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I. Kachaev, V. Dorofeev, A. Ekimov, V. Gotman, A. Ivashin,  
V. Kalendarev, Yu. Khokhlov, M. Kholodenko, V. Konstantinov,  
V. Matveev, M. Mikhasenko, V. Nikolaenko, A. Plekhanov, D. Ryabchikov,  
Yu. Salomatin, V. Sugonyaev, E. Volkov, A. Zaitsev

**Study of the Reaction  $\pi^- A \rightarrow \pi^+ \pi^- \pi^- A$   
at Large Statistics with VES Setup**

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## Abstract

Kachaev I. et al. Study of the Reaction  $\pi^- A \rightarrow \pi^+ \pi^- \pi^- A$  at Large Statistics with VES Setup: IHEP Preprint 2015-8. – Protvino, 2015. – p. 5, figs. 4, refs.: 9.

Partial Wave Analysis of the  $\pi^+ \pi^- \pi^-$  system produced by 29 GeV/c  $\pi^-$  beam on a beryllium target is presented. About  $30 \cdot 10^6$  events in the wide  $|t'|$  range  $0 \dots 0.8$  GeV<sup>2</sup>/c<sup>2</sup> are collected with upgraded VES setup. The size of the data sample is 2.5 times larger than one previously analyzed by VES. Data are analyzed using formalism of the density matrix with unlimited rank. We discuss status of the  $a_1(1420)$ ,  $a_2(1700)$ ,  $a_3(1875)$  states and a structure of exotic  $\rho(770)\pi$  P-wave with  $J^{PC} = 1^{-+}$ . Parameters of  $a_3(1875)$  are estimated as  $M = 1985 \pm 20$  MeV/c<sup>2</sup>,  $\Gamma = 200 \pm 50$  MeV/c<sup>2</sup> (preliminary).

## Аннотация

Качаев И.А. и др. Исследование реакции  $\pi^- A \rightarrow \pi^+ \pi^- \pi^- A$  на большой статистике на установке ВЕС: Препринт ИФВЭ 2015-8. – Протвино, 2015. – 5 с., 4 рис., библиогр.: 9.

Представлен парциально-волновой анализ конечного состояния  $\pi^+ \pi^- \pi^-$ , рождаемого  $\pi^-$  пучком с энергией 29 GeV/c на бериллиевой мишени. На модернизированной установке ВЕС набрано около  $30 \cdot 10^6$  событий данной реакции в широком  $|t'|$  диапазоне  $0 \dots 0.8$  GeV<sup>2</sup>/c<sup>2</sup>. Количество данных в 2.5 раза больше, чем то, которое было набрано установкой ВЕС ранее. Данные описываются с использованием формализма матрицы плотности неограниченного ранга. Обсуждается статус состояний  $a_1(1420)$ ,  $a_2(1700)$ ,  $a_3(1875)$  и структура экзотической  $\rho(770)\pi$  P-волны с  $J^{PC} = 1^{-+}$ . Получена оценка параметров  $a_3(1875)$  —  $M = 1985 \pm 20$  МэВ/c<sup>2</sup>,  $\Gamma = 200 \pm 50$  МэВ/c<sup>2</sup> (данные предварительные).

## 1. VES setup and events selection

We present preliminary results of the mass independent PWA of the  $\pi^+\pi^-\pi^-$  system on the data obtained after VES upgrade. We compare them with the data obtained before the upgrade and discuss the structures which can be considered resonant.

VES setup is full featured magnetic spectrometer which operates on mostly  $\pi^-$  beam (2% of  $K^-$ ) with the energy 37 GeV before upgrade and 29 GeV after upgrade. It is equipped with electromagnetic calorimeter and multicellular Cherenkov counter for particle identification. Description of VES setup before upgrade can be found in [1] and after upgrade in [2]. For charged  $3\pi$  system net result of the upgrade is much bigger acceptance (see Fig. 1) due to taking out of trigger hodoscope and much bigger statistics due to upgraded DAQ.

Diffractive production of the charged  $3\pi$  final state dominates at VES energies, so data selection is simple and the background is negligible. We require beam particle identified as  $\pi^-$ , three tracks with charges  $+-$  (identification as  $\pi$  mesons is done for old data only), total energy for charged tracks 27–31 (36–38) GeV and vertex of interaction inside the target. Energy threshold for clusters in the electromagnetic calorimeter which are not associated with tracks is 0.5 GeV. Analysis is done for  $M(3\pi) = 0.6$ –2.6 GeV/ $c^2$  in 20 MeV/ $c^2$  bins and four  $|t'|$  intervals 0–0.03–0.15–0.30–0.80 GeV<sup>2</sup>/ $c^2$ . We have about  $30 \cdot 10^6$   $3\pi$  events after upgrade and about  $12 \cdot 10^6$   $3\pi$  events before upgrade.

