CP-Violation in the Heavy Quark Systems

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We present a review of general picture in the sector of electroweak symmetry breaking with the CP-violation in the heavy quark interactions.

Introduction

After the precision measurements of electroweak parameters at LEP, FNAL and SLAC, the main progress in the experimental study of Standard Model (SM) is connected to investigations in the Higgs sector. The Higgs mechanism provides the basic renormalization properties of the SM as well as the masses of vector gauge fields and fermions. There are two directions in these studies. The first is an observation of neutral scalar Higgs particle in order to prove the completeness of SM. The second direction is an investigation of Yukawa couplings of Higgs scalar with the quark and lepton fields. When the number of generations is equal to three (as measured to the moment) or greater, these couplings can be complex, which inavitably leads to the violation of combined CP parity inverting both the charges (C) and space orientation (P). This violation after the transition to the observed mass-flavor states of fermions, manifests in the mixing of weak charged quark currents. Therefore, measuring the CKM elements in heavy quark decays tells us about the Higgs sector of SM.

Theoretically, two phenomenological approaches are developed in order to study the Yukawa sector. The first is a modelling of the mass matrices regardless of SM extensions. This approach is closely related with the pionering paper by Fritzsch [1] devoted to the mass matrix textures. The second way is based on restricting the Yukawa sector of SM extensions by observed regularities. These regularities of quark current mixing are quite definite and bright. Indeed, in the nature we deal with

- a single heavy major generation and two almost massless junior generations, and
- a small mixing of major generation with the junior generations.

These observations are refered to as the hierarchy of masses and hierarchy of mixings. The hierarchies are combined in the principle of democracy in the Yukawa interactions of quarks. According to this principle a leading contribution to the Yukawa interactions involves the only universal coupling $\lambda_{\rm Ferm}$ for all of three generations composed by equal-charge fermions, so that the mass matrix has the form, which can be transformed from the democratic basis to the heavy one in the following way:

$$M = \lambda_{\rm Ferm}\,\eta_{\rm vac} \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \Longrightarrow M_U = \lambda_{\rm Ferm}\,\eta_{\rm vac} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

due to the operation

$$U \cdot M \cdot U^\dagger = M_U,$$

where the rotation matrix U has the form

$$U = \begin{pmatrix} 1/\sqrt{3} & 1/\sqrt{3} & 1/\sqrt{3} \\ 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{6} & 1/\sqrt{6} & -2/\sqrt{6} \end{pmatrix}.$$

Since the mass matrices for the up-kind and down-kind fermions have the same form for the unit matrix of charged currents, then after the rotation the mixing Cabibbo–Kobayashi–Maskawa matrix (CKM) is transformed to the following:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \Longrightarrow U_{up} U^{\dagger}_{down} = I,$$

and in the limit of exact democracy we arrive to the single heavy quark generation and zero mixing of light junior generations with the major one.

The democracy is the approximate symmetry of Yukawa interactions. In the nature it is slightly perturbed, that leads to the observed picture of light junoir generations with small mixing of charged currents. We stress that an origin of democracy and its perturbations are guided by a structure beyond the SM.

Therefore, the study of the heavy quark mixing in weak interactions and the CP-violation informs us not only about the parameters of SM, but also about the dynamics underlying the Yukawa interactions beyond the SM.

1 CKM matrix in SM

The mixing of charged quark currents in the SM is described by the unitary CKM matrix with three real rotation angles and a single complex phase providing the violation of CP invariance. According to the observed hierarchy of mixing, the elements of this matrix is classified by the order of magnitude. Next step is a relation of mass-hierarchy regularities with the mixing.

1.1 Parametrizations & Textures of quark mass matrices

Wolfenstein [2] gave a general arrangement of unitary four-parameter matrix in terms of a small parameter $\lambda = |V_{us}| \approx 0.22$ representing the sine of Cabibbo angle in the mixing of two junior generations, so that

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix},$$

where numerically the constant A is about unit, while the ρ, η parameters are in the range of $0.2 - 0.3$. This representation of CKM matrix is completely phenomenological, and it does not involves any assumption on the nature of parameters.

