NATIONAL RESEARCH CENTRE
«KURCHATOV INSTITUTE»
Institute for High Energy Physics
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«Kurchatov Institute»

Preprint 2022-8

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Modified S-wave $\pi\pi$ scattering amplitude for multiparticle PWA

Submitted to *Ядерная Физика*

Protvino 2022

UDK 539.126.3 M–24

Abstract

Kachaev I.A. Modified S-wave $\pi\pi$ scattering amplitude for multiparticle PWA: NRC «Kurchatov Institute» – IHEP Preprint 2022-8. – Protvino, 2022. – p. 6, figs. 2, tables 2.

Suggested by Au, Morgan, Pennington (AMP) S-wave isospin $I = 0$ $\pi\pi$, KK scattering amplitude is good enough to describe experimental data for the moment. Still it has two disadvantages for use in multiparticle partial wave analysis (PWA), namely sharp drop at the KK threshold and unreasonable behavior at $M(\pi\pi) > 1.6\,GeV/c^2$. The drop is not seen in multiparticle systems.

We suggest the modified AMP amplitude, mAMP, for the only aim, namely to describe the broad part of S-wave $\pi\pi \to \pi\pi$ scattering in the wide $M(\pi\pi)$ range in multiparticle PWA. The mAMP amplitude describes threshold behavior of the $\pi\pi \to \pi\pi$ scattering and the wide structure at $M \sim 1400\,MeV/c^2$ reasonably well. It is assumed that narrow objects $f_0(980)$, $f_0(1500)$ are included in PWA separately. The amplitude does not describe $\pi\pi \to KK$ scattering. The mAMP amplitude is purely phenomenological.

Аннотация

И.А. Качаев. Модифицированная амплитуда S-волнового $\pi\pi$ рассеяния для многочастичного ПВА: Препринт НИЦ «Курчатовский институт» – ИФВЭ 2022-8. – Протвино, 2022. – 6 с., 2 рис., 2 табл.

Предложенная Ау, Морганом, Пеннингтоном (АМП) амплитуда S-волнового перерассеяния с изоспином $I = 0$ в каналах $\pi\pi$, KK удовлетворительно описывает известные на тот момент экспериментальные данные. Однако эта амплитуда имеет два недостатка, которые препятствуют её использованию в многочастичном парциально-волновом анализе (ПВА). А именно, узкий провал в районе порога открытия канала KK не наблюдается в многочастичных системах; поведение амплитуды при $M(\pi\pi) > 1.6\,GeV/c^2$ неестественно.

Мы предлагаем модифицированную амплитуду АМП (мАМП), предназначенную для единственной цели, а именно описания широкой части $\pi\pi \to \pi\pi$ S-волнового рассеяния в широком диапазоне $M(\pi\pi)$ в многочастичном ПВА. Амплитуда мАМП удовлетворительно описывает пороговые особенности $\pi\pi \to \pi\pi$ S-волнового рассеяния и широкую структуру при $M \sim 1400\,MeV/c^2$. Подразумевается, что узкие резонансы $f_0(980)$, $f_0(1500)$ вводятся в анализ дополнительно. Рассеяние $\pi\pi \to KK$ не описывается. Амплитуда мАМП имеет чисто феноменологический характер.

1. Introduction

To perform multiparticle PWA we need to know two-particle amplitudes, the *isobars* in the PWA terminology. For $\pi\pi$ system in P, D, F waves two-particle amplitudes are mostly described by known resonances $\rho(770)$, $f_2(1270)$, $\rho_3(1690)$ et al. Still in the $\pi\pi$ S-wave the situation is more complicated. In addition to narrow objects $f_0(980)$, $f_0(1500)$ there exists broad structure including at least threshold peculiarities, wide $f_0(500)$, $f_0(1370)$ and probably some other objects.

Scattering amplitude in the $\pi\pi$ S-wave was investigated in details in AMP article [1]. This article is concentrated on the construction of the scattering amplitude in the channels $\pi\pi$, KK with proper analyticity and unitarity. The best experimental data known at this time are used on input. The amplitude is presented in the form appropriate for later use.

When AMP parametrization of $\pi\pi$ scattering was used in three-particle PWA of $\pi^-\pi^-\pi^+$ system the following problems are found. The AMP amplitude describes $\pi\pi$ S-wave in a whole, in the full range of $M(\pi\pi)$. This leads to both physical and technical restrictions.

From the physical point of view amplitudes and phases of resonances in AMP amplitude are fixed once and forever. The authors of [1] used so called P-vector approach to describe production of the $\pi\pi$ system.

