NATIONAL RESEARCH CENTRE
«KURCHATOV INSTITUTE»
**Institute for High Energy Physics
of the National Research Centre
«Kurchatov Institute»**

M.M. Shapkin, S.A. Akimenko, A.V. Artamonov, A.M. Blik, V.S. Burtovoy, S.V. Donskov,
A.P. Filin, A.V. Inyakin, A.M. Gorin, G.V. Khaustov, S.A. Kholodenko, V.N. Kolosov,
A.S. Konstantinov, V.F. Kurshetsov, V.A. Lishin, M.V. Medynsky, Yu.V. Mikhailov,
V.F. Obraztsov, V.A. Polyakov, A.V. Popov, V.I. Romanovsky, V.I. Rykalin, A.S. Sadovsky,
V.D. Samoilenko, V.K. Semenov, O.V. Stenyakin, O.G. Tchikilev, V.A. Uvarov,
O.P. Yushchenko
(NRC "KURCHATOV INSTITUTE"-IHEP, PROTVINO),

V.A. Duk[1], S.N. Filippov, E.N. Gushchin, A.A. Khudyakov, V.I. Kravtsov,
Yu.G. Kudenko[2], A.Yu. Polyarush
(INR RAS, MOSCOW),

V.N. Bychkov, G.D. Kekelidze, V.M. Lysan, B.Zh. Zalikhanov
(JINR, DUBNA)

Study of the decay $K^+ \to \pi^+\pi^-\pi^+\gamma$ in the OKA experiment

[1] Also at University of Birmingham, Birmingham, United Kingdom
[2] Also at Moscow Institute of Physics and Technology, Moscow Region, Russia,
and at NRNU Moscow Engineering Physics Institute (MEPhI), Moscow, Russia

Abstract

Shapkin M.M. et al. Study of the decay $K^+ \to \pi^+\pi^-\pi^+\gamma$ in the OKA experiment: NRC «Kurchatov Institute» – IHEP Preprint 2018-8. – Protvino, 2018. – p. 10, figs. 11, tables 1, refs.: 10.

A high statistics data sample of the decays of K^+ mesons to three charged particles was accumulated by the OKA experiment in 2012 and 2013. This allowed to select a clean sample of about 450 events of $K^+ \to \pi^+\pi^-\pi^+\gamma$ decays with the energy of the photon in the kaon rest frame greater than 30 MeV . The measured branching fraction of the $K^+ \to \pi^+\pi^-\pi^+\gamma$, with $E_\gamma^* > 30$ MeV is $(0.71\pm0.05)\times10^{-5}$. The measured energy spectrum of the decay photon is compared with the prediction of the chiral perturbation theory to $O(p^4)$. A search for an up-down asymmetry of the photon with respect to the hadronic system decay plane is also performed.

Аннотация

Шапкин М.М. и др. Исследование распада $K^+ \to \pi^+\pi^-\pi^+\gamma$ в эксперименте ОКА: Препринт НИЦ «Курчатовский институт» – ИФВЭ 2018-8. – Протвино, 2018. – 10 с., 11 рис., 1 табл., библиогр.: 10.

Большой объем данных по распаду K^+ мезонов в три заряженных частицы был записан в эксперименте ОКА в 2012-2013 годах. Это позволило выделить чистую выборку из примерно 450 событий распада $K^+ \to \pi^+\pi^-\pi^+\gamma$, с энергией фотона в системе покоя каона больше 30 МэВ. Измеренный бренчинг рапада $K^+ \to \pi^+\pi^-\pi^+\gamma$ с $E_\gamma^* > 30$ MeV равен $(0.71 \pm 0.05) \times 10^{-5}$. Проведено сравнение измеренного энергетического спектра фотонов с вычислениями в киральной теории возмущений в порядке $O(p^4)$. Проведён поиск асимметрии верх-низ вылета фотона по отношению к плоскости распада адронной системы.

1. Introduction

The present experimental status of $K^+ \to \pi^+\pi^-\pi^+\gamma$ is rather meagre. It was observed in one experiment on statistics of 7 events [1]. The photon energies in these events were low, that did not allow to search for deviations from simple QED process of photon emission. The measured value of the $K^+ \to \pi^+\pi^-\pi^+\gamma$ branching ratio, with $E_\gamma^* > 5 MeV$ is $(1.04 \pm 0.31) \times 10^{-4}$ [2]. In the present analysis we have a possibility for more detailed study of this decay using larger data sample collected by the OKA experiment. The appropriate theory framework for such investigation is the chiral perturbation theory [3] (CHPT). To the lowest order in an expansion in momenta and meson masses, the radiative decays are completely determined [4] by the non-radiative amplitude for $K^+ \to \pi^+\pi^-\pi^+$. At next-to-leading order, a full-fledged CHPT calculation of nonleptonic weak amplitudes of $O(p^4)$ is required. Such a calculation is done in [5].

