STATE RESEARCH CENTER OF RUSSIA

INSTITUTE FOR HIGH ENERGY PHYSICS

IHEP 2008–4
IFO

A. Chikilev, L. Tkachenko

MINIMIZATION OF MECHANICAL STRAIN IN END PARTS OF COIL BLOCK FOR SUPERCONDUCTING MULTI-LAYER TYPE MAGNETS

Directed to *Superconductor Science and Technology*

Protvino 2008

UDC 538.082 M- 24

Abstract

Chikilev A., Tkachenko L. Minimization of Mechanical Strain in End Parts of Coil Block for Superconducting Multi-Layer Type Magnets: IHEP Preprint 2008–4. – Protvino, 2008. – p. 10, figs. 3, tables 3, refs.: 11.

A model of geometry for coil block in the end parts of superconducting multi-layer magnet has been considered to take into account finite dimensions of Rutherford type cable. A method for optimization of coil lengths for superconducting magnets is introduced. The length of end parts is shortened, which gives larger effective length of the magnet and simplifies assembly technique. Numerical results of coil ends optimization for the dipole superconducting magnet are shown, which have been developed with the help of program for modeling of coil blocks in the end parts.

Аннотация

Ткаченко Л.М., Чикилёв А.О. Минимизация механических напряжений в лобовых частях сверхпроводящей обмотки оболочечного типа: Препринт ИФВЭ 2008–4. – Протвино, 2008. – 10 с., 3 рис., 3 табл., библиогр.: 11.

Рассматривается модель геометрии лобовой части обмоточного блока сверхпроводящего слоевого магнита, в которой учитываются конечные размеры Резерфордовского кабеля. Представлен метод оптимизации длины обмоточных блоков сверхпроводящего магнита с целью увеличения его эффективной длины и упрощения технологии сборки магнита. Представлены численные результаты моделирования лобовых частей сверхпроводящего дипольного магнита, которые были получены с помощью программы моделирования лобовых частей обмоточных блоков.

Introduction

Superconducting multi-layer type magnets have the saddle-shaped coil, at the end parts of which occurred mechanical strains. A method for optimization of end part geometry of coil blocks is considered to decrease the mechanical strains and increase the effective length of the magnet. The classical "constant perimeter condition" [1] enables to specify the optimal position, from mechanical point of view, for only one side of a cable's turn in the coil ends, as a turn in this condition is modeled by an infinitely thin stripe without taking into consideration finite dimensions of a cable. This leads to overstrains in the coil block, when the grouped end method for stacking turns in the coil ends are implemented [2], in which it is supposed that the turns within a coil ends are laid directly on each other with no spaces between them. A model of cable geometry [3] allows one to take into account its finite dimensions in modeling of coil ends for superconducting magnets, and it was implemented in the BEND program [4]. Here, a modification of method for geometry optimization of coil ends is presented. A BENDM program for modeling of coil ends has been written on the base of developed method and the BEND program. In contrast to the BEND program, the turns in the coil ends may be constrained either to the inner or to the outer surface of cylinder.

Modeling and optimization of coil ends geometry

1.1. Main challenges for geometry optimization of coil ends

General view of ¼ part of a coil block for superconducting magnet is shown in **Fig. 1**. Angular position of coil blocks within straight section of the magnet is determined from requirements on the magnetic field quality in the central cross section. The Rutherford type cable with trapezoidal cross-section is used for winding of coil blocks.

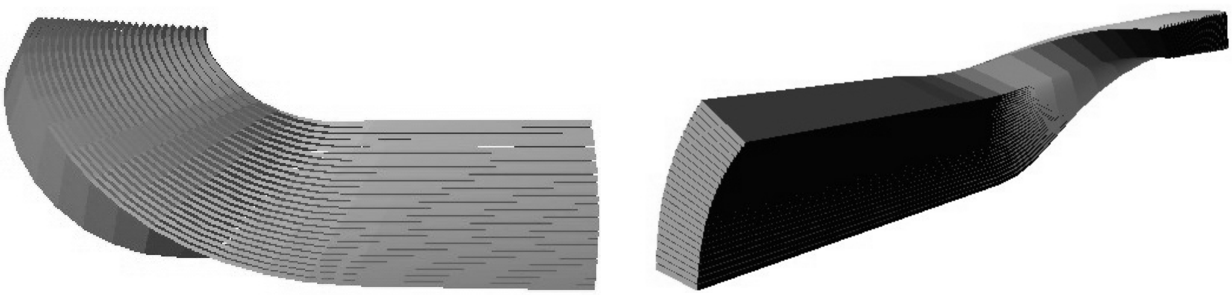

Fig. 1. General view of parts of a superconducting coil block.

