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Sampling-fluctuations in electromagnetic calorimeters

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Abstract

Denisov S.P., Goryachev V.N. Sampling-fluctuations in electromagnetic calorimeters: NRC «Kurchatov institute» – IHEP Preprint 2021–4. – Protvino, 2021. – p. 6, figs. 7, table 1, refs.: 5.

The paper presents the results of calculations of the sampling fluctuations of ionization energy loss in Pb-LAr electromagnetic sandwich calorimeters. The GEANT4 package was used to generate showers initiated by electrons with energies E_0 from 20 to 500 GeV. It is shown that the dependence of the sampling fluctuations on E_0 and the thickness of the lead absorber x can be described by the formula $kx^b/\sqrt{E_0}$. The exponent b weakly depends on the LAr gap width d and is close to 2/3 and k=0.1985–0.0363ln$d$ (mm).

Аннотация

Денисов С.П., Горячев В.Н. Сэмплинг-флуктуации в электромагнитных калориметрах: Препринт НИЦ «Курчатовский институт» – ИФВЭ 2021-4. – Протвино, 2021. – 6 с., 7 рис., 1 табл., библиогр.: 5.

В работе представлены результаты расчетов сэмплинг-флуктуаций ионизационных потерь энергии в электромагнитных Pb-LAr калориметрах типа “сэндвич”. Для генерации ливней, инициированных электронами с энергиями E_0 от 20 до 500 ГэВ, использовался пакет GEANT4. Показано, что зависимость сэмплинг-флуктуаций от E_0 и толщины свинцового абсорбера x может быть описана формулой $kx^b/\sqrt{E_0}$. Показатель степени b слабо зависит от ширины жидкоаргонового слоя d и близок к 2/3, а k=0.1985–0.0363ln$d$ (мм).

1. Introduction

The main contribution to the stochastic term $a/E^{1/2}$ of the energy resolution of the "sandwich" type electromagnetic (EM) calorimeters, consisting of alternating layers of passive and active medium, comes from the so-called sampling fluctuations (SF) associated with the statistical nature of the ionization energy losses by charged particles in EM showers and their distribution between active and passive layers [1,2]. The most popular passive material is Pb while W, U, Fe, Cu are used less frequently. The active layers are usually liquid argon, scintillator or silicon.

SF are defined by the physical processes occurring in the calorimeter and do not depend on its design limitations (short length or gaps between modules) or method of the signals detection. Thus SF determine the best possible value of the energy resolution that can be achieved for a given calorimeter. The purpose of our study is to investigate, by simulating EM showers, the SF dependence on the shower energy and the structure of a Pb-LAr sandwich calorimeter. The radiation length X_0 was chosen to be 6.37(Pb) and 19.55(LAr) g/cm^2 [1].

2. EM showers simulation and events selection

To simulate EM showers the GEANT4 10.01.p02 package (Physical list FTFP_BERT) [2] with the range cut of R_c=100 μm was used. The corresponding energy thresholds for e^+, e^- and γ are 232, 242, and 29 keV for Pb and 82, 83, and 2.0 keV for LAr.

EM showers are generated for electrons with energies E_0 of 20, 40, 80, 200, and 500 GeV for calorimeters with 400 Pb and LAr layers. The Pb layer thickness x varied from 0.2 to 0.9 X_0 with a step of 0.1 X_0 and the width d of the LAr gap is 2, 3, 4, 6 and 8 mm. Thus, the minimal total thickness of the calorimeters is 85.6 X_0 (at x=0.2 and d=2 mm). The transverse dimension of the calorimeters is 1x1 m^2. EM shower energy leakage from the calorimeters of such size can be neglected (see below). It was verified that changing R_c upto 700 μm weakly affect the SF values. For example the relative difference of SF's obtained with R_c=100 and 700 μm for the calorimeter of x/X_0=0.4 and d=4 mm is equal to (0.2±1.0)% at E_0=80 GeV.