2. Separated kaon beam and OKA experiment

The OKA experiment makes use of a secondary hadron beam at the U-70 Proton Synchrotron of NRC "Kurchatov Institute"-IHEP, Protvino, with enhanced fraction of kaons obtained by RF-separation with Panofsky scheme [6]. The OKA setup, Fig. 1, is a double magnetic spectrometer complemented by electromagnetic and hadron calorimeters and a Decay Volume. The first magnetic spectrometer, consisting of the magnet M_1 and surrounding 1 mm pitch PC's ($BPC_{(1Y)}$, $BPC_{(2Y,2X)}$, $BPC_{(3X,3Y)}$, $BPC_{(4X,4Y)}$) serves for the beam momentum measurement. It is supplemented by two threshold Cherenkov counters $\check{C}_1$, $\check{C}_2$ for kaon identification and by beam scintillation counters S_1, S_2, S_3, S_4. The 11 m long Decay Volume (DV) filled with helium contains 11 rings of guard system (GS). To reinforce GS, a gamma detector (BGD), made of lead glass blocks located behind the DV is used as a veto at large angles, while low angle particles pass through a central opening. The wide aperture spectrometric magnet, $SM_{(SP40A)}$, with a field integral of ~ 1 Tm serves as a spectrometer for the charged decay products together with corresponding tracking
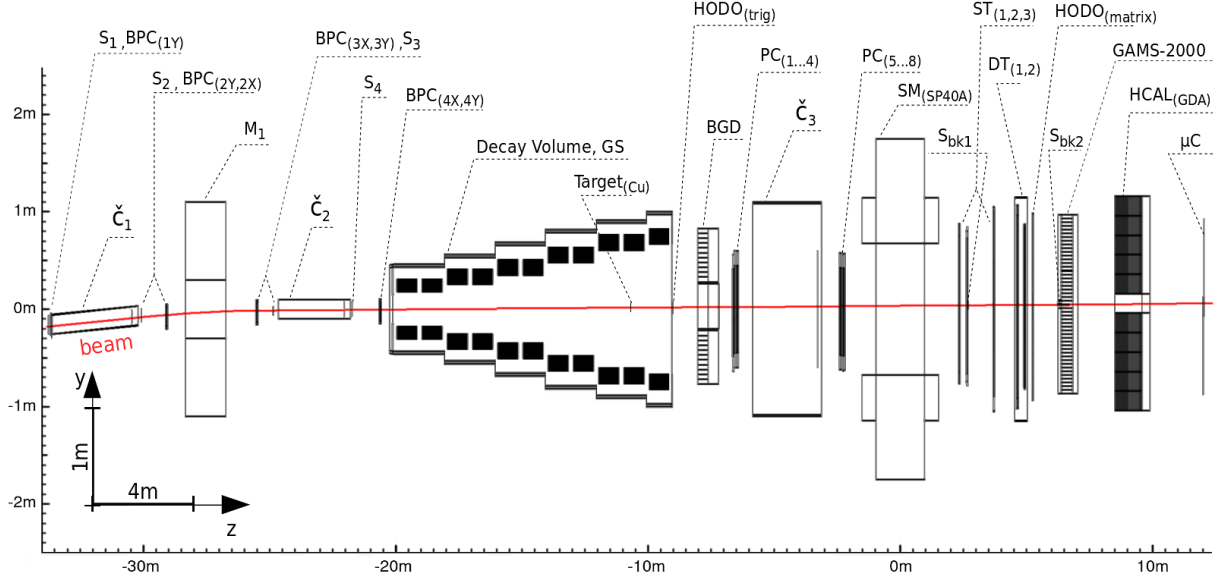
1

Figure 1. Schematic elevation view of the OKA setup, see the text for details.

chambers: 2 mm PC's ($PC_{1,...,8}$), 9 mm diameter straw tubes $ST_{(1,2,3)}$ and 30 mm diameter drift tubes $DT_{1,2}$. The matrix hodoscope $HODO_{(matrix)}$ is composed of 252 scintillator tiles with WLS+SiPM readout. It is used in the trigger, improves time resolution and links x–y projections of a track. Two scintillator counters S_{bk1}, S_{bk2} serve to suppress undecayed beam particles. At the end of the OKA setup there are two calorimeters: electromagnetic (GAMS-2000) made of lead glass blocks and a hadron one ($HCAL_{(GDA)}$)- 100 iron-scintillator sandwiches. Finally, four partially overlapping muon counters are located downstream the HCAL. The data acquisition system of the OKA setup [7] operates at ~ 25 kHz event rate with the mean event size of ~ 4 kByte. The detailes of the setup and the beam can be found elswhere [8].