Effective length is an important characteristic of multipole superconducting magnet. For the dipole magnet it equals to

$$L_{eff} = \frac{1}{B(0,0,0)} \int_{-\infty}^{\infty} B_y(0,0,z)dz \; .$$

Here the point of origin coincides with the dipole center and the axis z is a longitudinal axis of the magnet. If the length of the magnet is fixed, then the maximal effective length of the magnet is achieved, when the lengths of end parts for coil blocks is as small as possible. Sufficiently short coil ends are although convenient from technological point of view.

The main limiting factors in minimization of coil end lengths are mechanical characteristics such as the maximum strain, the smallest radius of curvature and the maximum twist in the coil ends [5]. The limits for these mechanical characteristics are defined by mechanical proprieties of superconducting cable, used for the production of the magnet.

1.2 Modeling of coil end geometry for superconducting magnet

Tight containment of the turns in the coil ends provides quick and successful training of the magnet. Spacers separate the coil blocks in the end parts of the magnet from each other. This leads to the grouped end method for stacking turns in the coil ends [2], in which the position of all turns in the coil block are determined by the specified position of any turn from the block, and, particularly, by the form of the spacer side, on which the first, i.e. the innermost, turn from the coil block is wound. The shape of this spacer side must be computed from requirement to minimize the mechanical strain in all turns of the block. Optimization of the coil block shape only from minimizing the mechanical strain in the innermost turn from the block does not give the optimal shape for other turns in the block. The neutral surface concept [6] in the solid body can be used to show it. The deformations and, consequently, the mechanical strains are equal to zero at this surface in the deformed body. As the coil block is pressed between spacers in the magnet, then one turn from the block can be placed nearly on this surface that optimally minimize the mechanical strain in it. The mechanical strains would increase, when moving off this surface both for the inner coils to the optimized coil and to the outer coils.

The “constant perimeter condition” [1] is generally used in modeling of the shape for superconducting cable in the coil ends of the magnet. The ruled surface is called the “constant perimeter surface”, when its geodesic parallels are of equal length. Such a surface is called a rectifying developable surface in differential geometry. The “constant perimeter condition” provides for the mechanically natural shape to the turn of an infinitely thin cable, that it does not change without acting of external forces. In general case if the “constant perimeter condition” is imposed on a side of the cable, then the other side of the cable satisfies to this condition only approximately. This leaks from the fact that the “constant perimeter condition” is fulfilled exactly only on the surface and does not take into considerations the finite dimensions of the turn. The more turns are in the block, the more this inaccuracy in specification of the form for the first turn, from which the other turns from the block are computed, affects on the form of the coil block. The mechanical strain in the coil block increases the much faster, the less precisely the form of the first turn from the block is specified. In this work the model of Rutherford cable with homogeneous and isotropic proprieties is considered, which gives an opportunity to take into considerations the finite dimensions of the cable and to specify the shape for the first turn for the coil block more precisely.

1.3 Physical model for superconducting cable

The physical model of superconducting cable [3] is used below for optimization of coil block geometry in the end parts of superconducting magnet. One turn from the coil block is modeled by a stripe in space, which is called “the guiding stripe”. From technological reasons the turns in the coil ends of multi-layer magnets are constrained either to the inner or to the outer surface of cylinder. Consequently, one side of the guiding stripe is placed either on the inner or on the outer surface of cylinder. This side is called “the base curve”. Another side is called “the free edge” of the guiding stripe. The shape of the guiding stripe in space is defined here from the Euler – Kirchhoff theory of thin rods. According to the symmetry it is enough to consider the guiding stripe only in the first quadrant. The optimal shape of the guiding stripe is determined from minimization of mechanical strain energy with the density along the strip per unit of the arc length s

$$\frac{1}{2}\left(a_1\kappa_1(s)^2 + a_2\kappa_2(s)^2 + a_3\tau(s)^2\right) . \qquad (1)$$

Here, the position of the guiding stripe in space is defined with the help of the base curve $\mathbf{r}(s)$. The curve $\mathbf{r}(s) = (x(s), y(s), z(s))$ is parameterized by arc length s, where $ds = \sqrt{dx^2 + dy^2 + dz^2}$. For the beginning of the curve $s = 0$ is taken the point of the base curve, where the straight part of the coil changes to the coil end. The ending point of the base curve $s = s_{fin}$ corresponds to turning point of the coil ends and it is the farthest point of the base curve from magnet center along axis z. The constant a_1 is the flexural rigidity of the turn about axis tangent to the guiding stripe and perpendicular to the base curve. The constant a_2 is the flexural rigidity of the turn about axis perpendicular to the guiding stripe. The constant a_3 is the torsional rigidity of the turn. The functions of arc length $\kappa_1(s)$ and $\kappa_2(s)$ are the components of curvature of $\mathbf{r}(s)$ perpendicular and tangent direction to the stripe at s, respectively, and the function $\tau(s)$ is the torsion of the strip at s.

