NATIONAL RESEARCH CENTRE
«KURCHATOV INSTITUTE»
Institute for High Energy Physics
of the National Research Centre
«Kurchatov Institute»

Preprint 2023-18

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Reduction of multiple scattering of positively charged ultra relativistic particles channeling in planar fields of single crystals

Submitted to *Nuclear Instruments and Methods in Physics Research Section A*

Protvino 2023

UDK 519.25.256 M–24

Abstract

Krutov A.M., Maisheev V.A. Reduction of multiple scattering of positively charged ultra relativistic particles channeling in planar fields of single crystals: NRC «Kurchatov Institute» – IHEP Preprint 2023-18. – Protvino, 2023. – p. 11, figs. 5.

Based on the theory of multiple scattering of positively charged particles channeling in the planes of a single crystal of silicon, calculations of the rms angle of scattering have been made for particles in the range of momenta 100 -7000 GeV/c. We show that these calculation results can be represented in a unified way using the found universal function. It was shown that there is a good agreement of this function with the experimental data. In general, the obtained results give a simple and clear description of the scattering process under consideration.

Аннотация

Крутов А.М., Маишеев В.А. Уменьшение многократного рассеяния положительно заряженных ультрарелятивистских частиц, каналирующих в плоскостных полях монокристаллов: Препринт НИЦ «Курчатовский институт» – ИФВЭ 2023-18. – Протвино, 2023. – 11 с., 5 рис.

На основе теории многократного рассеяния положительно заряженных частиц, каналирующих в плоскостях монокристалла кремния, выполнены расчеты среднеквадратичного угла рассеяния частиц в диапазоне импульсов 100 — 7000 ГэВ/с. Показано, что результаты этих расчетов можно представить в едином виде с помощью найденной универсальной функции. Показано, что имеется хорошее согласие этой функции с экспериментальными данными. В целом полученные результаты дают простое и наглядное описание рассматриваемого процесса рассеяния.

1. Introduction

Several years ago at CERN, an interesting effect of reduction of multiple scattering of positively charged particles during planar channeling in bent silicon crystals in these experiments.was observed and studied experimentally [1, 2, 3]. Moreover, the effect was observed in the plane transverse to the bending plane of the crystal. The value of the rms of the scattering angle of channeled and non-channeled particles was measured in this experiments. According to the measurements, the rms of the multiple scattering angle for channeled particles was up to 8 times less than for non-channeled particles.

The observed effect can be attributed to a large number of phenomena accompanying the channeling of particles in crystals. Such phenomena include Rutherford scattering, energy-loss processes, secondary electron emission, nuclear reactions, X-ray and γ-ray production. The cross sections of these processes depend on the impact parameters involved in collisions with individual target atoms. A characteristic common feature of such processes is the suppression of the probability of multiple Coulomb scattering in comparison with non-channeling particles. It is important to note that the quantitative description of such processes is of a specific nature, i.e., there are no universal relations for their unified mathematical representation. A description of many of these processes can be found in the literature [4, 5, 6, 7, 8].

In addition, some simple explanations of the observed effect can be found in [3]. The article proposes an explanation of this phenomenon as a result of two competing processes of dechanneling and scattering.

The effect of suppression of multiple scattering during the channeling of positively charged particles was considered theoretically in the article [9]. In the article, the distribution function of channeling particles over transverse energy in a sufficiently thick crystal is constructed based on an approximate description of the diffusion process in a thin crystal layer. Thus, by dividing the entire thickness of the crystal into a sufficiently large number of parts, it is possible to find the function of the distribution of particles over transverse energy in each of the mentioned parts. This is equivalent to finding the distribution function for different crystal thicknesses. Then, it is convenient to move from the transverse energy distribution function to distribution functions over the oscillation amplitudes of channeling particles. With the help of such a function, it is possible to determine the number of particles approaching to the region of atomic nuclei. Taking into account the number of such particles and their residence time near the nuclei, it is possible to find the dependence of multiple scattering angles for different thicknesses.