3. The analysis procedure

The study of the decay $K^+ \to \pi^+\pi^-\pi^+\gamma$ is performed with the data set accumulated in 2012 and 2013 runs with 17.7 GeV/c beam momentum. Two triggers were used. The first one selects beam kaons decays inside the OKA setup: $\mathtt{Tr_{Kdecay}} = \mathtt{S_1 \cdot S_2 \cdot S_3 \cdot S_4 \cdot \check{C}_1 \cdot \bar{\check{C}}_2 \cdot \bar{S}_{bk}}$ and, in addition, requires an energy deposition in GAMS-2000 e.m. calorimeter higher than MIP: $\mathtt{Tr_{Gams}} = \mathtt{Tr_{Kdecay}} \cdot (\mathtt{E_{Gams}} > \mathtt{MIP})$. The second one, $\mathtt{Tr_{HODO}} = \mathtt{Tr_{Kdecay}} \cdot (2 \leq \mathtt{Mult} \leq 4)$, includes additionally a requirement on multiplicity in the Matrix hodoscope. The beam intensity ($\mathtt{S_1 \cdot S_2 \cdot S_3 \cdot S_4}$) was $\sim 2 \cdot 10^6$ per spill, the fraction of kaons in the beam was $\sim 12.5\%$, i.e. the kaon intensity was $\sim$250k/spill. The total number of kaons entering the DV corresponds to $\sim 3.4 \times 10^{10}$. For the estimation of the background contribution to the selected data set, a sample of the Monte Carlo events with six main decay channels of charged kaon ($\pi^+\pi^0$, $\pi^+\pi^0\pi^0$, $\pi^+\pi^-\pi^+$, $\mu^+\nu$, $\pi^0\mu^+\nu$, $\pi^0 e^+\nu$) mixed accordingly to the

2

branching fractions, with the total statistics about equal to that of the recorded data sample is used. The Monte Carlo events are passed through full OKA simulation and reconstruction procedures. Monte Carlo sample for the signal events is produced in the same way as for the background. As the input, the weight, proportional to the square of the absolute value of the matrix element for the decay $K^+ \to \pi^+\pi^-\pi^+\gamma$, given in [4] is used.

3.1. Event selection

To select $K^+ \to \pi^+\pi^-\pi^+\gamma$ decay channel in off-line analysis a set of requirements is applied:
– the momentum of the beam track is required to be measured;
– the number of the secondary charged tracks is required to be equal to three and the net charge of them is equal to +1;
– the decay vertex should have good χ^2 and should be with a margin incide the DV ;
– the charged tracks are not identified as electrons in the GAMS-2000 electromagnetic calorimeter;
– the missing mass squared to each positive pion $M^2_{miss}(\pi^+) = (P_{K^+} - P_{\pi^+})^2$ should be greater than 0.07 GeV^2;
– the event should contain one and only one photon with the energy $E_\gamma > 0.5$ GeV – the invariant mass of the photon with each pion $(M(\pi\gamma))$should be greater than 0.17 GeV ;
– the square of the transverse momentum of the $\pi^+\pi^-\pi^+\gamma$ system is less than 0.001 GeV^2;
– the ratio of the momentum of the $\pi^+\pi^-\pi^+\gamma$ system to the momentum of the beam track (X_b) should be within the range 0.95-1.05.
The cut on the square of the missing mass to each positive pion $M^2_{miss}(\pi^+)$ is used for the suppression of the background from the decay $K^+ \to \pi^+\pi^0$ with $\pi^0 \to e^+e^-\gamma$. The distribution on $M^2_{miss}(\pi^+)$ is shown in Fig 2 for the data, main sources of the background and the signal. The distribution for the background reproduces well the peak at the square of π^0 mass, visible in the data.

The main source of the background for the decay $K^+ \to \pi^+\pi^-\pi^+\gamma$ is the decay of kaon to three charged pions when the pions produce hadron showers in the electromagnetic calorimeter and, because of fluctuations, a part of a shower is not associated with a charged track by the reconstruction program. To suppress such kind of background a cut on $M(\pi\gamma)$ is done, Fig 3. The reconstructed momentum of the $\pi^+\pi^-\pi^+\gamma$ system should be equal to the momentum of the beam particle. This motivates the last two cuts listed above. The distributions of the square of the transverse momentum of the $\pi^+\pi^-\pi^+\gamma$ system and the ratio of absolute value of its momentum to the momentum of the beam particle is shown in Fig. 4 and Fig. 5.