In the AMP amplitude there is a zero at $M(\pi\pi) \approx 1\,GeV/c^2$ due to $f_0(980)$ object and the threshold in the KK channel. If this zero is not seen in the multiparticle system it should be compensated by the pole in P-vector. It is impossible in the multiparticle PWA if in it a connection of $\pi\pi$ system with production channel is described by a set of coupling coefficients, as it is done in the Illinois PWA program [2].

From the technical point of view the AMP amplitude is parametrized via a set of poles and background polynomial of 4th degree. It is known that polynomial parametrization rapidly makes senseless outside of the range of definition. Due to this fact AMP amplitude has unphysical maximum at $M(\pi\pi) \sim 1.6\,GeV/c^2$.

2. The K-matrix method

The AMP amplitude is constructed in K-matrix formalism [3, 4]. Formally the transition from initial to final state $S_{ij} = \langle j|S|i\rangle$ is described by the unitary scattering operator, $SS^+ = S^+S = I$.

Still it is much simple to work with Hermitian matrices then with unitary. Let us construct the Hermitian matrix K which describes S exactly. After separation of no-interaction part one can define the transition operator T as

$$S = I + 2iT \tag{1}$$

where I is identity operator and factor $2i$ is introduced for convenience. From unitarity of S and the definition of T we have

$$T - T^+ = 2i\,T^+T = 2i\,TT^+ \tag{2}$$

Introducing the inverse operator T^{-1} we have

$$(T^+)^{-1} - T^{-1} = 2i\,I \quad \text{so} \quad (T^{-1} + i\,I)^+ = T^{-1} + i\,I \tag{3}$$

Now we can introduce operators K and M, from (3) they are Hermitian

$$K^{-1} = T^{-1} + i\,I \quad \text{and} \quad M = K^{-1} \tag{4}$$

It is known that if time-reversal invariance is hold than K and M matrices are not only Hermitian but are also real and so symmetric. Explicit definition for T is

$$T = K\,(I - iK)^{-1} = (I - iK)^{-1}\,K \tag{5}$$

So defined transition amplitude T is not Lorentz invariant. Let us construct it in Lorentz invariant form. For decay of particle with mass m to two particles with masses m_a, m_b phase space is $\rho = 2q/m$ where q is breakup momentum

$$\rho = \sqrt{\left[1 - \left(\frac{m_a + m_b}{m}\right)^2\right]\left[1 - \left(\frac{m_a - m_b}{m}\right)^2\right]} \tag{6}$$

Phase space is normalized as $\rho \to 1$ at $m \to \infty$. By definition phase space is considered a diagonal matrix, so in two channel case it is

$$\rho = \begin{pmatrix} \rho_1 & 0 \\ 0 & \rho_2 \end{pmatrix} \tag{7}$$

Lorentz invariant amplitude $\hat{T}$ is defined as

$$T_{ij} = \{\rho_i^*\}^{1/2}\,\hat{T}_{ij}\,\{\rho_j\}^{1/2} \tag{8}$$

or in matrix form

$$T = \{\rho^+\}^{1/2}\, \hat{T}\, \{\rho\}^{1/2} \tag{9}$$

Below the threshold of the channel its phase space become complex. Complex conjugation in (8) is required for proper analytic continuation into the region below the threshold of some channels.

Let us note a significant difference between T and $\hat{T}$. In the one channel case

$$S = e^{2i\delta}, \quad T = e^{i\delta} \sin\delta, \quad \hat{T} = \frac{1}{\rho} T \tag{10}$$

where δ is phase shift. For any δ the amplitude T is confined in the circle with the centre $(0, i/2)$ and the radius $1/2$. The dependence $T(s)$ is named the Argand diagram. The amplitude $\hat{T}$ has other normalization and is used as Dalitz plot amplitude in PWA programs. On the threshold of the reaction in the case of S-wave scattering $T \to 0$ but $\hat{T} \to const.$

Now we can define Lorentz invariant matrix $\hat{K}$ as

$$K = \{\rho^+\}^{1/2}\, \hat{K}\, \{\rho\}^{1/2} \tag{11}$$

If $\hat{K}$ is taken to be real and symmetric, then K is Hermitian and S unitary even some ρ becomes imaginary below the threshold of some channels. From (4) we have

$$\hat{K}^{-1} = \hat{T}^{-1} + i\rho \tag{12}$$

Explicit form of $\hat{T}$ is

$$\hat{T} = \hat{K}\left(I - i\rho\hat{K}\right)^{-1} = \left(I - i\hat{K}\rho\right)^{-1} \hat{K} \tag{13}$$

3. The original AMP amplitude

In [1] (3.18) $\hat{K}$ matrix (named below K for consistency with [1]) has been parametrized as sum of poles and polynomial background (this is so called K solution)

$$K_{ij} = \frac{(s - s_0)}{4m_K^2} \sum_p \frac{f_i^p f_j^p}{(s_p - s)(s_p - s_0)} + \sum_{n=0} c_{ij}^n \left(\frac{s}{4m_K^2} - 1\right)^n \tag{14}$$