Fritzsch, Xing [3] and Rasin [4] considered a classification of parametrizations under the limits of small mixings and mass hierarchy. They found **9** variants resulted in all of the best form

$$V_{CKM} = \begin{pmatrix} s_u s_d c + c_u c_d e^{-i\tilde{\delta}} & s_u c_d c - c_u s_d e^{-i\tilde{\delta}} & s_u s \\ c_u s_d c - s_u c_d e^{-i\tilde{\delta}} & c_u c_d c + s_u s_d e^{-i\tilde{\delta}} & c_u s \\ -s_d s & -c_d s & c \end{pmatrix},$$

where we have used the notations; $s_{u,d} = \sin\theta_{u,d}$, $c = \cos\theta_3$ and so on. This Fritzsch–Xing parametrization is different from the standard form given by Kobayashi and Maskawa. The advantage of Fritzsch–Xing form is due to the transparent relations between the regularities of quark mass-matrices and the mixing parameters. Indeed, Fritzscha and Xing shown that, for example, the limit of small mixing of major generation with the junior ones, i.e. the decoupling of heavy generation, takes a simplest form for the infinitely heavy major generation. So,

$$\sin\theta_3 \to 0 \quad \Leftrightarrow \quad m_{\text{heavy}} \to \infty \quad \Longrightarrow \quad \text{decoupling of heavy generation.}$$

The other relation considers the limit of massless junior generation and its connection to the decoupling and CP-violation, so that

$$\sin\theta_{u,d} \to 0 \quad \Leftrightarrow \quad m_{u,d} \to 0 \quad \Longrightarrow \quad \begin{cases} \text{decoupling of junior generation,} \\ \text{no CP-violation.} \end{cases}$$

Some other conditions making the preference for the Fritzsch–Xing parametrization are also discussed in [3].

The above form of mixing matrix can be easily obtained by the diagonalization of mass matrices given by [5]

$$M = \begin{pmatrix} \mu_1 & \bar{\mu} + \Delta & \bar{\mu} \\ \bar{\mu} + \Delta & \mu_2 & \bar{\mu} - \Delta \\ \bar{\mu} & \bar{\mu} - \Delta & \mu_3 \end{pmatrix}, \tag{1}$$

where the complex parameters are arranged, so that $\mu_{1,2,3} \approx \bar{\mu} \gg \Delta$. Further, one can get the form of (1) by a transformation of quite symmetric original mass matrix [5]

$$M_{RL} = \begin{pmatrix} v_1 & v_2 & v_3 \\ v_2 & v_3 & v_1 \\ v_3 & v_1 & v_2 \end{pmatrix}. \tag{2}$$

Matrix (2) possesses the permutational symmetry of indices, that does not change the eigen-values. Its form can be derived from the $\mathbb{Z}_3$ symmetry in the Higgs vacuum sector, which is symbollically shown in Fig. 1. This symmetry implies that the fermions are coupled to the scalar fields, which have the same vacuum expectations except the variation of complex phase rotated by the angle $\frac{2\pi}{3}$. Thus, we deal with

$$\left.\begin{array}{r} \text{three real parameters \& one complex phase} \\ \Im m\, v_1 \neq 0, \quad v_{2,3} \in \Re \end{array}\right\} \Longrightarrow \begin{array}{l} \boxed{\mathbb{Z}_3} \text{ symmetry of vacuum:} \\ \{a = e^{\frac{2\pi}{3}},\ 1,\ a^{-1} = e^{-\frac{2\pi}{3}}\}. \end{array}$$

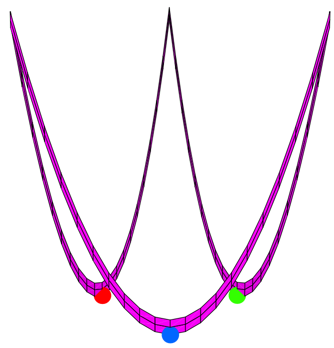

Fig. 1. Three equivalent positions in the Higgs sector composing the real $\mathbb{Z}_3$ symmetric vacuum.

It is worth to stress that this picture of vacuum structure is quite general, and it is can be consistent not only with the quark sector, but also with the charged leptons and neutrinos. So, if the only admissible complex phase of v_1 is small, i.e. the phase is given by the unit element of $\mathbb{Z}_3$, then we arrive to the situation with quarks and charged leptons, while in the case of phase close to the value given by the basis element of $\mathbb{Z}_3$ we get the most probable picture in the neutrino sector with the almost degenerate neutrinos and large mixings:

$$|\Im m\, v_1| \ll |v_{1,2,3}| \approx v \quad \Longrightarrow \boxed{\text{Hierarchy}} \text{ of charged fermion masses \& mixings.}$$

$$v_1 = |v_1| e^{\frac{2\pi}{3}},\ |v_{1,2,3}| \approx v \quad \Longrightarrow \text{Almost } \boxed{\text{degenerate}} \text{ neutrinos \& } \boxed{\text{large mixing.}}$$