The invariant mass distribution of the $\pi^+\pi^-\pi^+\gamma$ system after application of all the cuts listed above is shown in Fig. 6. We see clear separation of the signal, peaking around the nominal value of the kaon PDG [2] mass and the background concentrating at higher masses. The number of events in the signal region is about 450. As almost all the photons in the selected $K^+ \to \pi^+\pi^-\pi^+\gamma$ events have the energy in the kaon rest frame greater
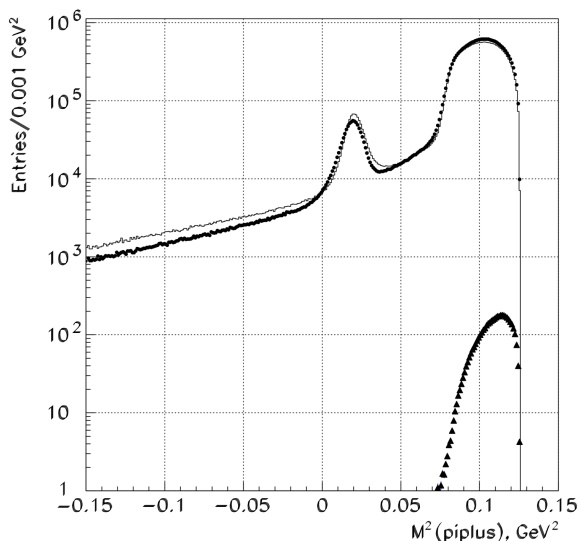
Figure 2. Square of the missing mass to each positive pion for the data (black circles), main sources of the background (histogram) and the MC signal (black triangles).
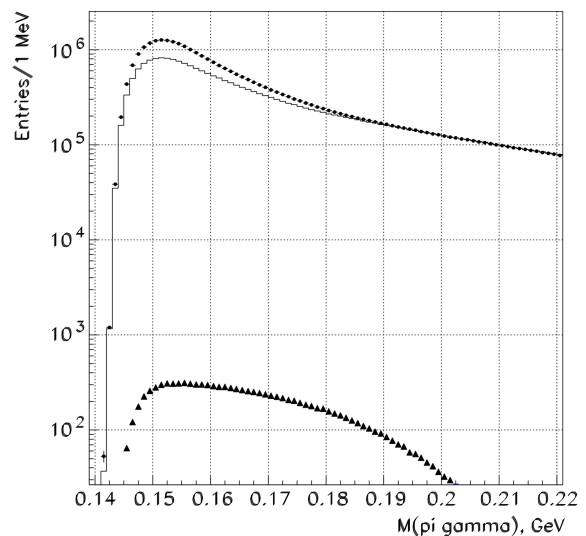
Figure 3. The invariant mass distribution of the charged pions and photons in the electromagnetic calorimeters for the data (black circles), main sources of the background (histogram) and the MC signal (black triangles).

than 0.03 GeV, in the following analysis we apply an additional explicit cut $E_\gamma^* > 0.03$ GeV.

3.2. Measurement of the branching ratio of the $K^+ \to \pi^+\pi^-\pi^+\gamma$ decay

The invariant mass distribution of the $\pi^+\pi^-\pi^+\gamma$ system in the signal region is shown in Fig. 7. The Monte Carlo distributions for the main sources of the background and the signal events are shown together with the data. We define the mass range of 0.486-0.504 GeV as a signal region. The number of the decays of $K^+ \to \pi^+\pi^-\pi^+\gamma$ is determined as the difference of the number of the data events in the signal region and the expected number of events from the background. The expected background contribution to the signal region is determined using Monte Carlo events for six main channels of the kaon decays described in the above section. To determine the branching fraction of the decay $K^+ \to \pi^+\pi^-\pi^+\gamma$ we use the decay $K^+ \to \pi^+\pi^-\pi^+$ as the normalization channel. We expect the cancellation of many ambiguities in the ratio of the branching fractions $\mathrm{Br}(K^+ \to \pi^+\pi^-\pi^+\gamma)/\mathrm{Br}(K^+ \to \pi^+\pi^-\pi^+)$. The invariant mass spectrum of the system of three charged pions for the same selection criteria as for the charged part of the $\pi^+\pi^-\pi^+\gamma$ decay is shown in Fig 8.