On the basis of the proposed theory [9], the rms angles of multiple scattering 180 and 400 GeV/c particles channeled in the (111) planes of silicon single crystals were calculated. The calculation results turned out to be in good agreement with the measurements. The obtained theoretical relations are not simple and for calculations with one given momentum and a crystal thickness of the order of 10 cm, it takes about several hours on a laptop.

In this work, on the basis of a complex theoretical representation of the process of multiple scattering positively charged relativistic particles channeling in crystals, we find a very simple description of this process in the particle momentum range from 100 to 7000 GeV/c and crystal thicknesses up to $\sim$ 10 cm.

2. About general theory

So far, calculations of multiple scattering during channeling have been performed for particle momenta of 180 and 400 GeV/c. Here we want to calculate the rms angles of multiple scattering in a wider range of momenta. For these purposes, we use the theory from the article [9].

The theory of the process is built mainly on the following provisions: 1) we use a simplified consideration of the process of multiple scattering of particles by atoms[10], namely, we assume that charged particles scatter in the screened potential of the atom only when they are near the atomic center (at a distance less than $b_{max} = 137^2 r_e Z^{-1/3}$, where r_e is the classical electron radius and Z is the ordinal number of the atom. 2) we assume that the channeling particle in the transverse plane of the crystal bending makes harmonic oscillatory movement. In this motion, the particle scatters only when it approaches close enough to the atomic center. 3) we use in the calculations a mathematical model of the process[9], which allows us to calculate the function of particle distributions over oscillatory amplitudes at different crystal thicknesses.

According to theory [9], the rms angle of multiple scattering on a crystal thickness L equals

$$\sigma_s^2(L) = \frac{4\pi N_a Z^2 e^4}{p^2 c^2} \int_0^L \int_0^{\xi_{max}} \frac{dN}{d\xi}(\xi, l) F_m(\xi) d\xi dl, \quad (1)$$

where $\frac{dN}{d\xi}$ is the distribution function (normalized on unit) over relative oscillatory amplitude $\xi = 2x_m/d$ (x_m, d are the oscillatory amplitude and interplanar distance, e is the electron charge, p is the particle momentum, c is the velocity of light, and $N_a = 8.35\frac{8}{a^3}$ (a is the side of crystal cell). Function $F_m(\xi)$ takes into account the probability of interaction with atomic center. For amplitudes smaller then $d/2 - b_{max}$ this function is equal 0.

$$F_m(x_m) = \frac{1}{2\pi}\int_0^{2\pi} \ln\{\frac{d/2 - x_m|\sin t|}{b_{max}}\}\tau(x_m|\sin t| - d/2 + b_{max})dt + J_1(x_m)J_2(x_m)\ln((x_m - d/2 + R_A)/b_{min}), \quad (2)$$

$$J_1(x_m) = \frac{1}{2\pi}\int_0^{2\pi} |\sin t| \ \tau(x_m|\sin t| - d/2 + R_A)dt, \quad (3)$$

$$J_2(x_m) = \frac{1}{\sqrt{2\pi}\sigma_T}\int_{x_m - d/2 + R_A}^{R_A} \exp -x^2/(2\sigma_T^2)dx, \quad (4)$$

where σ_T is the rms amplitude of thermal oscillations and $R_A \approx (2.5-3)\sigma_T < b_{max}$ and $\tau(t)$-function is equal to 1 or 0 when $t > 0$ or $t \leq 0$.

In this article, we made the following changes compared to the work [9]: 1) the function $F_m(x_m)$ was changed, namely, a term was added that takes into account that the atomic centers are distributed according to the normal law near the distance $d/2$ from the center of the planar channel, which corresponds to taking into account the thermal motion of atoms. In specific calculations, this increased the result is an average of 25 percent. 2) we used the Molière potential, which was calculated according to the article [11]. Compared to the previously used potential obtained from X-ray measurements this better described the results of the available experimental data.