The guiding stripe is most conveniently represented in the local coordinate system which is defined by Frenet frame, at each point of the base curve, consisting of the tangent to the base curve $\mathbf{t}(s) = d\mathbf{r}/ds$, the principal normal $\mathbf{n}(s)$ and the binormal $\mathbf{b}(s) = \mathbf{t}\times\mathbf{n}$ to the curve. The right-hand triple of vectors for this coordinate system $\mathbf{t}(s)$, $\mathbf{n}(s)$ and $\mathbf{b}(s)$ are related with the total curvature of the stripe $\kappa(s) = |d^2\mathbf{r}/ds^2|$ and with the torsion of the turn by the formulas of Frenet–Serret of classical differential geometry:

$$\frac{d\mathbf{t}}{ds} = \kappa(s)\mathbf{n}, \quad \frac{d\mathbf{n}}{ds} = -\kappa(s)\mathbf{t} + \tau(s)\mathbf{b}, \quad \frac{d\mathbf{b}}{ds} = -\tau(s)\mathbf{n} .$$

Substituting into (1) the components of curvature of the stripe by directions through the total curvature $\kappa_1(s) = \kappa(s)\cos(\varphi(s))$ and $\kappa_2(s) = \kappa(s)\sin(\varphi(s))$, it gives:

$$\left(\{a_1\cos^2(\varphi(s)) + a_2\sin^2(\varphi(s))\}\kappa^2(s) + a_3\tau(s)^2\right). \qquad (2)$$

The integral of density (2) by the arc length s along the base curve from the beginning of the coil $s = 0$ to the returning point in the top of the coil ends of this curve $s = s_{fin}$ is the total mechanical strain energy of the turn of the cable in the coil ends. At the beginning point of the base curve $s = 0$ the guiding stripe must coincide with the initial position in the coil, where its straight part of the coil changes to the coil ends. The base curve is placed on the outer or on the inner cylinder depending on to which cylinder the coil is constrained. The condition of smooth return is imposed on the guiding stripe at the top of the coil ends of the magnet at a point $s = s_{fin}$. The minimization of this energy functional gives the optimal shape for the turn by Dirichlet's principle. The process of minimization of mechanical strain is divided into iterative process consisting of two steps.

1.4 Adaptation of the constant perimeter condition

At the first step of iterative process the guiding stripe is approximated with the help of "constant perimeter condition". The flexural rigidity a_2 of the turn about axis perpendicular to its guiding stripe is much larger then the flexural rigidity a_1 of the turn about axis tangent to the guiding stripe and perpendicular to the base curve. Supposing that the turn has the shape of an infinitely thin stripe, it gives: $a_2/a_1 = \infty$. Fulfillment of this condition is guaranteed by the absence of the bend of the guiding stripe around the axis perpendicular to it, i.e. by the equality $\varphi(s) \equiv 0$. The detailed description of the first step is given in [3]. The minimization of the mechanical strain energy functional with the density (2), in which $\varphi(s) \equiv 0$, gives the base curve lying on the cylinder. It is known from differential geometry that a rectifying developable surface [7], satisfying the "constant perimeter condition", is uniquely defined for a determined smooth curve. The surface is called a rectifying developable if and only if it can be flattened out into a plane without any stretching or tearing. The points of this ruled surface $\mathbf{R}(s,u)$ are computed from the points of the base curve $\mathbf{r}(s)$ by a shift along the ruling vector $\mathbf{p}(s)$ on the distance u:

$$\mathbf{R}(s,u) = \mathbf{r}(s) + u\,\mathbf{p}(s), \qquad u \in (-\infty, +\infty). \tag{3}$$

The ruling vector for this surface at a point s is the Darboux vector:

$$\mathbf{p}(s) = \frac{\tau(s)\,\mathbf{t} + \kappa(s)\,\mathbf{b}}{\sqrt{\tau(s)^2 + \kappa(s)^2}}.$$

Particularly, the intermediate position of the free edge of the guiding stripe, defined by the geodesic parallel curve to the base curve, is given from (3) by substituting $u = \pm H/\sin\theta(s)$, where the sign depends on to which cylinder the turn is constrained. Here H is the width of a cable, and $\theta(s)$ is the angle between the direction $\mathbf{p}$ and tangent $\mathbf{t}$ vectors to the base curve.