By definition $m_K = 1/2(m_{K+} + m_{K0})$. Here s_0 is known Adler zero near the threshold of $\pi\pi$ system. We have found that amplitude calculated according to this formula does not match fig. (5.3) in [1]. If we use parameters stated in the article proper formula which corresponds to this figure is

$$K_{ij} = \frac{(s - s_0)}{4m_K^2} \left[\sum_p \frac{f_i^p f_j^p}{(s_p - s)(s_p - s_0)} + \sum_{n=0} c_{ij}^n \left(\frac{s}{4m_K^2} - 1\right)^n\right] \tag{15}$$

We believe that it is formula (15) that describes the original K solution of AMP amplitude.

For the $\hat{M}$ matrix in [1] (3.20) there exists the following parametrization (so called M solution)

$$M_{ij} = \frac{a_{ij}}{(s - s_0)} + \sum_p \frac{f_i^p f_j^p}{(s_p - s)} + \sum_{n=0} c_{ij}^n \left[\frac{s}{4m_K^2} - 1 \right]^n \tag{16}$$

Again, we believe that there is a mistake in the sign in [1] here and the proper formula for M solution is

$$M_{ij} = \frac{a_{ij}}{(s - s_0)} - \sum_p \frac{f_i^p f_j^p}{(s_p - s)} + \sum_{n=0} c_{ij}^n \left[\frac{s}{4m_K^2} - 1 \right]^n \tag{17}$$

Note that for M solution parameters of T matrix tends to zero at $s \to \infty$ while for K solution they tends to unitary limit. The reason of this behavior is that outside of its scope polynomial background tends to infinity. We consider the behavior of M solution more appropriate for our aims and use it. Parameters of original M solution from [1] are listed in table 1

s_0	s_1	f_1^1	f_2^1	a_{11}	a_{12}	a_{22}
-0.0074	0.9828	0.1968	-0.0154	0.1131	0.0150	-0.3216

c_{11}^0	c_{11}^1	c_{11}^2	c_{11}^3	c_{11}^4
0.0337	-0.3185	-0.0942	-0.5927	0.1957
c_{12}^0	c_{12}^1	c_{12}^2	c_{12}^3	c_{12}^4
-0.2826	0.0918	0.1669	-0.2082	-0.1386
c_{22}^0	c_{22}^1	c_{22}^2	c_{22}^3	c_{22}^4
0.3010	-0.5140	0.1176	0.5204	-0.3977

Table 1. Table of coefficients of original AMP amplitude

s_0	s_1	f_1^1	f_2^1	a_{11}	a_{12}	a_{22}
-0.0074	0.9828	0	0	0.1131	0	-0.3216

c_{11}^0	c_{11}^1	c_{11}^2	c_{11}^3	c_{11}^4
0.0337	-0.3185	-0.0942	-0.5927	0
c_{12}^0	c_{12}^1	c_{12}^2	c_{12}^3	c_{12}^4
0	0	0	0	0
c_{22}^0	c_{22}^1	c_{22}^2	c_{22}^3	c_{22}^4
0.3010	-0.5140	0.1176	0.5204	0

Table 2. Table of coefficients for modified amplitude mAMP

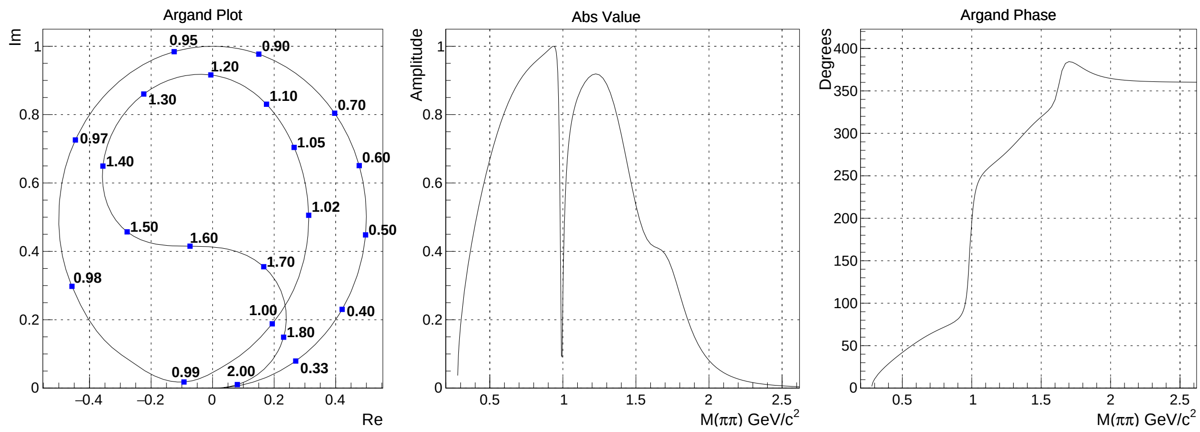

Figure 1. Original AMP amplitude

4. Our modifications

Our aim is to construct the scattering amplitude $\pi\pi \to \pi\pi$ which is near to the original AMP amplitude, is smooth in the region $M(\pi\pi) \approx 1\, GeV/c^2$ and smoothly tends to zero at $M(\pi\pi) > 1.5\, GeV/c^2$.