In order to illustrate the framework of symmetric vacuum state, we give the example with two generations involving the $\mathbb{Z}_2$ symmetry, which results in two-parametric mass matrix. It can be represented in the symmetric form transformed to the matrix similar to (1)

$$M_{RL} = \begin{pmatrix} v_2 & v_1 \\ v_1 & v_2 \end{pmatrix} \Longrightarrow \begin{pmatrix} v-a & v \\ v & v+a \end{pmatrix}, \tag{3}$$

where the real parameters $v_{1,2} \in \Re$ are close to each other, and we have introduced the notations $v = v_2$, $a = v \tan 2\theta$, so that we get the hierarchy of masses

$$m_{1,2} = |v_1 \pm v_2| \quad \Longrightarrow \quad m_1 \ll m_2, \tag{4}$$

while mixing angle of generations is related to the mass ratio

$$\cos 2\theta = \frac{v_2}{v_1}, \quad \Leftrightarrow \quad \tan 2\theta = 2\frac{\sqrt{m_1 m_2}}{m_2 - m_1} \approx 2\sqrt{\frac{m_1}{m_2}},$$

that approximately leads to

$$\sin\theta \approx \sqrt{\frac{m_1}{m_2}}.$$

Similar relations for three generations appear for the mixing of junior generations, while the mixing with the major one is suppressed as m_2/m_3 and the CP-violation phase is fixed in the case of $\mathbb{Z}_3$. So, we get

$$\left|\frac{V_{ub}}{V_{cb}}\right| = \sqrt{\frac{m_u}{m_c}}, \qquad \left|\frac{V_{td}}{V_{ts}}\right| = \sqrt{\frac{m_d}{m_s}},$$

and the complex phase is given by

$$\cos\tilde{\delta}_{\mathbb{Z}_3} = -\frac{5}{8}.$$

The Wolfenstein parameter λ has a dependence on the phase and masses of light generations.

The status of experimental measurements for the elements of CKM matrix is presented in Table 1 taken from [6]. We see that the most accurate determination is given for the element V_{ud}. Next, the element V_{cb} is measured with a low uncertainty, while the extraction of V_{ub} involves model estimates, which can lead to underestimation of systematic errors. This was recently demonstrated by M.Voloshin [7], who considered the influence of factorization breaking in the calculation of hadronic matrix elements for the four quark operators, that can result in the difference of lifetimes for D^0 and D_s mesons. These nonfactorizable effects can change the form of end-point spectra in the decays of B mesons due to the $b \to u$ current, which is important in the theoretical description of corresponding decays providing the extraction of V_{ub}. Finally, the ratio of V_{ts}/V_{td} is still constrained, but determined, because of difficulties in the measuring of rapid B_s oscillations.

In the heavy quark sector news come from the data acquisition at Belle and BaBar experiments searching for the CP-violation in B decays [8], which we discuss below.

Table 1. Present knowledge of the CKM matrix: Experimental determination [from Faccioli, 2000].