Fig. 1 shows the results of comparing our calculations with the measurement results. It can be seen that the calculations using the improved program and the Molière potential are in good agreement with the measurements. Experimental results were obtained for crystal bending radii of 220 and 63 meters, respectively. for momenta of 180 and 400 GeV/c. We also present the results of calculations for radii smaller than those in the experiment. and for very large ones (of the order of 10000 m). It can be seen that the calculated angles of multiple scattering differ insignificantly for different radii. Therefore, below we will present the calculations performed for very large radii (actually infinite). Such crystals practically do not differ from straight (unbent) crystals. It should be noted that during experiments the presence of crystal curvature is an important factor, since during measurements it makes it possible to separate the channeling and dechanneling fractions.

3. Calculations for different momenta

In order to understand and study the process of multiple scattering in planar channeling of relativistic particles we calculated the rms angle over a wide range of particle momenta from 100 to 7000 GeV/c. As already stated we chose an almost infinite bending radius for calculations. We also chose the maximum thickness of the crystals equal to 12 cm. In the calculations, we calculated the change in the amplitude function of the distribution of particles over a relatively small thickness equal to 0.1 mm in our case. Initial distribution functions (both in transverse energy and in amplitude of oscillations) were found under the assumption that the beam at the entrance to the crystal is uniformly distributed over the coordinate and over the entrance angle.

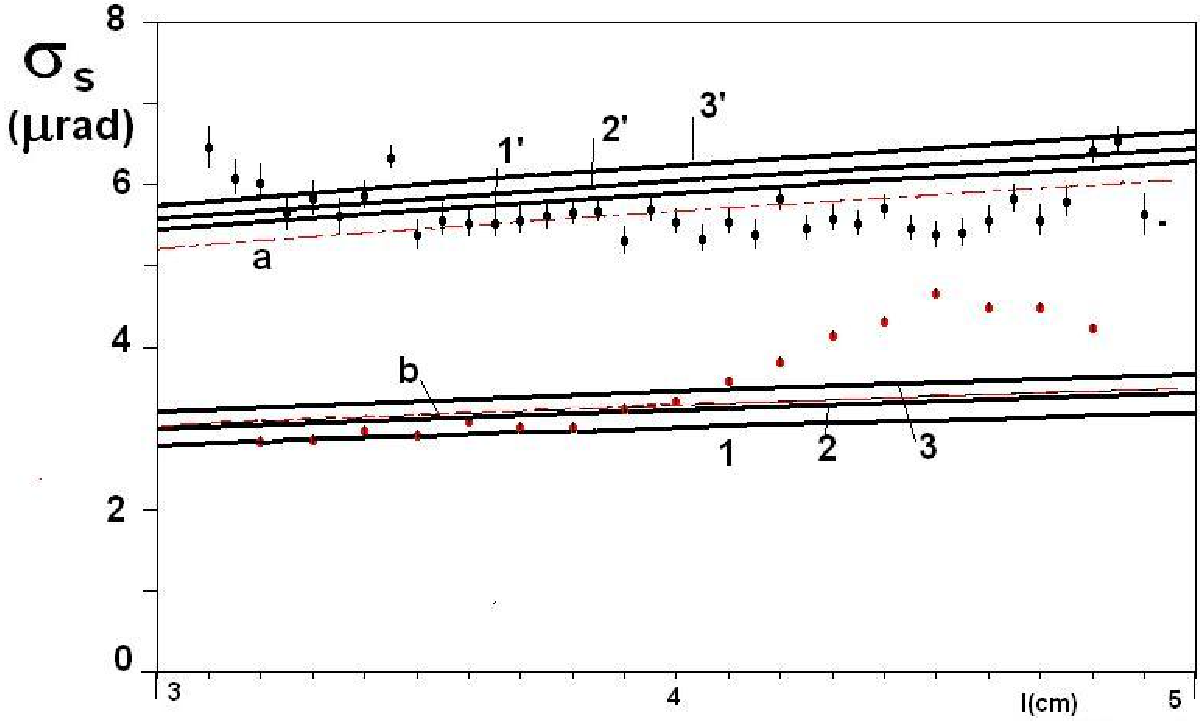

Figure 1. Calculations of the rms angle of multiple scattering of positively charged particles channeling in (111) planes of silicon single crystal. The curves $1, 2, 3$ are calculated for 400 GeV/c particles and bending radii equal to ∞, 63 and 30 meters, correspondingly. The curves $1', 2', 3'$ are calculated for 180 GeV/c particles and bending radii equal to ∞, 220 and 63 meters, correspondingly. The dotted lines a and b and experimental points are taken from paper [3].