For each set of E_0, x, d parameters 10^4 events are generated. For each event, the values of ionization energy loss in each of 400 layers of LAr are calculated. The typical distribution of the total energy E released in LAr is shown on the left side of Fig. 1. All such distributions have tails in the region of low E growing with E_0 increase. It turns out that the tails are associated with photonuclear reactions: they disappear if strong interactions are "turned off" in the simulation. Events with anomalously low energy are often observed during the calibration of calorimeters in electron beams, but they, as a rule, are associated only with bremsstrahlung of electrons in the matter in front of the calorimeter and are excluded by introducing a cutoff on the signal amplitude (see, for example, [4, 5]). In the present work a similar procedure was used: only events with E from the interval $<E>\pm 3RMS$ were accepted to estimate the SF. The $N(E)$ spectra for selected events are well fitted by Gaussian. An example of such spectrum is shown on the right side of Fig. 1. Its parameters are $<E>=59.34$, $\sigma=0.281$ and $RMS=0.272$ GeV. For the spectrum on the left side of Fig. 1 $<E>=59.17$ and $RMS=0.472$ GeV.

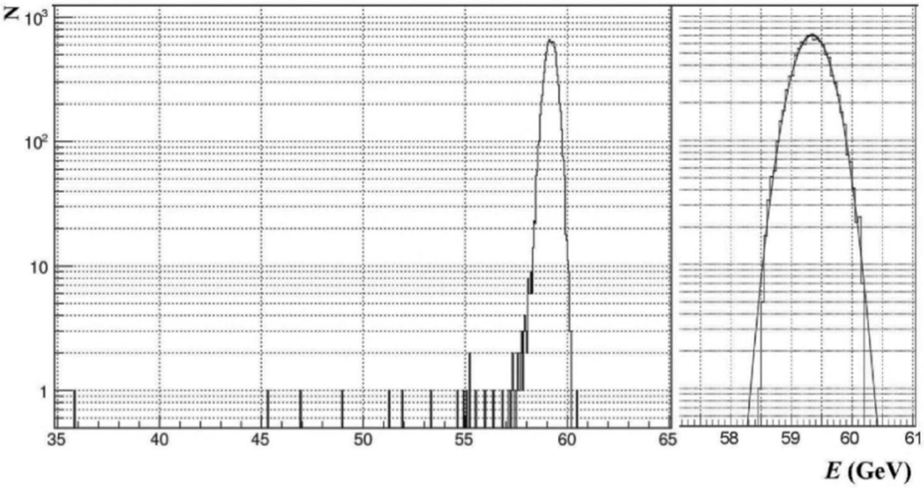

Fig. 1. Spectra of ionization energy loss in liquid argon of a calorimeter with 50 Pb layers of 0.6 X_0 thick and 4 mm LAr gaps (31.4 X_0 in total). Left – for all events, right – for events from the interval of $<E>\pm 3$ RMS fitted by a Gaussian. E_0=500 GeV.

The examples of the SF dependence on the calorimeter thickness l_c is shown in Fig. 2. It can be seen that $\sigma/<E>$ reaches a plateau at l_{cp} from 22 to 30 X_0. Thus fluctuations due to shower energy leakage from calorimeters with l_c more than 30 X_0 can be neglected. The datain Fig. 2 make it possible to estimate the calorimeter thickness at which $\sigma/<E>$ exceeds the plateau level by a certain value. For example, if 10% is chosen as the acceptable excess, then the dependence of l_c (0.1) on E_0 can be represented by the formula:

$$l_c(0.1) = (11.2\pm0.2)+(2.53\pm0.05)\ln E_0 \qquad (1)$$

(see Fig. 3). The l_{cp} (E_0) function also increases logarithmically with growing E_0:

$$l_{cp}=(14.8\pm0.3)+(2.46\pm0.05)\ln E_0. \qquad (2)$$

In both formulas E_0 is measured in GeV and l_c и l_{cp} in X_0.