The branching fraction of the decay $K^+ \to \pi^+\pi^-\pi^+\gamma$ is determined in the following
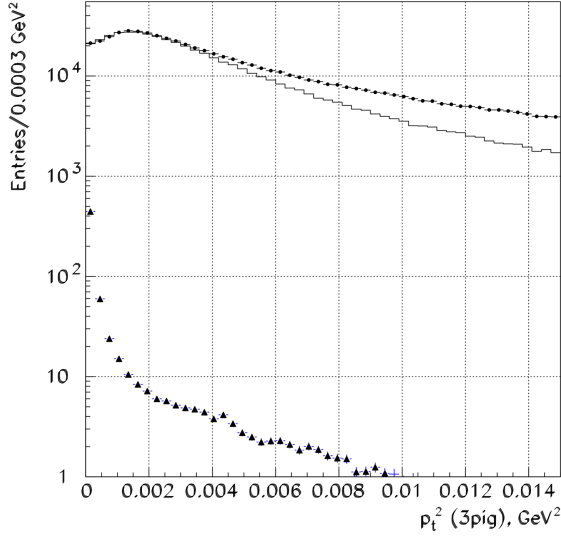
Figure 4. The distribution of the square of transverse momentum of the $\pi^+\pi^-\pi^+\gamma$ system for the data (black circles), main sources of the background (histogram) and the MC signal (blue triangles)
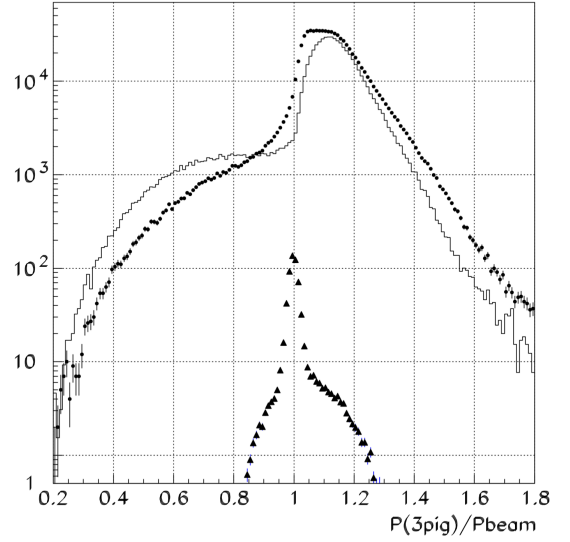
Figure 5. The ratio of the absolute value of the momentum of the $\pi^+\pi^-\pi^+\gamma$ system to the momentum of the beam particle for the data (black circles), main sources of the background (histogram) and the MC signal (blue triangles)

way:

$$Br(3\pi\gamma) = Br(3\pi)_{PDG} \times \epsilon(3\pi) \times N(3\pi\gamma)_D/\epsilon(3\pi\gamma)/N(3\pi)_D,$$

where $Br(3\pi)_{PDG}$ is the branching fraction of the decay $K^+ \to \pi^+\pi^-\pi^+$ from PDG [2]; $\epsilon(3\pi)$ and $\epsilon(3\pi\gamma)$ are the reconstruction efficiencies for the decays $K^+ \to \pi^+\pi^-\pi^+$ and $K^+ \to \pi^+\pi^-\pi^+\gamma$, determined from the Monte Carlo; $N(3\pi\gamma)_D$ and $N(3\pi)_D$ are the numbers of the decays of $K^+ \to \pi^+\pi^-\pi^+\gamma$ and $K^+ \to \pi^+\pi^-\pi^+$ in the data. Efficiencies $\epsilon(3\pi)$ and $\epsilon(3\pi\gamma)$, determined from the Monte Carlo, are 0.12 ± 0.002 and 0.024 ± 0.001, respectively. The obtained result is

$$Br(3\pi\gamma) = (0.71 \pm 0.04(stat.)) \times 10^{-5}.$$

The main source of the systematic error is the uncertainty in the estimate of the background contribution to the signal region (see Fig. 6,7). For the estimation of this uncertainty we varied the normalization of the background distribution. The variations are quadratically summed with that due to the changes of the cuts listed in the selection criteria. The estimated value of the systematic uncertainty for the branching fraction is 0.03×10^{-5}. The final result is

$$Br(3\pi\gamma) = (0.71 \pm 0.04(stat.) \pm 0.03(syst.)) \times 10^{-5}, E_\gamma^* > 0.03 GeV.$$

The theory prediction in the framework of the CHPT is $Br_{th} = (0.665 \pm 0.005) \times 10^{-5}$ [5].
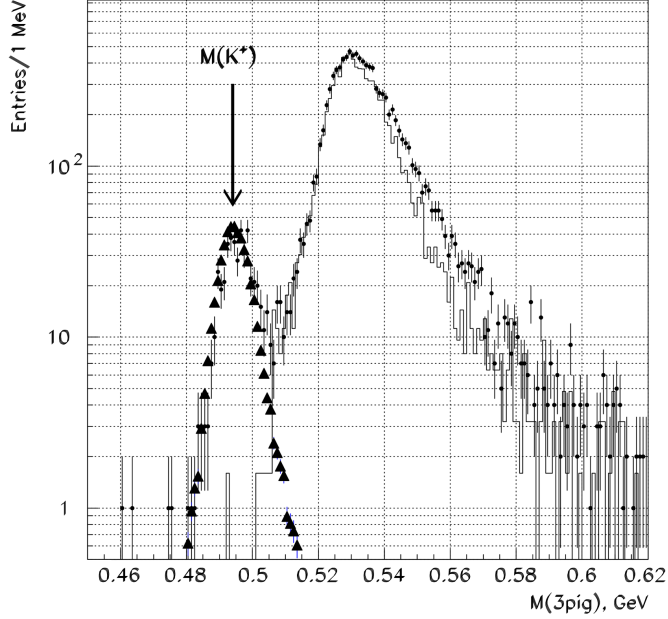
5

Figure 6. The invariant mass distribution of the $\pi^+\pi^-\pi^+\gamma$ system for the data (black circles), main sources of background (histogram) and signal decays of the kaons (black triangles)

3.3. Measurement of the photon energy spectrum

To perform a measurement of the photon energy spectrum, we split the data sample into four parts with photon energies in the rest frame of the kaon lying in the ranges 0.03-0.04, 0.04-0.05, 0.05-0.06 and 0.06-0.07 GeV. After that we apply the procedure of the previous section to each of the four data samples. The $\pi^+\pi^-\pi^+\gamma$ invariant mass spectra for the listed above ranges of the photon energy are shown in Fig 9 together with expected signal and background contributions.