Fig. 2 presents the results of calculations for wide range particle momenta and for crystal thickness from 0 up to 12 cm. It is useful to compare the obtained results with the process of multiple scattering of particles in an amorphous medium.

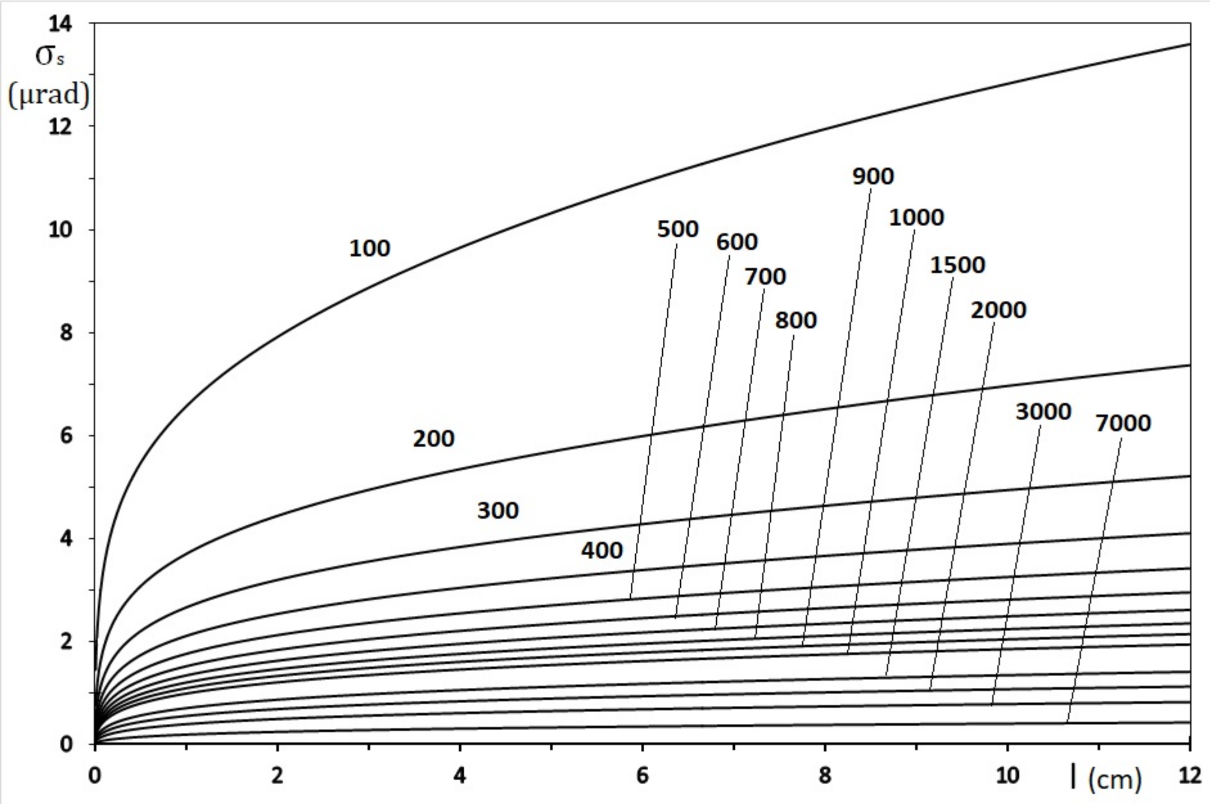

Figure 2. Calculations of the rms angle of multiple scattering of positively charged particles with different momenta (numbers near curves in GeV/c) channeling in (111) planes of silicon single crystal.