The rectifying developable surface is uniquely determined for a given smooth curve. The rectifying developable surface to the base curve at a point $s = 0$ intersects with the cross-section of the magnet by a straight line $\mathbf{R}(0,u)$, $u \in (-\infty, +\infty)$. The line $\mathbf{R}(0,u)$ is directed by the radius to the cylinder. Generally, the side edges of turns in the straight section of the magnet have similar but not the same direction. So the intermediate position of the beginning for the guiding stripe lies on the straight line $\mathbf{R}(0,u)$, and, in general case, it does not coincide with the initial position of a turn at its beginning from the straight part of the coil block. The intermediate position of the guiding stripe has to be turned so that its beginning coincides with the position in the turn, in which the straight part of the coil changes to the coil end.

1.5 Determination of the turn shape from rotation of the constant perimeter surface

At the second step of iterative process the free edge of the guiding stripe is determined with the help of rotation of its intermediate position, obtained at the first step on a small angle $\varphi(s)$, until its beginning coincides with the prescribed position in the start of the coil end from the straight part of the coil. The density of mechanical energy for a turn, rotated on the angle $\varphi(s)$, is given by the formula (2), in which the total curvature $\kappa(s)$ is determined at the first step, and the torsion of the turn is given by the formula: $\tau(s) = \tau_0(s) + d\varphi/ds$, where $\tau_0(s)$ is the torsion, determined at the first step. The optimal angle of rotation $\varphi(s)$ is defined by using of Lagrange method. The Lagrange function (2) is written in the following form:

$$L(\varphi, \frac{d\varphi}{ds}, s) = \left(\frac{a_3}{a_1}(\frac{d\varphi}{ds} + \tau_0(s))^2 + \{\frac{a_2}{a_1} - 1\}\sin^2(\varphi(s))\kappa^2(s) \right).$$

From where the Lagrange equation is obtained for determination of optimal angle for rotation of rectifying developable surface:

$$2\frac{a_3}{a_1}(\frac{d^2\varphi}{ds^2} + \frac{d\tau_0}{ds}) - \{\frac{a_2}{a_1} - 1\}\sin(2\varphi(s))\kappa^2(s) = 0.$$

The following boundary conditions are imposed on the angle of rotation $\varphi(s)$: At the beginning of the coil block end from the straight part the guiding stripe must coincide with the initial position of a turn: $\varphi(0) = \varphi_0$. In the point $s = s_{fin}$ the condition of smooth, returning of the turn in the coil block end, is imposed: $(d\varphi/ds)(s_{fin}) = 0$. The considered here method is approximated, that leads to necessity of smoothing the obtained solution for the given boundary problem to provide the smooth transition of the guiding stripe into the straight part of the coil block by additional condition $(d\varphi/ds)(0) = 0$.

1.6 Taking into considerations finite dimensions of a cable in the process of optimization

It is suggested in [4] to approximate the function of rotation $\varphi(s)$ by a polynomial, the coefficients of which must be visually determined by user of the optimization program for the coil block ends in the process of minimization of mechanical strain, curvature and the torsion in the coil block. This gives an opportunity not to define the rigidities of the turn a_1, a_2 and a_3, but it leads to considerable slowing down of the process of optimization for the coil ends. In the introduced here method the rigidities are taken from [6], where they are determined for a thin elastic rod. The considered in [6] rod consists of homogeneous isotropic material and it has the rectangular cross-section. The flexural rigidities and the torsional rigidity are given by the following formulas:

$$a_1 = \frac{E d^3 H}{12}, \qquad a_2 = \frac{E H^3 d}{12}, \qquad a_3 = \frac{\mu H d^3}{3}.$$

Here E is Young's elasticity modulus, μ is modulus of elasticity in shear, H is the width of a cable, d is the mid thickness of a cable. The following constants are used in the process of minimization:

$$\frac{a_2}{a_1} = \frac{H^2}{d^2}, \qquad \frac{a_3}{a_1} = 4\frac{\mu}{E} = \frac{2}{1+\nu},$$ where ν is the Poisson ratio.