The obtained values of the branching fractions for the given energy ranges are listed in the table 1 and shown in the Fig. 10. For comparison, we also give the $O(p^4)$ CHPT predictions from [5].

4. Search for a photon up-down asymmetry

In a decay in the beauty sector $B^+ \to K^+\pi^+\pi^-\gamma$ the LHCb experiment has found significant up-down asymmetry of the photon with respect to the hadronic system decay plane [9]. This observable was proposed in [10], it is both P and T-odd. We perform an analogous study for the radiative kaon decay to three charged pions and photon to search for a New Physics effects. In Fig 11 we show the distribution of the cosine of the angle of
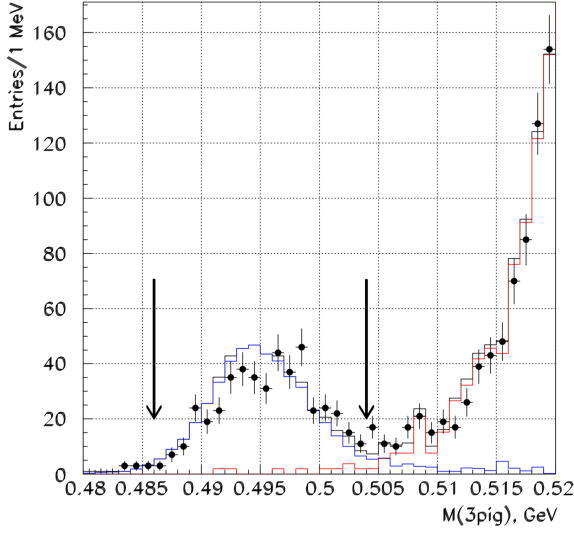
6

Figure 7. The invariant mass distribution of the $\pi^+\pi^-\pi^+\gamma$ system for the data (black circles), the signal decays (blue histogram), the main sources of the background (red histogram) and the sum of the contributions of the signal decays and the background (black histogram). The arrows show the signal region used in the analysis.
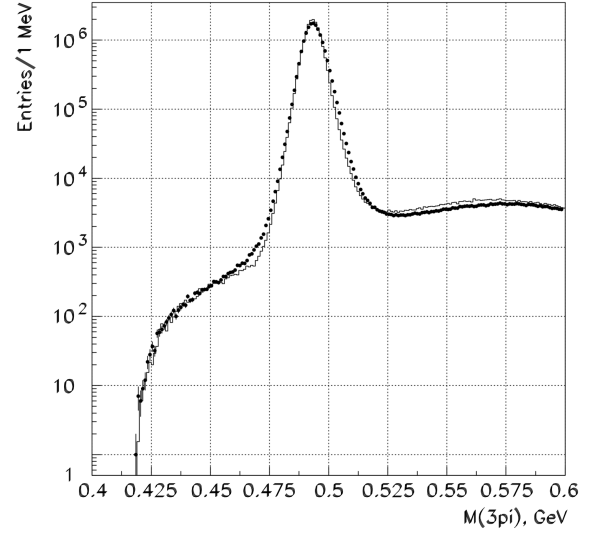
Figure 8. The invariant mass distribution of three charged pions for the data (black circles) and for the Monte Carlo for the six main channels of the charged kaon decay (histogram).

the photon direction with respect to the pion system decay plane:

$$cos(\theta) = n_\gamma \cdot [p_f(\pi) \times p_s(\pi)]/|[p_f(\pi) \times p_s(\pi)]|,$$

Table 1. Values of the branching fractions for the decay $K^+ \to \pi^+\pi^-\pi^+\gamma$ in the intervals of the photon energy in the kaon rest frame. The uncertainties of the data given in the table are the statistical ones.