The theory of multiple scattering of charged particles in amorphous media was considered in many articles. From a practical point of view we can refer to the recommendation of Data Particle Group [12] to use a Gaussian approximation for the central 98% of the projected angular distribution, with the rms equal to

$$\sigma_0 = \frac{13.6[MeV]}{\beta c p}\sqrt{l/X_0}[1 + 0.038\ln(l/X_0)], \tag{5}$$

where p and βc are the momentum and velocity of the incident particle, c is the velocity of light, l, X_0 are the thickness of the scattering amorphous medium and its radiation length. The rms angle in this equation is from a fit to Moliere distribution for singly charged particles with $\beta = 1$ for all Z, and is accurate to 11% or better for $10^{-3} < l/X_0 < 100$.

The paper [3] presents the results of measurements of the rms angle of multiple scattering of positively charged unchanneled particles moving in a single crystal of silicon:

$$\sigma_n^2 = [\frac{\varepsilon}{\beta cp}]^2 \frac{l}{X_0}[1 + \omega \ln(l/X_0)]^2, \quad (6)$$

where ε and ω are free parameters for the approximation. The best fit is for $\varepsilon = 13.35$ MeV and $\omega = 0.063$.

From Eqs. (5)-(6) it follows that the rms scattering angle of non-channeled particles (moving in amorphous or crystalline media) inversely proportional to their energy E. The paper [3] states that for positively charged particles in planar channeling, the inverse law proportionality is violated. The results presented on Fig. 2 also demonstrate this violation. The same paper shows that it is possible that the scattering angle has the form $\sim 1/E^h$, where coefficient h is equal to $\approx 0.76 \pm 0.04$ from the measurements and $\approx 0.070 \pm 0.01$ from theory.

Next, we take the function $\mathcal{K}(p) = p^h \sigma_s$ and find the coefficient h for which the curves from Fig. 2 closest to each other. Fig. 3 shows the $\mathcal{K}(E)$ family of functions at different energies and for $h = 0.81$. It can be seen that, at particle momenta in the range from 200 to 7000 GeV/c, the curves almost coincide. There is a noticeable difference for almost all crystal lengths, only for momentum 100 GeV/c.

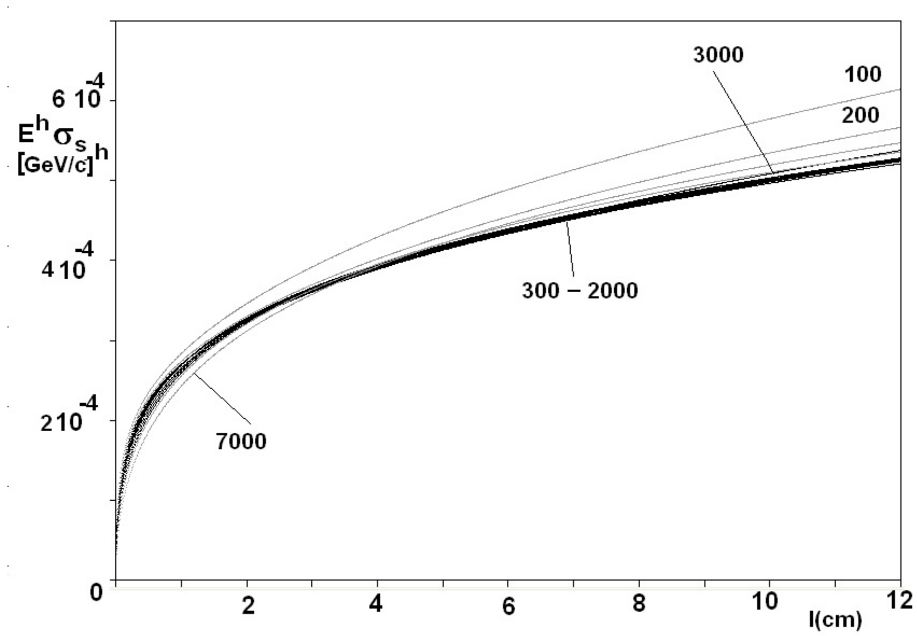

Figure 3. Family of $\mathcal{K}(E, l)$ functions in the particle momentum range 100-7000 GeV/c (numbers near curves in GeV/c). $h = 0.81$.