For the determination of these constants it is sufficient to assign only the Poisson ratio, which is presumed to be equal to its mediate value for metals $\nu = 0.3$.

1.7 The method of optimization of shape for the coil block

The following step is to find the optimal position of the guiding stripe in the coil block, which gives the minimization of mechanical strains in the whole coil block by its absolute value.

The grouped end method is applied for stacking turns in the coil ends. The positions of turns in the coil block are computed from the position of the modeled turn by stacking them tightly against each other, by their side edges without either gaps inside or bulges outside of the prescribed volume. The minimization of mechanical strain is carried out by the position of the guiding stripe in the coil block to find the optimal shape for the whole coil block. The position of the guiding stripe is determined in that way, at which the maximum value of mechanical strain attains its minimum for the whole coil block. In this process the guiding stripe may be either placed on lateral surface of any turn from the coil block or even inside any turn from the block. So the minimization is carried out by the connected set of positions for the guiding stripe.

The considered here model of coil ends for superconducting magnet has a clear physical sense. With the help of the guiding stripe an analogue for the neutral surface in the coil block is modeled approximately. The determination of neutral surface is convenient for consideration of problems in the theory of elasticity [6]. The optimal position for the neutral surface is determined in the process of minimization of mechanical strain in the coil block ends by the position of the guiding stripe. Nevertheless, this model is an approximation, as the neutral surface in the coil ends of superconducting magnet do not coincide with a position of the guiding stripe in general case. Moreover, the guiding stripe is just an analogue for the neutral surface.

The mechanical strains in the coil block of a superconducting magnet are determined by the surface of the spacer, on which the first turn from the coil block is wound. So it is acceptable to add virtual turns from inside to the block for correct optimization of the shape for this surface. In some cases, the proper minimum of maximum mechanical strain is not attained, when the position of the guiding stripe is determined only in the real turns of the coil block. The virtual turns are introduced only on the stage of optimization of the form for the coil ends to model the optimal shape for the surface of the spacer, on which the coil block is wound. The minimization of mechanical strain is carried out only in real turns from the block. The position of the guiding stripe is determined both in real turns from the block and in virtual turns, added from inside to the coil block. In general case, this approach gives an opportunity to determine the optimal position for the guiding stripe, at which the proper minimum of maximum mechanical strain in the coil ends is attained.

1.8 Minimization of lengths for the coil ends

The ending step of optimization process is minimization of dimensions for the coil ends of superconducting magnet without loss of possibility to wind the coil blocks of superconducting magnet with negligible deformation of used superconducting cable. The assurance factors for superconducting cable are the following mechanical characteristics, obtained from empirical analysis for the given type of cable: the maximum mechanical strain $|\delta L|_{bad}$, the smallest radius of curvature R_{bad}, and the maximum twist $|T_w|_{bad}$ in the coil ends. The possibility of coil end production with the specified linear dimension l for the coil block is defined from inequality $U(l) \leq 1$, where

$$U(l) = \max\left(\frac{|\delta L|_{\max}}{|\delta L|_{bad}}, \quad \frac{R_{bad}}{|R_{cv}|_{\min}}, \quad \frac{|T_w|_{\max}}{|T_w|_{bad}} \right).$$

Here $|\delta L|_{max}(l)$ is the greatest mechanical strain in the coil block; $|R_{cv}|_{min}(l)$ is the smallest radius of the curvature in the coil block; $|T_w|_{max}(l)$ is the greatest twist in the coil block. The length of the coil block end, at which the mechanical characteristics are not exceeding the assurance factors for the given type of cable is determined from equality $U(l) = 1$.

2. The coil ends for the model of dipole magnet for synchrotron SIS 300

2.1 Description of geometry for the coil of the dipole magnet

The developed here method was applied for the optimization of the coil ends for the alternative 1-meter model of the fast cycling dipole magnet for the synchrotron SIS 300, the FAIR project, designed in co-working of GSI (Germany) and IHEP [8]. The main challenge in modeling of the coil ends for this 6 Tesla magnet was attaining the maximal effective length, when the total length of the coil block is fixed. The Rutherford type cable with insulation has the following dimensions: the bases of trapezoid 1.558 mm and 1.794 mm, 0.9° keystone angle, the width of the cable is 15.35 mm.

General views of cross-section and coil ends for dipole magnet are shown in **Fig. 2**.

