STATE RESEARCH CENTER OF RUSSIA
INSTITUTE FOR HIGH ENERGY PHYSICS

IHEP 2006–17

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SINGLE CHARGED HIGGS BOSONS PRODUCTION IN THE LIKE-SIGN LEPTON $(e^-e^-, \mu^-\mu^-)$ COLLISIONS [1]

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[1] Based partly on the talk presented by the author at $e-e-$ 99: the Electron-Electron Linear Collider Workshop, December 10-12, 1999, Santa Cruz, CA , USA.

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Protvino 2006

UDK 539.1.01 M–24

Abstract

Tikhonin F.F. Single Charged Higgs Boson Production in the Like-Sign Lepton ($e^-e^-, \mu^-\mu^-$) Collisions: IHEP Preprint 2006–17. – Protvino, 2006. – p. 5, refs.: 8.

The production of nonstandard Higgs boson at l^-l^- ($l = e$ or μ) collisions is considered. With some simplifications completely analytical expression for the cross-section is presented. This expression is very convenient for the comparison of different models. Numerical results are briefly commented

Аннотация

Тихонин Ф.Ф. Рождение одиночных заряженных бозонов Хиггса в соударениях лептонов с одинаковыми зарядами ($e^-e^-, \mu^-\mu^-$): Препринт ИФВЭ 2006–17. – Протвино, 2006. – 5 с., библиогр.: 8.

Рассматривается рождение нестандартного бозона Хиггса в l^-l^- ($l = e$ или в μ) столкновениях. При некоторых упрощающих предположениях для сечения рождения получено полностью аналитическое выражение, которое весьма удобно для сравнения различных моделей. Кратко комментируются количественные результаты.

1. Introduction

Among the various electroweak symmetry breaking schemes the Higgs mechanism plays central role. However, to confirm the relability of this scheme, it is urgently necessary to find the fundamental scalar, e.i. Higgs boson. As it is well known this is the main goal of the LHC project. It is hopefully expected, that the fundamental scalar will be revealed and investigated to some extent at this machine. However for the thorough study and understanding of the Higgs properties, in our opinion, lepton colliders are much more suitable. By the name "lepton colliders" we mean electron (positron) colliders as well as the MUON one.

The $\mu^+\mu^-$ colliders were suggested many years ago [1], [2], [3]. The muons being as much as ≈ 200 times heavier than electrons don't waste their energy to the synchrotron radiation, because the intensity of the latter is proportional to the $mass^{-4}$ radiated particle. This idea has attracted much attention in recent years due to several reasons. From the point of view of machine design consideration it was revealed, that ionization cooling concept offers the possibility of making a high luminosity accelerator. Due to efforts of more than 100 physicists from about half hundread institutions during the last decade very significant progress was made towards the muon cooling. The latest news on the subject came from Daniel Kaplan. Namely, one of the conclusions of his talk at the IHCEP'06 is "*Future looks bright for muon colliders...*"[4].

On the other hand side the physics potential of muon collider recently was enriched by possibility to build the "Higgs boson factory" in analogy with the "Z boson factories" and therefore such facility might provide a unique instrument for particle physics research. In addition, while at the linear electron (positron) colliders the maximum attainable energy is restricted by the values of 1–2 TeV, the muon collider has no such a restriction. Now the design concept of accelerators at the 10 TeV and even at the 100 TeV are under the discussion (see the website http://pubweb.bnl.gov/people/bking/heshop/ for Workshop "Studies on Colliders and Collider Physics at the Highest Energies: Muon Colliders at 10 TeV to 100 TeV", which has been held 27 September – 1 October, 1999 at Montauk, New York, USA). Therefore, the electron and muon colliders will be treated on the equal footing.