If we do not take into account the slowly changing logarithmic term in Eqs. (5)-(6), then we can assume that the angle σ_s^2 is proportional to the thickness of the crystal. Let's look at the function

$$\mathcal{M}(p, l) = (\sigma_s(p, l)p^h)^w. \quad (7)$$

Then we can choose the parameter w such that the function $\mathcal{M}(p,l)$ is close to proportional depending on the crystal thickness l.

Fig. 4 illustrates the $\mathcal{M}(p,l)$ family of functions for different energies. The best value of the parameter $w = 3.73$. It can be seen that in the range 200 - 2000 GeV/c, the family of functions approximately represents a proportional dependence on the crystal thickness, and all curves practically coincide with each other. Curves for 100, 3000 and 7000 GeV/c are slightly different from most of the curves already mentioned.

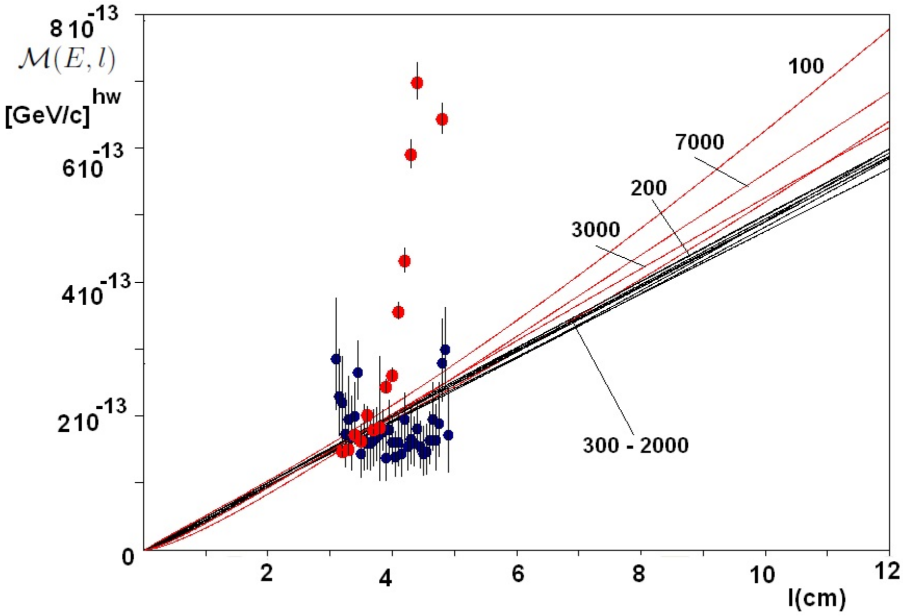

Figure 4. Family of $\mathcal{M}(E,l)$ functions (where $h = 0.81, w = 3.73$) in the particle momentum range 100-7000 GeV/c (numbers near curves in GeV/c). The red and blue circles (colors in the online version) are taken from experiments in 400 and 180 GeV/c beams, correspondingly.

Thus, we can write an approximate equality

$$\mathcal{M}(p,l) = (\sigma_s(p,l)p^h)^w \approx C(p)l, \tag{8}$$

where $C(p)$ is a constant coefficient, and this coefficient is practically independent of the particle momentum (in the range 200 - 2000 GeV/c). From here we find the approximate dependence of the rms angle of the multiple scattering of positively charged particles during channeling in the (111) plane of a silicon single crystal.

$$\sigma_s(p,l) = \frac{\mathcal{M}(p,l)^{1/w}}{p^h} \approx \frac{(Cl)^{1/w}}{p^h} \tag{9}$$

Fig. 4 illustrates the behavior of the function $\mathcal{M}(p,l)$ for parameters $w = 3.73$ and

$h = 0.81$. For the parameter $C(p)$ one can write

$$C(p) = \frac{\sigma_s^w p^{hw}}{l}. \tag{10}$$

Fig. 5 shows the calculation of the parameter $C(p)$ depending on the thickness for different particle energies It can be seen that this parameter for particles in the energy range of 300 - 2000 GeV and for thicknesses of more than a few millimeters about the same. We find that we can set $C(p) = (4.95 \pm 0.2)10^{-14}\,(GeV/c)^{hw}/cm$ for the specified range of energies and for thicknesses greater than 1 cm.This result is valid for $h = 0.81$ and $w = 3.73$.

