.Ф.Тихонин.  
О структуре электрогравитационных взаимодействий.

Оригинал-макет подготовлен с помощью системы **L<sup>A</sup>T<sub>E</sub>X**.

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Подписано к печати 22.08.2006. Формат 60 × 84/8.  
Офсетная печать. Печ.л. 1,25. Уч.-изд.л. 0,9. Тираж 80. Заказ .  
Индекс 3649.

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ГНЦ РФ Институт физики высоких энергий  
142281, Протвино Московской обл.