Now some words about the like-sign option of lepton colliders are in order. One of the distinguished features of such a colliders is the absence of the (annoing simetimes) annihilation channels. Absence of s-channnel diagrams might be helpful, for example, in the case of a search for anomalous triple gauge boson couplings. Good possibilities for such an investigations deliver two processes, $l^+l^- \to \nu\bar{\nu}\gamma$ and $l^+l^- \to \nu\bar{\nu}Z$. First of them probes the $W\gamma W$ vertex, while the latter is sensitive to the anomalous WZW vertex independently of the first one, in contrary to the case of process $l^+l^- \to W^+W^-$. The possibilities are due to the diagram with photon attached to the W boson exchanged in t-channel in the first case and analogous diagram with the Z in the latter case. But both thees processes suffer from the "intrinsic background", namely, s-channel diagrams. This effect is completely absent in the case of l^-l^- collisions. Of course, the vertices, which can be studied in the latter case are completely differ from that of the first option (e.i. l^-l^+). Namely, the vertices, which can be probed in the l^-l^- collisions are ZZZ, $ZZ\gamma$ and $Z\gamma\gamma$. While aformentioned vertices exist in the Standard Model, the latter one are absent. Nevetheless, it is expedient to search for their effect in the hope to find out some hints on a "new physics". In particular, much efforts have been devoted to this aim at the LEP during the time of operating this engine. Therefore, the l^-l^- choice is the good opportunity for such a type of investigations.

In the case of the Higgs boson searches and investigations there is strict difference between both the choices, as for the standard Higgs as well for those, stemming from the various extentions of the Standard Model. First, in the case of l^-l^+ collisions Higgs boson can be produced in two competitive ways, by the "Bjorken process", $l^-l^+ \to l^-l^+H^0$, and by the "fusion process", $l^+l^- \to \nu\bar{\nu}H^0$. First of them falls with the energy, but, nevertheless, dominates in the region of center of mass energies up to $\sqrt{s} \cong 400$ GeV, and beyond this region the "fusion process" dominates, growing as $\approx \log(\frac{\sqrt{s}}{M_H})$. Distinctive feature of the l^-l^- collisions is the absence of the "Bjorken process".

2. Calculation

We consider the single charged Higg boson production. Such bosons arise in the numerous extentions of the Standard Model. Properties of the Higgs boson very different depending on the model chosen so we consider the general case of coupling it to the other particles. The analytical expression for the "Bjorken process" is well known, we show here, that the analytical result for the differential cross-section of the "fusion prosess",

$$l^-l^- \to H^-l^-\nu_l \quad , \tag{1}$$

can be obtained too. Let us denote by p_1 and p_2 the 4-momenta of the initial leptons, by k_1 and k_2 the 4-momenta of the final leptons and by q_H the 4-momentum of outgoing boson. Also, introduce $P = p_1 + p_2 - q_H$. The t-channel and the u-channel exchange diagrams contribute to this process. In both cases we have the WZ' – fusion process. The Z' stands for the any type of Z-bosons, including the standard one. In the last case it is well known, that this coupling is absent at the tree level in the standard Model, but it can be induced at this order in models including Higgs triplet representations or it can be generated at the one-loop level in multi-higgs doublet models (for the recent discussion see, for example, paper [5] and references therein) The most difficult part of the cross-section calculation is the phase space integration. Integrals, meeted in the course of calculation are of the following type

$$\mathcal{I}^{1;\mu;\mu\nu;\dots}_{m,n} = \frac{1; k_1^\mu; k_2^\mu; k_1^\mu k_2^\nu; \dots \cdot \delta^{(4)}(P - k_1 - k_2)}{\Big[(p_1 - k_1)^2 - M_W^2\Big]^m \cdot \Big[(p_2 - k_2)^2 - M_{Z'}^2\Big]^n} \cdot \frac{d^3k_1}{2k_1^0} \cdot \frac{d^3k_2}{2k_2^0} \ . \tag{2}$$

where μ, ν are the Lorentz indicies and m, n are the exponents of propagators. The comprehensive list of integrals involved can be found in the paper [6].

With the aid of this integrals the fully analytical differential (with respect to the angle of produced Higgs boson and its energy) cross section can be obtained. Considerable simplisication of the cross section can be achieved due to the following observation. Matrix element squired for the given process contains scalar productions of the momenta k_i, p_i, etc. It has been noted, that the relation

$$\int (k_1\cdot p_2)(k_2\cdot p_1)d\Phi \ \cong \ \int (p_1\cdot p_2)(k_1\cdot k_2)d\Phi, \tag{3}$$

where $d\Phi$ is the phase space volume element of the final-state particles, is fulfilled with good accuracy (of the order of $\leq\ 1\%$) . Owing to this observation we can write the cross-section in the following simple form

$$\frac{d\sigma}{d^3q_H} = \frac{1}{8\pi^5}\frac{1}{s}\frac{1}{q_H^0}\Big(\frac{g_1}{2c_1}\Big)^2\Big(\frac{g_2}{2c_2}\Big)^2 g^2_{Z'H^\pm W^\mp}(g_V^2\ +\ g_A^2)(g_V'^2\ +\ g_A'^2)P^2(p_1p_2)\mathcal{I}_{2,2}\ , \tag{4}$$

where $\frac{d^3q_H}{2q_H^0} = \frac{\pi}{4}\sqrt{x^2 - \frac{4m_H^2}{s}}\,dxd(\cos\theta_H)$. In turn, q_H^0 is the energy of the Higgs boson produced and θ_H - is the production angle of it with respect to the initial beams.