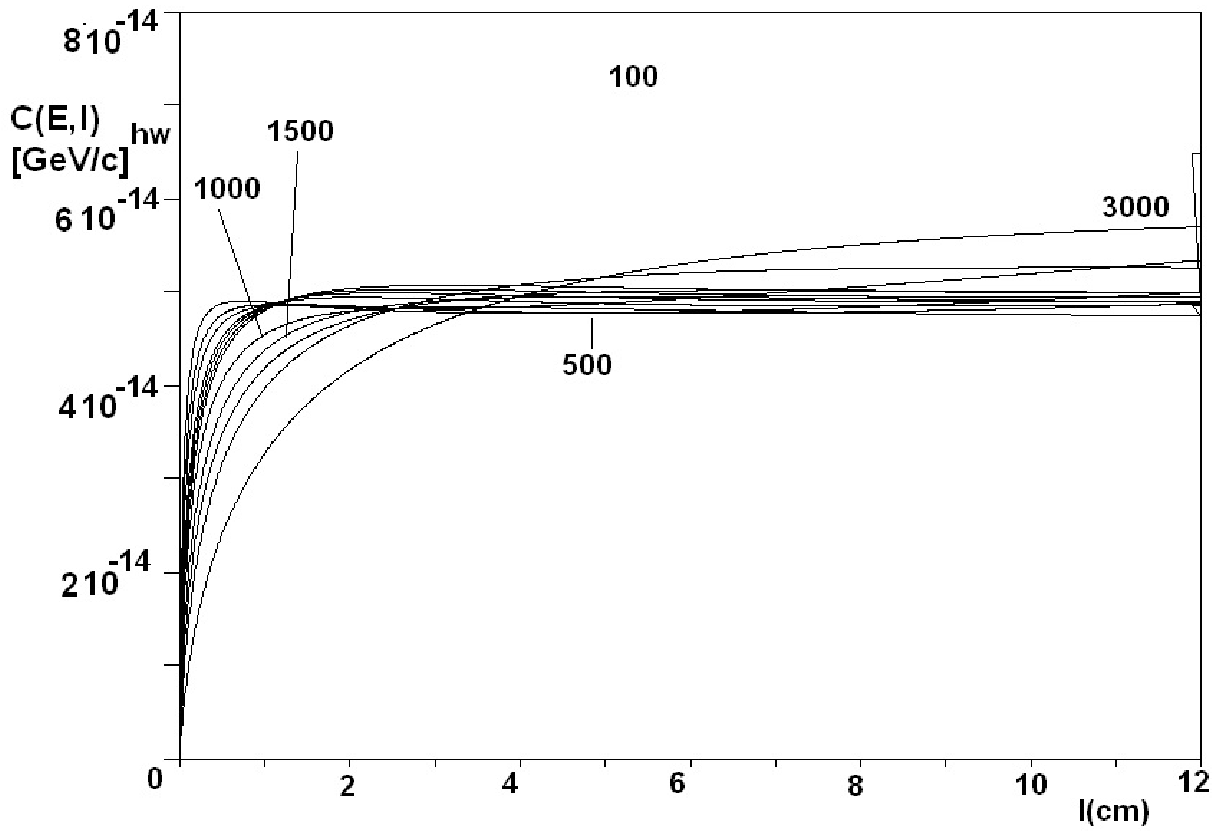

Figure 5. On definition of C constant (see text).

The results obtained here give a fairly simple quantitative description of the process of multiple scattering of positively charged particles with momenta in the range (200 - 2000) GeV/c during channeling in (111) planes of a silicon single crystal. The accuracy of calculating functions using an approximate Eq. (9) is better than one percent compared to the general theory for momenta of more than 200 and up to 2000 GeV/c and crystal thicknesses of more than a few millimeters.