## 2. Method of the analysis

Our method of the analysis is based on Illinois PWA [3]. We are using extended likelihood event by event fit with positive definite density matrix as parameters. No restrictions are placed on the rank of the matrix. Amplitudes are constructed using the isobar model, sequential decay of 3-particle system via  $\pi\pi$  subsystem, with relativistic corrections according to [4]. Each wave has quantum numbers  $J^P L M^\eta R$ , where  $J^P$  is spin-parity for  $3\pi$  system,  $M^\eta$  is projection of spin and exchange naturality,  $R$  is the

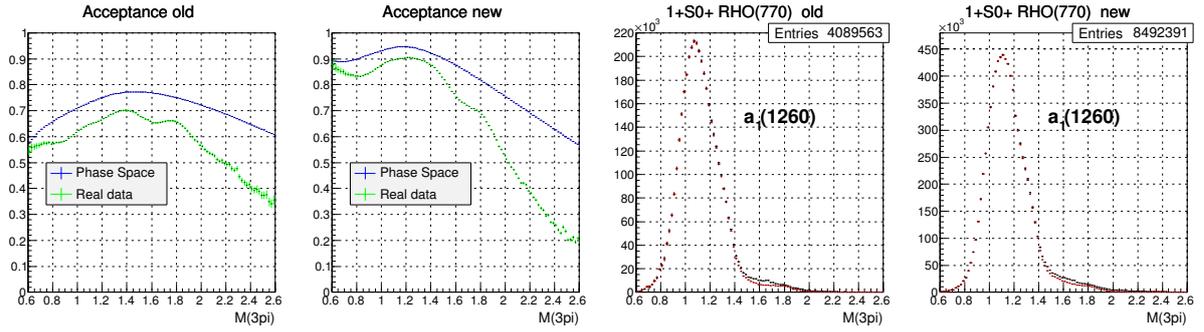


Figure 1. VES geometric acceptance before and after upgrade; largest wave  $1^+S0^+\rho(770)$  for  $|t'| < 0.03 \text{ GeV}^2/c^2$  in old and new data.

known resonance in  $\pi\pi$  system and  $L$  is orbital momentum in the  $R\pi$  decay. Isospin and G-parity  $I^G = 1^-$  are the same for all  $3\pi$  charged states. To describe broad part of  $\pi\pi$   $S$ -wave we use modified M solution from [5]. To make this amplitude broad we drop 4-th order terms and coupling to  $K\bar{K}$ . We name this pseudo state  $\varepsilon$ . It should describe among other things  $f_0(1400)$  and possible  $\sigma(600)$ . Narrow  $f_0(975)$  and  $f_0(1500)$  are included separately. Purely geometric (not GEANT) model of the acceptance is used.

## 2.1. Coherent part of the density matrix

Let us define coherent part of the density matrix  $\rho$  as the largest part of the matrix which has rank one and so behaves like vector of amplitudes. It can be constructed as follows. Let us decompose hermitian matrix  $\rho$  with dimension  $d$  into its eigenvalues and eigenvectors:

$$\rho_{ij} = \sum_{k=1}^d e_k V_k^i V_k^{j+} \quad \text{where} \quad \begin{cases} e_k \text{ is } k\text{-th eigenvalue} \\ V_k \text{ is } k\text{-th eigenvector} \end{cases}$$

Let  $e_1 > e_2 > \dots > e_d > 0$ . Let us define the “leading term” with matrix elements  $\rho_{ijL} = e_1 V_1^i V_1^{j+}$  as coherent part of density matrix and the rest, namely  $\rho_S = \rho - \rho_L$ , as incoherent part of  $\rho$ . In addition let it be  $e_1 \gg e_2$ . Then decomposition described is stable with respect to small variations of  $\rho$  matrix elements. This statement can be justified as follows. Eigenvalues of a hermitian matrix are always stable with respect to small variations of matrix elements; elements of eigenvector have this property if corresponding eigenvalue is well separated from the others. The condition  $e_1 \gg e_2$  is often met for  $3\pi$  system. Experience shows that resonances tend to concentrate in  $\rho_L$ . Results for full  $\rho$  are drawn on histograms below as black points with errors (upper points), for  $\rho_L$  as red one (lower points).