Now, let us introduce the variable $x\ =\ \frac{2q_H^0}{\sqrt{s}}$, which we will use in the calculation. Variables x and $\cos\theta_H$ are allowed to vary in the ranges

$$\frac{2M_H}{\sqrt{s}}\ \leq\ x\ \ \leq\ 1\ +\ \frac{M_H^2}{s}\ , \qquad -1\ \leq\ \cos\theta_H\ \leq\ +1\ . \tag{5}$$

Explanations for the other entities in the equation (4) are as follows.

$$\begin{aligned}\frac{g_1}{2c_1} &= \frac{g}{2\sqrt{2}}\quad for\ \ vertex\ \ (Wl\nu_l)\ ;\\ \frac{g_2}{2c_2} &= \frac{g}{2\cos\theta_{EW}}\quad for\ \ vertex\ \ (Z'll)\ .\end{aligned} \tag{6}$$

$g_V\ =\ -g_A\ =\ 1$ for vertex $(Wl\nu_l)$ $g_V\ =\ -\frac{1}{2}\ +\ \sin^2\theta_{EW}\ ,\ g_A\ =\ -\frac{1}{2}$ for (Zll) – vertices. Constants g'_V and g'_A for vertex $(Z'll)$ are not well defined, they are fully model-dependent. One representative example will be given below. The strength of $Z'H^\pm W^\mp$ interaction is also model-dependent, one example of which can be extracted, for example, from the paper [7].

$$g_{Z'H^\pm W^\mp}\ \cong\ 0.45(gM_W), \tag{7}$$

where the (gM_W) is the well known constant at vertex $(H^0W^+W^-)$ (g is the electroweak constant, M_W is the mass of the W-boson and H^0 is the fundamental scalar of the Standard Model) .

Integral $\mathcal{I}_{2,2}$ (see Eq.(2)) can be expressed in terms of the invariant variables in the following way:

$$\begin{aligned}\mathcal{I}_{2,2} \;&=\; \frac{\pi}{2}\frac{1}{(p_1P)(p_2P)}\frac{1}{M_c^4}\frac{1}{w^2}\Big\{\Big(\widetilde{M}_W \,+\, \widetilde{M}'_Z\Big)\frac{w}{1-w} \,-\, 3\Big(1 \,-\, c^2\Big) \\ &+\; \Big[cM_cw \,+\, \frac{3}{2}\Big(1 \,-\, c^2\Big)\Big]\frac{1}{\sqrt{w}}\ln\frac{1\,+\,\sqrt{w}}{1\,-\,\sqrt{w}}\;, \end{aligned} \tag{8}$$

$$\begin{aligned} m_W \;&=\; 1 \,+\, \frac{M_W}{(p_1P)}, \qquad m'_Z = 1 \,+\, \frac{M'_Z}{(p_2P)}, \\ \widetilde{M}_W \;&=\; m_W^2 \,-\, 1, \qquad \widetilde{M}'_Z \;=\; m'^2_Z \,-\, 1, \\ M_c \;&=\; m_W + m'_Z \,+\, c, \qquad c = 1 \,-\, \frac{P^2\;(p_1p_2)}{(p_1P)(p_2P)}, \\ w \;&=\; 1 \,-\, \frac{\widetilde{M}_W + \widetilde{M}'_Z}{M_c^2}. \end{aligned} \tag{9}$$