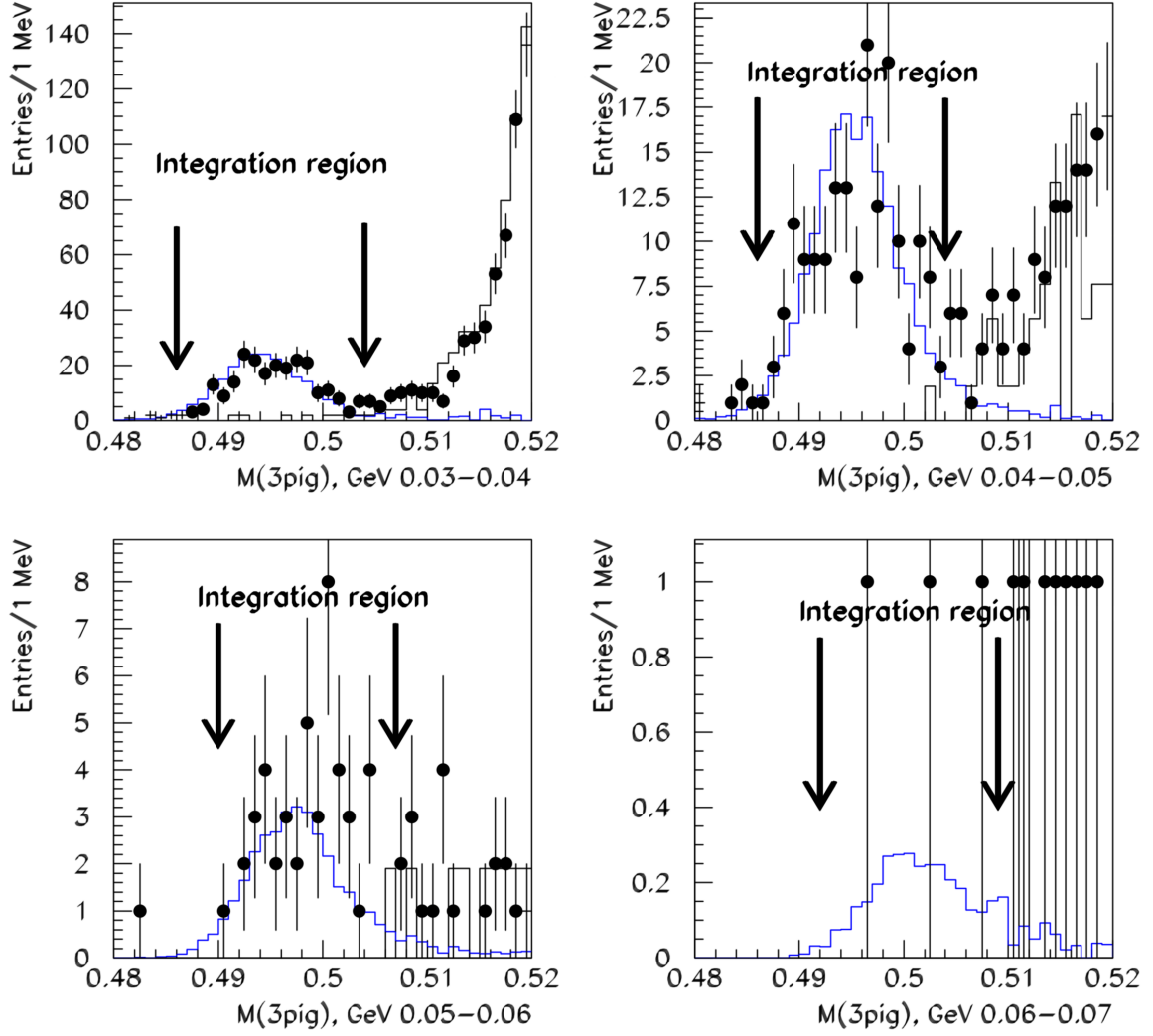| Energy interval (GeV)	Branching fraction(data)	Branching fraction(CHPT [5])
0.03-0.04	$(5.17 \pm 0.34) \cdot 10^{-6}$	$(4.93 \pm 0.05) \cdot 10^{-6}$
0.04-0.05	$(1.55 \pm 0.12) \cdot 10^{-6}$	$(1.44 \pm 0.01) \cdot 10^{-6}$
0.05-0.06	$(0.35 \pm 0.05) \cdot 10^{-6}$	$(0.269 \pm 0.003) \cdot 10^{-6}$
0.06-0.07	$(0.11 \pm 0.06) \cdot 10^{-7}$	$(0.136 \pm 0.002) \cdot 10^{-7}$

Figure 9. Invariant mass distributions of the $\pi^+\pi^-\pi^+\gamma$ system for the data (black circles), signal decays (blue histogram) and main sources of the background (black histogram) for the different energy ranges of the photon in the decay $K^+ \to \pi^+\pi^-\pi^+\gamma$. The top left distribution is for the photon energy range 0.03-0.04 GeV, the top right one is for the range 0.04-0.05 GeV, the bottom left one is for the 0.05-0.06 GeV and the last one is for the 0.06-0.07 GeV. The arrows show the signal regions used in the analysis.