Fig. 4 shows a comparison of the results of calculations and measurements for the universal function $\mathcal{M}(p,l)$. The measurements were performed in beams of positively charged particles with momenta of 180 and 400 GeV/c for silicon crystal thicknesses from 3 to 5 cm. Based on the theory, we have shown that in a wide range of particle energies, the function defined by the equation $\mathcal{M}(p,l)$ (see Eqs. (7)-(8)) is proportional to the crystal thickness and practically does not depend on energy. In addition, it is necessary to satisfy the condition $h = 0.81, w = 3.73$. Note that the measurement results for 400 GeV are many red large dots of 17 elements. It can be seen from Fig. 4 that the first 10 points are located near the theoretical curve, while the remaining 7 points quite different from what is expected. Also in Fig. 1 it can be seen that the measured last 7 points are located along the arc. This fact is also noted in the work [3]. We agree with authors of this paper that this behavior of the experimental data can be explained by the presence of possible microdefects (micro scratches, etc.) on both side surfaces of the single crystal.

4. Discussion

In this paper, we conducted a numerical study of an analytical model of multiple scattering of positively charged particles channeling in (111) planes of a silicon single crystal. We found that there is a fairly simple function $\mathcal{M}(E,l)$ (see Eq. (7)). We have shown how, knowing this function, it is possible to calculate approximate the rms angle of multiple scattering for a particle with momentum p. However, we found that this is true when the particle momentum belongs to the range of 100 - 7000 GeV/c (200 -2000 GeV/c with an accuracy better than 1 percent). The function $\mathcal{M}(E,l)$ is defined by two parameters h and w. We found that $h = 0.81$ and $w = 3.73$. If we ignore the logarithmic term (see equation 5), then for an amorphous substance there is also a function $\mathcal{M}(E,l)$, but for it $h = 1$ and $w = 2$.

The processes of multiple scattering of particles in amorphous media and in planar channeling in crystals have a common nature, it is Coulomb scattering on nuclei. However, at channeling the process under consideration has features. Thus, as the particle beam propagates in the crystal, its intensity decreases due to diffusion processes (dechanneling). Moreover, the particles with the highest transverse energy are eliminated from the channeling regime. Another peculiarity is that scattering is possible only when particles move near nuclei. These features exactly lead to the reduction of multiple scattering at planar channeling.

Available experimental data obtained at momenta of 180 and 400 GeV/c confirm the existence of the universal function $\mathcal{M}(E,l)$ (see Fig. 4).

In addition, Eq. (9) can be represented as

$$\sigma_s(p,l) \approx \frac{(Cl)^{1/w}}{p^h} = [\frac{(CX_0)^{1/(wh)}}{p}]^h (l/X_0)^{1/w} = [\frac{m_s}{p}]^h (l/X_0)^{1/w}, \qquad (11)$$

where $m_s = (CX_0)^{1/(wh)} \approx 0.083 MeV/c$ is a dimensional constant that has the same meaning as the constant equal to 13.6 MeV in Eq. (5).

In this article, channeling was considered only in the (111) planes of silicon. We think that our research can be applied to other planes and other single crystals. Besides, our study indicates the possibility of a rigorous mathematical simplification of the theoretical approach proposed in the article [9].

5. Conclusion

Calculations of multiple scattering during channeling of positively charged particles performed using analytical theory in a wide range of particle momenta give a new look at the process and present a simple and clear description of it.

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Received November 23, 2023.

Препринт отпечатан с оригинала-макета, подготовленного авторами.

Крутов А.М., Маишеев В.А.
Уменьшение многократного рассеяния положительно заряженных ультрарелятивистских частиц, каналирующих в плоскостных полях монокристаллов.

Оригинал-макет подготовлен с помощью системы **LaTeX.**

Подписано к печати 28.11.2023 Формат 60 × 84/16. Цифровая печать.
Печ.л. 1. Уч.-изд.л. 1,2. Тираж 60. Заказ 22. Индекс 3649.

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П Р Е П Р И Н Т 2023–18,
НИЦ «Курчатовский институт» – ИФВЭ, 2023