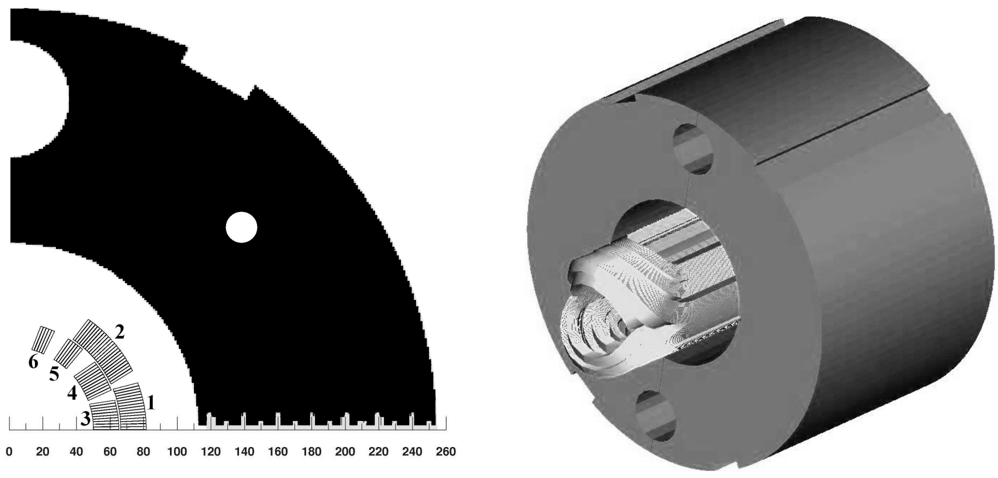

Fig. 2. General views of cross-section and coil ends for dipole magnet.

The geometry of the coil block in the cross-section of magnet is defined by the number of turns in the block N, the inner radius of the layer R, the initial angle of the block φ, and the inclination angle α - both from the median plane. The main geometric parameters of the coil, used for the construction of the coil ends geometry [8] is shown in the **Table 1.**

Table 1. Geometric parameters of the coil cross-section.

Block	N	R, mm	φ, deg.	α, deg.
1	16	66.35	0.22	0
2	23	66.35	24.37	32.38
3	10	50	0.23	0
4	11	50	20.39	25.23
5	6	50	47.24	50.40
6	5	50	65.76	64.64

The geometry of the coil block end is determined by the following parameters: the inclination angle for the first turn β (counted from the magnet center) in the YZ plane to the axis Z; the half axis of the upper ellipse for the first turn E and the half-straight part of the coil block S.

2.2 Minimization of lengths for the coil block ends of dipole magnet

The optimization of alternative geometry for the coil ends of superconducting dipole magnet was carried out by using of the BENDM program which was modified from the BEND program [4] with the help of developed method. The turns in the coil block ends of considered magnet are constrained to the inner cylinder. The guiding stripe in the BEND program is placed on lateral surface of any cable from the coil block. The position of the guiding stripe in the BENDM program is determined from the condition of strict minimum on the mechanical strain in the coil block.

For the determination of optimal lengths for the coil ends the following norm was used to estimate the badness of mechanical characteristics for the fixed value of half axis E:

$$U(E) = \max\left(\frac{3}{2}\frac{|\delta L|_{\max}}{|\delta L|_b},\quad 2\frac{R_b}{|R_{cv}|_{\min}},\quad \frac{3}{2}\frac{|T_w|_{\max}}{|T_w|_b} \right)$$

This norm differs from the norm of the BEND program. Here the greatest strain in the coil block $|\delta L|_{max}(E)$ is estimated by evaluation of maximum stretch in the block; $|R_{cv}|_{min}(E)$ is the smallest radius of the curvature in the coil block; $|T_w|_{max}$ is the greatest twist in the coil block. The assurance factors of superconducting cable have been obtained from empirical analysis [9]: $|\delta L|_b = 0.3$ for the maximal mechanical strain, $R_b = 2.5$ *mm* for the minimal radius of curvature and $|T_w|_b = 3.5°/mm$ for the maximum twist in the coil block of the magnet. In modeling of the coil ends for the dipole magnet for synchrotron SIS 300 the stronger criteria are used:

$$|\delta L|_{bad} = \frac{2}{3}|\delta L|_b = 0.2, \quad R_{bad} = 2R_b = 5mm, \quad |T_w|_{bad} = \frac{2}{3}|T_w|_b = 2.3°/mm.$$