where n_γ is the unit vector of photon direction in the 3-pion rest frame, $[p_f(\pi) \times p_s(\pi)]$ is the vector product of the momenta of the fastest and slowest pions in the same frame. For comparison, we show the same distribution for the Monte Carlo signal events with the matrix element from [5]. The observed asymmetry in the data

$$A = (N(cos\theta > 0) - N(cos\theta < 0))/N_{total} = 0.03 \pm 0.05 \pm 0.03$$

8
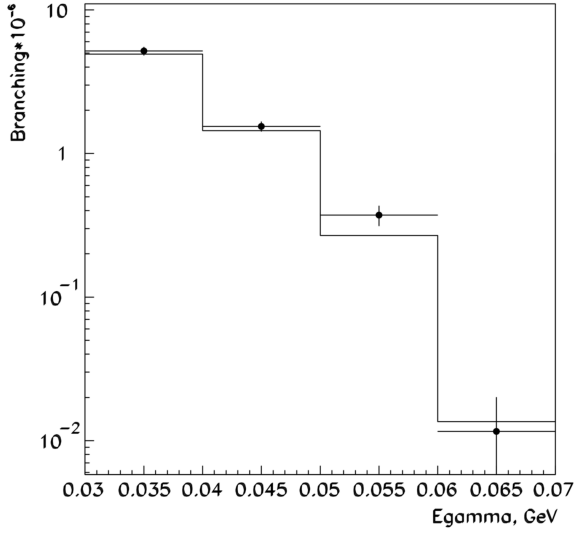
Figure 10. Measured branching fractions for the decay $K^+ \to \pi^+\pi^-\pi^+\gamma$ in the photon energy ranges in the kaon rest frame for the data (black circles) and the CHPT prediction from [5] (histogram).
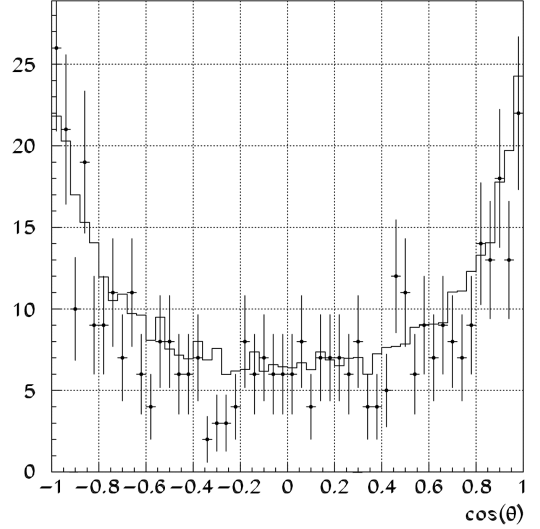

Figure 11. The distribution (black circles) of the cosine of the angle of the photon direction with respect to the hadronic system decay plane(see the text). The histogram shows the Monte Carlo prediction for the decay $K^+ \to \pi^+\pi^-\pi^+\gamma$ with the matrix element from [5].

is consistent with zero within statistical error.

Conclusions

The decay $K^+ \to \pi^+\pi^-\pi^+\gamma$ is studied on statistics of 450 events. The measured branching fraction is $Br(3\pi\gamma) = (0.71 \pm 0.05) \times 10^{-5}$ for $E^*_\gamma > 0.03$ GeV. The photon energy spectrum for this decay is also determined. The measured branching fraction and the energy spectrum agree well within the errors with the calculations in the framework of the chiral perturbation theory. The measured up-down asymmetry of the photon with respect to the decay plane of the hadronic system is 0.03 ± 0.06. No sign of P and T-odd effects is observed.

Acknowledgements

References

[1] V.V. Barmin et al., Sov. J. Nucl. Phys. 50, 421 (1989).

[2] C. Patringnani et al. (Particle Data Group), Chin. Phys. C. 40, 100001 (2016).

[3] S. Weinberg, Physica. 96A, 327 (1979);
J. Gasser and H. Leutwyler, Ann. Phys. (N.Y.) 158, 142 (1984); Nucl. Phys. B250, 465 (1985);
H. Leutwyler, Ann. Phys. (N.Y.) 235, 165 (1994).

[4] G. D'Ambrosio, G. Ecker, G. Isidori and H Neufeld, Phys. Lett. B380, 165 (1996).

[5] G. D'Ambrosio, G. Ecker, G. Isidori and H Neufeld, Z. Phys. C76 Issue 2, 301 (1997).

[6] V. I. Garkusha et al., IHEP preprint, IHEP 2003-4.

[7] S. V. Donskov et al., Instrum. Exp. Tech. 59 no.4, 519 (2016).

[8] A. S. Sadovsky et al., Eur.Phys.J.C(2018) 78:92.

[9] R. Aaij et al. (LHCb Collaboration), Phys. Rev. Lett. 112, 161801 (2012).

[10] M. Gronau, Y. Grossman, D. Pirjol and A. Ryd, Phys. Rev. Lett. 88, 051802 (2002).