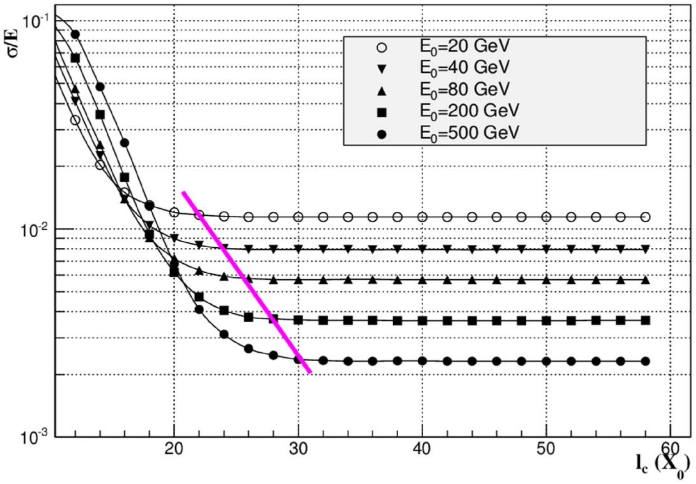

Fig. 2. Dependences of SF on thickness of the calorimeter with x/X_0=0.2 and d=4 mm. Energy E_0=500 GeV. The straight line reflects dependence (2).

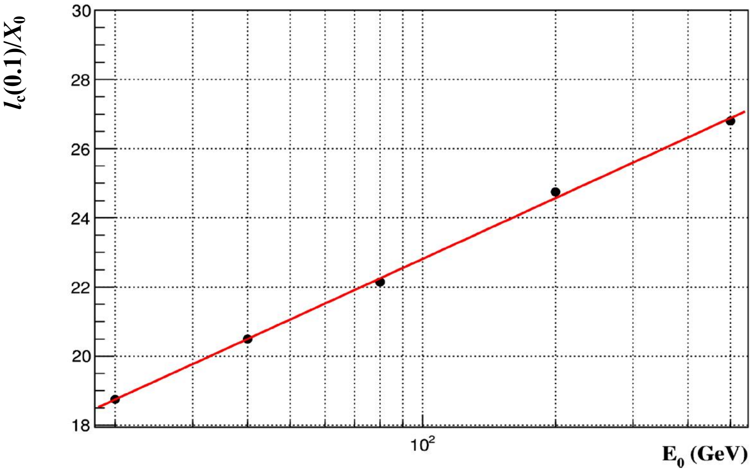

Fig. 3. Dependence of the calorimeter thickness at which $\sigma/<E>$ increases by 10% compared to the plateau value on E_0 (x/X_0=0.2 and d=4 mm). Straight line – formula (1).

3. Dependence of sampling-fluctuations on the energy and structure of the calorimeter

The dependence of SF on E_0 is well fitted by the power function

$$\frac{\sigma}{<E>}=\frac{c}{E_0^n} \tag{3}$$

for all x and d (an example is shown in Fig. 4). The exponent n does not depend on x and d and its average value 0.4988±0.0005 is close to 0.5 expected for the statistical nature of processes in an EM shower. The dependence of the coefficient c on the absorber thickness x also follows a power law

$$c=kx^{\mathrm{b}}. \tag{4}$$

The values of c calculated with n=0.5 are given in Table 1 and Fig. 5. It turned out that the exponent b weakly depends on d (Fig. 6) and its average value is 0.666±0.007 and the coefficient k can be represented by the logarithmic function k=(0.1985±0.0003)–(0.0363±0.0002)ln$d$ (mm) (Fig. 7). It follows from Table 1 that the contribution of the SF to the stochastic parameter a of the calorimeters energy resolution varies from 4% (x/X_0=0.2, d=8 mm) to 16% (x/X_0 = 0.9, d = 2 mm).

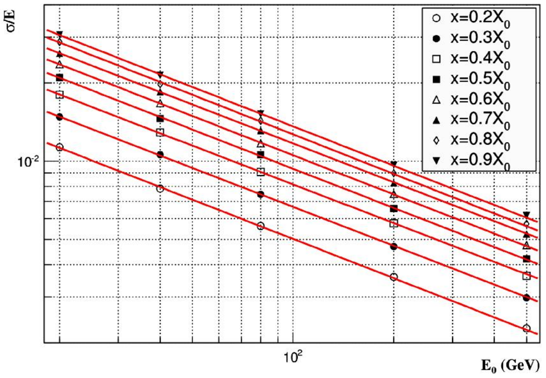

Fig. 4. SF *vs* shower energy for d=4 mm.