## 3. Fit results

In Fig. 1 one can see wave  $1^+S0^+\rho(770)$  for low  $|t'|$  region in both old 37 GeV and new 29 GeV data. The wave contains huge contribution from  $a_1(1260)$  and a shoulder at

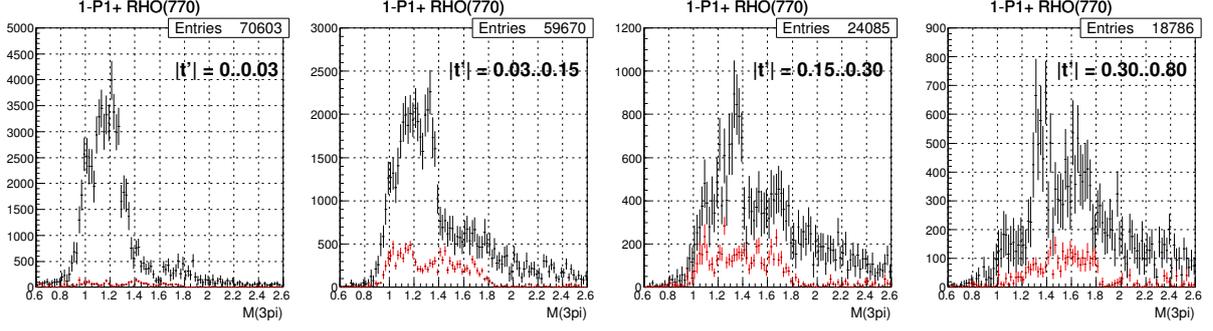


Figure 2. Exotic wave  $1^-P1^+\rho(770)$  in all four  $|t'|$  regions, old 37 GeV data.

$M(3\pi) \approx 1.7 \text{ GeV}/c^2$  which can correspond to  $a_1(1700)$ . Two conclusions can be drawn here: first, the structure of the wave is approximately the same in both old and new data; next, data for the coherent part of the density matrix fill the whole wave. The same is true for all other largest waves, like  $0^-S0^+\varepsilon$  and  $2^-S0^+f_2(1270)$ .

Probably the wave with exotic quantum numbers  $J^{PC} = 1^{-+}$  has the most controversial status in the whole  $3\pi$  PWA. Corresponding objects  $\pi_1(1300)$  and  $\pi_1(1600)$  are long discussed. The wave  $1^-P1^+\rho(770)$  is shown in Fig. 2, 3 for all four  $|t'|$  intervals both for old and new data. The wave is small, no more than 2–5% of the total number of events for old and new data in all  $|t'|$  regions. This wave does not correspond to coherent part of density matrix — results for  $\rho_L$  are 2–10 times smaller than for the whole  $\rho$ . Prominent feature of the new data is that this wave is two times larger than in old data with respect to total number of events for  $|t'| < 0.03 \text{ GeV}^2/c^2$  and is slightly more structured in other  $|t'|$  regions. The reason can be that our model of the setup is still too crude. Given this data existence of both  $\pi_1$  objects looks questionable.

Now we discuss some other possibly resonant waves. Only new 29 GeV data are shown. The new data have better quality although old data mostly lead us to the same conclusions. In Fig. 4a one can see wave  $2^+D1^+\rho(770)$  for medium  $|t'| = 0.03 \dots 0.15 \text{ GeV}^2/c^2$  region with well known  $a_2(1320)$ . State  $a_2(1700)$  was discussed in this wave. One can see that the state  $a_2(1320)$  is in the coherent part of the density matrix  $\rho_L$  and

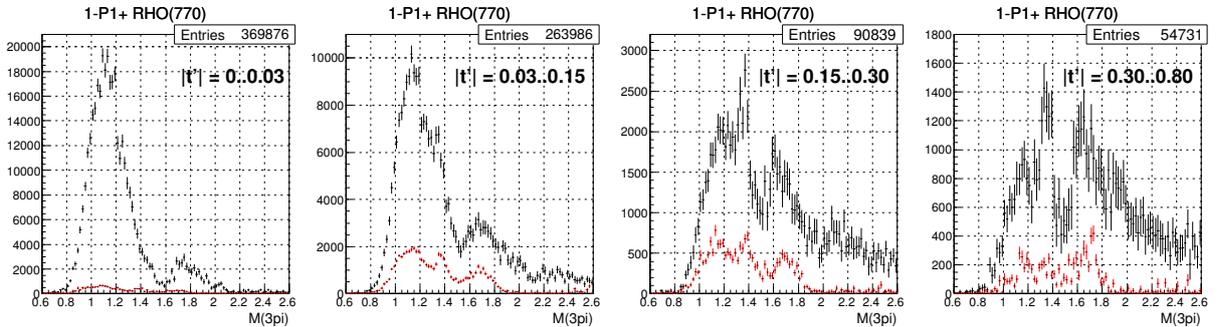


Figure 3. Exotic wave  $1^-P1^+\rho(770)$  in all four  $|t'|$  regions, new 29 GeV data.

there is nothing in this wave outside  $a_2(1320)$  region, especially in the  $\rho_L$ . We can't see anything which could be interpreted as  $a_2(1700)$ .