We require the proper behavior at $M(\pi\pi) \approx 1\, GeV/c^2$ by setting to zero the connection with KK channel and the connection with $f_0(980)$ pole. To suppress an unphysical shoulder in the original amplitude at $M(\pi\pi) \sim 1.7\, GeV/c^2$ we set to zero coefficients at the 4th degree of the background polynomial.

The parameters of the modified amplitude are given in the table 2. The original amplitude is shown in the figure 1, the modified one in the figure 2. The figures show the values, from left to right: Argand plot of the matrix element T_{11}, namely $\pi\pi \to \pi\pi$, its absolute value and Argand phase.

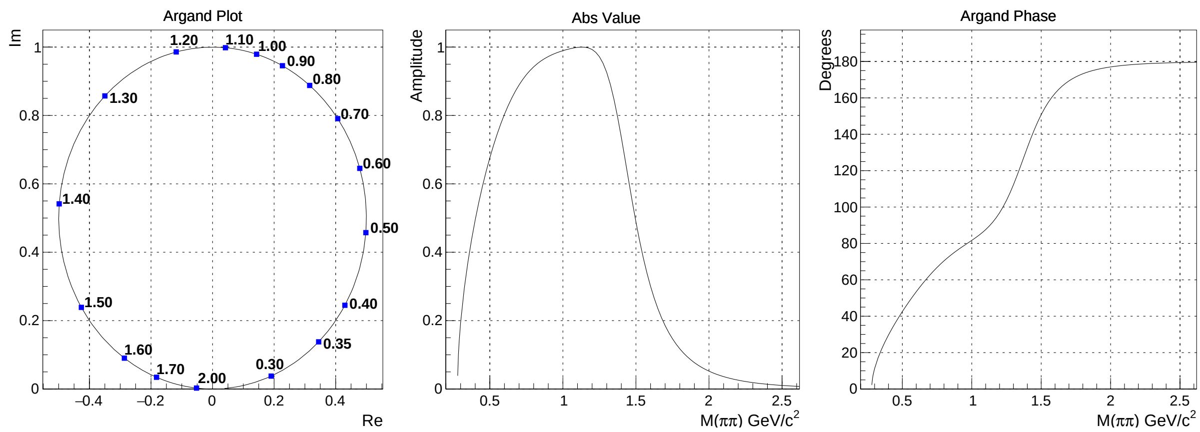

Figure 2. Modified AMP amplitude

5. Results

The amplitude mAMP is constructed, suitable to describe $\pi\pi \to \pi\pi$ scattering in S wave with $I = 0$ in multiparticle PWA. It is not suitable to describe KK channel.

It is unitary, smooth in the broad range of $M(\pi\pi)$, is near to AMP M solution for $M(\pi\pi) < 1.6\ GeV/c^2$ except at $M(\pi\pi) \sim 1\,GeV/c^2$, smoothly tends to zero at $M(\pi\pi) \to \infty$. It describes the threshold behavior of $\pi\pi$ scattering and the broad structure at $M \sim 1400\,MeV/c^2$ according to the data known at the time of writing of [1]. Narrow resonances $f_0(980)$, $f_0(1500)$ intentionally are not described, they should be entered into the analysis separately.

The amplitude is built according to formula (17) with coefficients from table 2. This amplitude was used in [5, 6] and the other publications of the VES group and also in [7]. An early version of the amplitude was used in [8].

The author thanks the members of the VES group for initiating this work, useful discussions and use of this amplitude in the analysis.

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Received September 30, 2022

Препринт отпечатан с оригинала-макета, подготовленного авторами.

Качаев И.А., эксперимент ВЕС
Модифицированная амплитуда S-волнового $\pi\pi$ рассеяния для многочастичного ПВА.

Оригинал-макет подготовлен с помощью системы LaTeX.

Подписано к печати 03.10.2022 Формат $60 \times 84/16$. Цифровая печать.
Печ.л. 0,6. Уч.-изд.л. 0,8. Тираж 60. Заказ 10. Индекс 3649.

НИЦ «Курчатовский институт» – ИФВЭ
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П Р Е П Р И Н Т 2022–8,
НИЦ «Курчатовский институт» – ИФВЭ, 2022