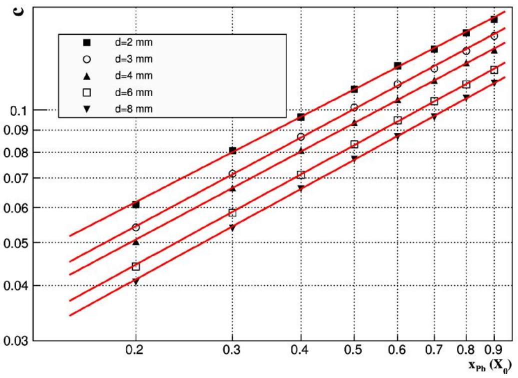

Fig. 5. Parameter *c vs* absorber thickness.

Table 1. Values of the parameters c in formula (3). Errors do not exceed 0.5%.

x/X_0	***d*, мм**				
	2	3	4	6	8
0.2	0.0609	0.0542	0.0503	0.0442	0.0407
0.3	0.0807	0.0716	0.0665	0.0584	0.0539
0.4	0.0962	0.0869	0.0809	0.0712	0.0662
0.5	0.1113	0.1013	0.0936	0.0835	0.0773
0.6	0.1258	0.1142	0.1055	0.0946	0.0869
0.7	0.1371	0.1239	0.1166	0.1046	0.0963
0.8	0.1492	0.1359	0.1276	0.1141	0.1062
0.9	0.1602	0.1469	0.1366	0.1231	0.1148

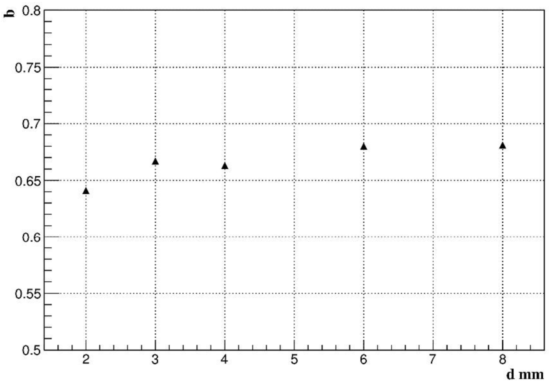

Fig. 6. Parameter b *vs* LAr gap width d.

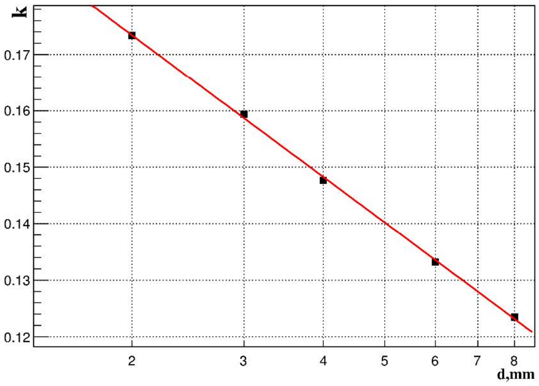

Fig. 7. Parameter k *vs* LAr gap width d.

4. Conclusion

The dependence of the sampling fluctuations of the ionization energy loss in the active layers of the Pb-LAr sandwich calorimeter on the energy E_0 of the primary electron and on the calorimeter thickness l_c and structure are investigated using GEANT4 package. The l_c values at which fluctuations reach a plateau and are determined only by SF are obtained (formula (2)). The calorimeter thickness at which fluctuations exceed the plateau level by 10% follows a logarithmic dependence (formula (1)).

The SF dependence on E_0 can be well described by the formula $\sigma/<E>=c(d,x)/\sqrt{E_0}$ where d is the Pb absorber thickness and x is the LAr gap width. The coefficient $c(d,x)$ follows the power law of $k(d)x^b$. The power b rather weakly depends on d and its average value is close to 2/3. The parameter k can be represented by the logarithmic function $k=(0.1985\pm0.0003)-(0.0363\pm0.0002)\ln d$ (mm).

Since SF in calorimeters with the same ionization energy losses in active and passive layers are close the results obtained can be used to estimate SF in calorimeters made of other materials. For example, SF in calorimeters with the same thickness of scintillator and LAr layers are close since the ionization energy losses per unit length in these materials are similar.

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