Until the end of this section all waves are shown for  $|t'| < 0.03 \text{ GeV}^2/c^2$ . In Fig. 4 b the wave  $1^+P0^+f_0(975)$  with discussed  $a_1(1420)$  is shown. Good narrow resonant like structure can be seen at  $M(3\pi) \approx 1.45 \text{ GeV}/c^2$ . The same structure, albeit less prominent and never reported, can be seen in our old data. This structure has some peculiarities — its coherent and incoherent parts are approximately of the same magnitude; coherent part is severely wider than the peak itself. These features make difficult resonant interpretation of given structure. Various attempts to interpret this structure exist [6, 7].

In Fig. 4 c the wave  $0^-S0^+f_0(1500)$  is shown. This is probably a decay mode  $\pi(1800) \rightarrow f_0(1500)\pi$  which was studied before but is much more pronounced in the new data. For this wave at  $M(3\pi) \sim 1.8 \text{ GeV}/c^2$  coherent part fills the whole wave which supports resonant interpretation of the peak.

In Fig. 4 d one can see the wave  $3^+S0^+\rho_3(1690)$ . A peak at  $M(3\pi) \sim 1.9 \text{ GeV}/c^2$  is clearly seen. The peak is even more pronounced in  $\rho_L$ . The mass-dependent fit for this wave was done. Full  $\rho$  matrix data were used. The data were described with relativistic Breit-Wigner for  $S$ -wave  $\rho_3(1690)\pi$  system plus incoherent linear background, ignoring nonzero width of  $\rho_3(1690)$ . Resonance parameters obtained are  $M = 1985 \pm 20 \text{ MeV}/c^2$ ,  $\Gamma = 200 \pm 50 \text{ MeV}/c^2$ . The object  $a_3(1875)$  is listed as "further states" (namely, dropped from the list of known states) in [8] and was last observed in [9]. Our analysis can be a ground to re-establish this state.

## 4. Conclusions

The Mass-independent PWA is done for old 37 GeV and new 29 GeV  $\pi^+\pi^-\pi^-$  data collected with VES setup. Preliminary results are shown. Large PWA waves look alike for 37 GeV and 29 GeV data. Some small waves are seen much better in the new data. Decay modes  $\pi(1800) \rightarrow f_0(1500)\pi$ ,  $a_3(1875) \rightarrow \rho_3(1690)\pi$  are seen in  $0^-S$  and  $3^+S$  waves. For  $a_3(1875)$  we obtain preliminary parameters

$$M = 1985 \pm 20 \text{ MeV}/c^2 \text{ (syst)} \quad \Gamma = 200 \pm 50 \text{ MeV}/c^2 \text{ (syst)}$$

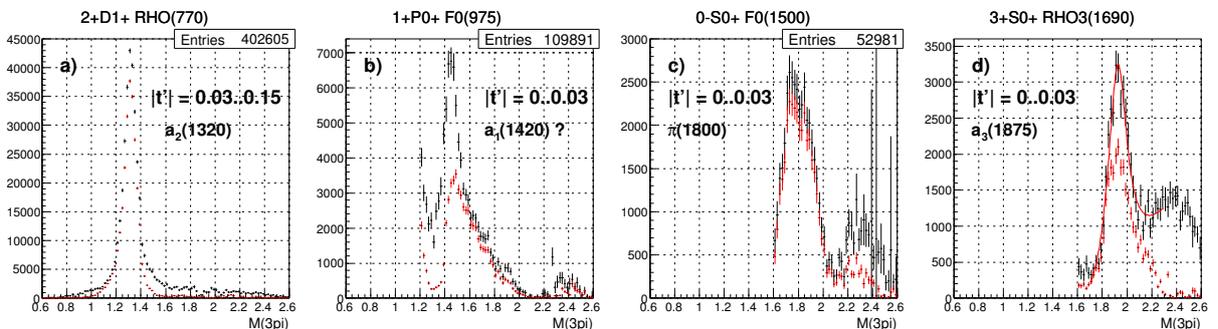


Figure 4. Waves  $2^+D1^+\rho(770)$ ,  $1^+P0^+f_0(975)$ ,  $0^-S0^+f_0(1500)$ ,  $3^+S0^+\rho_3(1690)$ , new data.

State  $a_2(1700)$  is not seen in  $2^+D1^+\rho\pi$ . Interpretation of  $f_0(975)\pi$  in  $1^+S$  wave at  $M \sim 1.45$  GeV/ $c^2$  is controversial while the signal itself is surely seen. The wave  $1^-P1^+\rho(770)$  with  $J^{PC} = 1^{-+}$  is small, no more than 2–4% from total number of events in all  $|t'|$  regions both in old and new data. Its coherent part is 2–10 times smaller.

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И.А. Качаев и др.

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ФГБУ ГНЦ ИФВЭ НИЦ «Курчатовский институт»  
142281, Московская область, город Протвино, площадь Науки, дом 1

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