The maximum value of stretch in the coil block was computed by finding the maximum stretch in "the hard way" for the turns in the block. The turn is bent by "the hard way" around the axis, perpendicular to one of its "lateral surfaces" and it is bent by "the easy way" about the axis, tangent to one of its "lateral surfaces" and perpendicular to its direction. The geometry of the turn is described by two outer curves, which are the corners of the side of the turn far from the mandrel, and by two inner curves, which are the corners of the side of the turn constrained to the cylinder. Every curve consists of fifty points in longitudinal direction. So, fifty sections describe the geometry of the turn in the coil ends. The stretch in "the hard way" δL is computed for the four corners of the turn from the coil block by the following formula:

$$\delta L = \frac{2H}{|x_1| + |x_3|}\left[\Theta_{12} + \Theta_{23} - \pi\right].$$

Here $|x_1|$ and $|x_3|$ are the lengths of vectors x_1 and x_3; Θ_{12} and Θ_{23} are the angles between vectors (x_1, x_2) and (x_2, x_3) respectively; H is the width of the cable. Vectors x_1, x_3 connect the point on the outer curve of the turn with the preceding and following points on this curve respectively, when the stretch is evaluated on the outer curve of the turn. For the inner curve, vectors x_1, x_3 are defined the same way. For both cases of defining the stretch on the inner and on the outer curve of the turn, vector x_2 is placed in the section of the turn on the lateral side of the turn and it connects the point on the outer curve of the turn with the point on the inner curve of the turn.

The estimation of the length of coil end, at which the mechanical characteristics in the coil block are not exceeding the assurance factors for the chosen type of cable, is defined from inequality $U(E) \leq 1$. Particularly, in the process of the optimization of coil geometry for the dipole magnet the second coil block of the dipole was divided into two blocks in the coil ends to decrease the mechanical strain in coil block ends. The optimal length of half axis E and the optimal number of turns in the block n in the second obtained coil block were determined from the equality $U(E) = 0.7$, as it is shown in **Fig. 3**. At the final stage of optimization the spacer thickness in the coil ends were determined to suppress lower integral field multipoles and obtain the maximal effective length for the magnet [9]. It turns out, that the optimal geometry for n = 11 is the most appropriate from technological point of view.

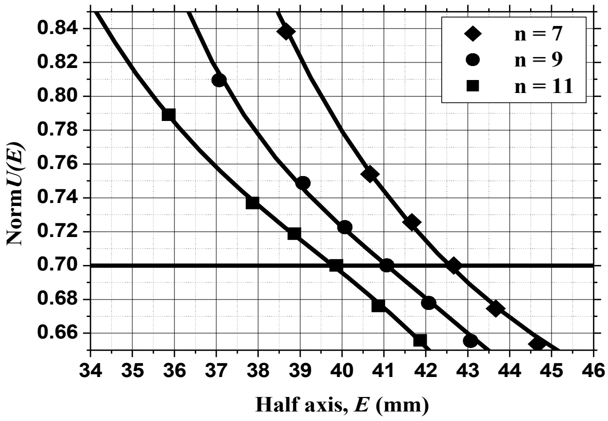

Fig. 3. The norm $U(E)$, determined for the second coil block with n turns.

The comparison between old geometry [10] (marked by index o) and new alternative geometry, obtained with the help of the BENDM program (marked by index n) is shown in the **Table 2**. The half-straight part of the coil block S from the magnet center was determined with the help of Roxie program [11] in the process of optimization of the field quality for the dipole magnet.

Table 2. Comparison of optimization results for coil ends.

Block	N	β_o, deg.	E_o, mm	S_o, mm	β_n , deg.	E_n, mm	S_n, mm
1	16	35.67	160.0	290.0	46.62	74.0	383.8
2	11	44.87	100.0	315.0	53.88	39.9	392.0
3	12	55.45	85.0	240.0	56.86	21.5	383.0
4	10	37.65	130.0	334.9	57.41	66.5	407.3
5	11	43.47	90.0	280.4	61.88	48.9	380.4
6	6	54.94	50.0	272.8	71.79	31.4	372.8
7	5	74.65	28.0	256.6	76.95	18.2	345.6

In the Roxie program the coil ends are modeled with the method, in which the edges of the turns at the cylinder are modeled by pseudo hyper-ellipse on cylinder. For comparison of modeling methods of geometry for the coil ends the output of coil ends geometry from the Roxie program is used, for obtaining of which the half axis E and the angles β from the Table 2 are substituted. The maximal mechanical strains by its absolute value $|\delta L|_{max}$ in the coil ends are compared. For the coil block ends, obtained with the help of BENDM program, the maximal mechanical strain is $|\delta L|_{max}$ = 0.2. For the coil block ends, modeled by the Roxie method for given in the Table 2 values of the half axis E_n and angles β_n, the maximal mechanical strain is $|\delta L|_{max}$ = 0.44. For the old geometry of the dipole magnet [10] the maximal mechanical strain in the coil ends is $|\delta L|_{max}$ = 0.73. The comparison of the old and new geometry has shown that the introduced here method of optimization gives a considerable opportunity to increase the effective length of the magnet, what follows from the **Table 3.**

Table 3. The main geometric parameters of the dipole magnet.