Let us remind, that masses of W^- and Z' - bosons are denoted by corresponding capital letters, M_W and $M_{Z'}$, correspondingly. It is evident, that the form of cross-section (4) is very easy-tractable. But the most important thing is that it allows for to compare different models without cumbersome calculations, because all the difference between them embodied in the constants g'_V, g'_A and $g_{Z'H^\pm W^\mp}$. In particular, from this expression the different approximations can be obtained. For example, for the well-known Weizsacker-Williams approximation it is enough to put

$$\mathcal{I}_{2,2} \;=\; \frac{\pi}{4}\frac{1}{M_W^4 s^2}\frac{1}{\overrightarrow{q}^{\,4}_H}, \tag{10}$$

where $\overrightarrow{q}_H$ is the 3-momentum of the Higgs. Integrating Eq.(4) with this $\mathcal{I}_{2,2}$ with respect to x and $\cos\theta$ in the limits above we obtain well known expression

$$\sigma \;=\; \frac{1}{16}\Big(\frac{\alpha}{\sin^2\theta_W}\Big)^3\frac{1}{M_W^2}\Big[(1+\frac{M_H^2}{s})\ln\frac{s}{M_H^2} - 2 + 2\frac{M_H^2}{s}\Big] \tag{11}$$

Cross-section obtained in the Eq.(4) is valid only for the t-channel diagram. The case for both the t- and u-channel diagrams, is too complicated to be placed here, but the full expression may be evaluted approximately. It has been checked that the u-channel diagram contribution approximately is equal to that of t-channel and interference term is very small (and negative). So, to obtain cross-section for the $l^-l^- \to H^-l^-\nu_l$ it is enough to multiply the expression from Eq.(4) by a factor of ≈ 2.

Unfortunately, all the models with nonstandard Higgs sector have such the parameters sets, which always inevitabely diminish cross-sections. Let's consider one example, representing the "hybrid" model of two papers [7] and [8]. For $Z'H^\pm W^\mp$ coupling we use the value from the paper [7] (see Eq.(7) of the present paper), while coupling constants of the Z' - boson to electron are borrowed from the latter:

$$g'_V \;=\; \frac{2}{\sqrt{10}}\cos\theta_{E_6}\;, \qquad g'_A \;=\; -\,\frac{1}{\sqrt{10}}\cos\theta_{E_6} \;-\; \frac{1}{\sqrt{6}}\sin\theta_{E_6}, \tag{12}$$

where θ_{E_6} is the angle, stemming from the superstring theories with the E_6 group as a group of grand unification. Coefficient involved in the expression (4) runs in the following limits

$$0.12 \;\leq\; \left(g'^2_V \;+\; g'^2_A\right) \;\leq\; 0.54 \tag{13}$$

Gathering these coefficients in the cross-section and comparing it with the case of W-W fusion, we see that this cross-section loses a factor of 3.7 for the maximum value of Eq.(13). The situation is worsened with the growing mass of Z'.

It might happens, however, that the vertex $ZH^{\pm}W^{\mp}$ exists in some models with highly nonstandard Higgs sector, or at the loop induced level, as it was mentioned above. Then for the usual Z - boson instead of elusive Z' and for the some representative values of $\sqrt{s} = 500\ GeV$ and $M_H = 150\ GeV$ (and using again the maximum value from Eq.(13)) we obtain the total cross-section value as large as $\sigma \cong 40 fb$. Proceeding in the same way, we will be able to probe many extentions of Higgs sector of the Standart Model using the expression given in the text (see the Eq.(4)).

3. Conclusion

The fully analytical expression for the charged Higgs boson production cross section at the e^-e^- collider is obtained. Some simplifications were made to make calculation easy. This result is very convinient for the aim of comparision between different models, because all the constants, characterising models are factorised out. In view of the simplicity of the Eq.(4) we don't give much numerical results, restricting ourselves to only one illustrative example.

Acknowledgements

I am grateful to Prof. C.A.Heusch and Nora Rogers for the enjoyable workshop and for the financial support.

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Received August 18, 2006.

Препринт отпечатан с оригинала-макета, подготовленного автором.

Ф.Ф.Тихонин.
Рождение одиночных заряженных бозонов Хиггса в соударениях лептонов с одинаковыми зарядами $e^-e^-, \mu^-\mu^-$.

Оригинал-макет подготовлен с помощью системы **LaTeX.**

Подписано к печати 22.08.2006. Формат $60 \times 84/8$.
Офсетная печать. Печ.л. 0,85. Уч.-изд.л. 0,7. Тираж 80. Заказ .
Индекс 3649.

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ПРЕПРИНТ 2006–17, ИФВЭ, 2006