Geometry	Old	New
The length of the coil ends, mm	213.5	123.6
Difference between geometrical and effective length, mm	250.7	137.3
The maximal mechanical strain $|\delta L|_{max}$	0.73	0.2 (0.44)

Conclusion

A method for optimization of coil ends geometry for superconducting multilayer magnets is presented, based on the model of coil ends, which gives an opportunity to take into account the finite dimensions of the Rutherford cable. The minimization of lengths for the coil block ends is carried out for the geometry of the short dipole model to obtain the maximal effective length of the magnet. The implementation of the grouped end method for stacking coils in the end parts of the magnet provides for the tight containment of the turns in the coil ends that gives an opportunity to improve the training of the superconducting magnet and simplify its production technology. The BENDM program enables to determine the optimal shape for the coil end, when the longitudinal length of the coil block is specified. The comparison of geometries for coil block ends has shown the advantage of the models, in which the mechanical proprieties of superconducting cable and its finite dimensions are taken into consideration.

Acknowledgments

The mechanical optimization of the coil ends for the dipole magnet would not have been possible without use of the BEND program. The authors would like to thank Joe M. Cook and Jeffrey S. Brandt for provision of the BEND program and Joe M. Cook for the right on modernization of this program.

This article was fulfilled in the frames of the spade-work for ROSATOM State Corporation by the contract № Н 4д.47.03.08.125.

References

[1] H.I. Rosten. The Constant Perimeter End. RL-73-096, Rutherford Laboratory, Chilton, 1973-9.

[2] R.C. Bossert, et al. Analytical Solutions to SSC Coil End Design. FERMILAB-Conf-89/054, SSC-209, FNAL, 1989.

[3] J.M. Cook. An Application of Differential Geometry to SSC Magnet End Winding. FERMILAB-TM-1633, FNAL, 1990.

[4] J.M. Cook. Strain Energy Minimization in SSC Magnet Winding. FERMILAB-TM-1703, FNAL, 1990.

[5] R.C. Bossert, J.M. Cook and J.S. Brandt. End Designs for Superconducting Magnets. FERMILAB-TM-1757, FNAL, 1991.
[6] L.D. Landau and E. M. Lifshitz. Theory of Elasticity. Pergamon Press. New York. 1970.
[7] P.L. Walstrom. Twisted Coil Geometry in Plasma Confinement Devices. Journal of Fusion Energy, Vol. 6, No. 3, 1987.
[8] S. Kozub et al. Geometry Modernization of a Dipole Magnet for SIS 300. MT-20, Philadelphia, USA, 2007-9.
[9] J. Brandt, et al. Coil End Design for the SSC Collider Dipole Magnet. FERMILAB-TM-1753, FNAL, 1991.
[10] S. Kozub, et al. Magnetic and Thermal Characteristics of a Model Dipole Magnet for the SIS 300. IEEE Transactions on Applied Superconductivity, V. 17, I. 2 -P. 2, June 2007, p. 1161-63.
[11] S. Russenschuck (Editor): "ROXIE: Routine for the Optimization of magnet X-sections, Inverse Field Calculation and Coil End Design", First International ROXIE users meeting and workshop, 6-18.03.1998, — Proceedings. CERN 99-01. ISBN 92-9083-140-5.

March 25, 2008.

Л.М. Ткаченко, О.А. Чикилёв.
Минимизация механических напряжений в лобовых частях сверхпроводящей обмотки оболочечного типа.

Оригинал к печати подготовлен авторами.

Подписано к печати 1.04.2008. Формат 60 × 84/8. Офсетная печать.
Печ.л. 1,25. Уч.– изд.л. 1,1. Тираж 40. Заказ . Индекс 3649.

ГНЦ РФ Институт физики высоких энергий,
142284, Протвино, Московской обл.

Индекс 3649

П Р Е П Р И Н Т 2008-4, ИФВЭ, 2008