$\lvert V_{ij}\rvert$ etc.	from	value
G_F	*muon lifetime*	$1.16639(1)\cdot 10^{-5} GeV^{-2}(\hbar c)^3$
$\lvert V_{ud}\rvert$	*nuclear super-allowed decays*	$0.9740 \pm 0.0001_{exp} \pm 0.0010_{th}$
$\lvert V_{ud}\rvert$	*neutron decay*	$0.9738 \pm 0.0016_{exp} \pm 0.0004_{th}$
$\lvert V_{ud}\rvert$	*pion β decay*	$0.9670 \pm 0.0160_{exp} \pm 0.0008_{th}$
$\lvert V_{us}\rvert$	*K_{e3} decays*	$0.2200 \pm 0.0017_{exp} \pm 0.0018_{th}$
$\lvert V_{us}\rvert$	*hyperon semileptonic decays*	$0.21 - 0.24$
$\lvert V_{cd}\rvert$	*neutrino charm production*	0.225 ± 0.012
$\lvert V_{cs}\rvert$		1.04 ± 0.16
$\lvert V_{cs}\rvert$	*D_{e3} decays*	$1.02 \pm 0.05_{exp} \pm 0.14_{th}$
$\lvert V_{cs}\rvert$	*hadronic W decays*	0.99 ± 0.02
$\lvert V_{ub}\rvert$	$B \to \rho\ell\bar{\nu}$	$(3.25 \pm 0.30_{exp} \pm 0.55_{th})10^{-3}$
$\lvert V_{ub}/V_{cb}\rvert$	*inclusive $B \to X_u\ell\bar{\nu}$ (CLEO, ARGUS)*	$0.088 \pm 0.006_{exp} \pm 0.007_{th}$
$\lvert V_{ub}\rvert^2$	*inclusive $b \to u\ell\bar{\nu}$ (LEP)*	$(16.8 \pm 5.5_{exp} \pm 1.3_{th})10^{-6}$
$\lvert V_{cb}\rvert$	$B \to D^*\ell\bar{\nu}$	$(42.8 \pm 3.3_{exp} \pm 2.1_{th})10^{-3}$
$\lvert V_{cb}\rvert$	*inclusive $b \to c\ell\bar{\nu}$*	$(41.2 \pm 0.7_{exp} \pm 1.5_{th})10^{-3}$
$\lvert V_{tb}\rvert^2/\sum_i \lvert V_{ti}\rvert^2$	*top quark decays*	$0.93^{+0.31}_{-0.23}$
	effective FCNC processes	
$\lvert V_{td}V_{tb}\rvert$	*$B_d^0/\bar{B}_d^0$ and $B_s^0/\bar{B}_s^0$ oscillations*	$(8.1 \pm 0.7_{exp} \pm 0.6_{th})\cdot 10^{-3}$
$\lvert V_{ts}/V_{td}\rvert$		> 4.6
$\lvert V_{ts}V_{tb}/V_{cb}\rvert^2$	*inclusive $b \to s\gamma$*	$0.94 \pm 0.11_{exp} \pm 0.09_{th}$
$\mathrm{Im}(V_{ij})$	*CP-violation measurements:*	$\lvert\epsilon_K\rvert = (2.271 \pm 0.017)\cdot 10^{-3}$
		$\epsilon'_K/\epsilon_K = (19.0 \pm 4.5)\cdot 10^{-4}$
		$\sin 2\beta = 0.48^{+0.22}_{-0.24}$

1.2 Unitarity triangle

The unitarity of mixing matrix in the SM provides us with useful conditions on the values of matrix elements due to zero nondiagonal elements of $V_{CKM}\cdot V^\dagger_{CKM} = 1$. In B decays the corresponding condition can be studied

$$V_{ud}V^*_{ub} + V_{cd}V^*_{cb} + V_{td}V^*_{tb} = 0, \qquad \times\frac{1}{V_{cd}V^*_{cb}},$$

which, after the multiplication to the factor shown above, can be represented as the equality for three vectors in the complex $\boxed{\rho,\,\eta}$ plane of Wolfenstein parameters as shown in Fig. 2.

In terms of re-scaled Wolfenstein parameters, $\bar{\rho} = c\rho$, $\bar{\eta} = c\eta$, $c = (1-\lambda^2/2)$, the ratios $|V_{ub}|/|V_{cb}|$ and $|V_{td}|/|V_{ts}|$ are

$$\frac{|V_{ub}|}{|V_{cb}|} = \frac{\lambda}{c}\sqrt{\bar{\rho}^2+\bar{\eta}^2}, \qquad \frac{|V_{td}|}{|V_{ts}|} \approx \frac{\lambda}{c}\sqrt{(1-\bar{\rho})^2+\bar{\eta}^2}, \tag{5}$$

which should be compared with the predictions based on the relations of mixing parameters with the masses of fermions, i.e. on the regularities coming from the principle of democracy

$$\left|\frac{V_{ub}}{V_{cb}}\right| = \sqrt{\frac{m_u}{m_c}}, \qquad \left|\frac{V_{td}}{V_{ts}}\right| = \sqrt{\frac{m_d}{m_s}}.$$

The position of triangle vertex in accordance with the current experimental 1σ-data on various parameters of CKM matrix [6] is shown in Fig. 3, where we also present the results of theoretical expectations coming from the relations between the mass ratios of quarks and the mixings. We see that allowing a light variation of CP-violating phase in the theoretical model leads to a contour close to the present data on the unitarity triangle.

We can draw a conclusion that within the current accuracy we do not see any contradiction of measurements with the mass-matrix hierarchy, moreover the CP-violation phase is close to the value following from the $\mathbb{Z}_3$ symmetry of vacuum.

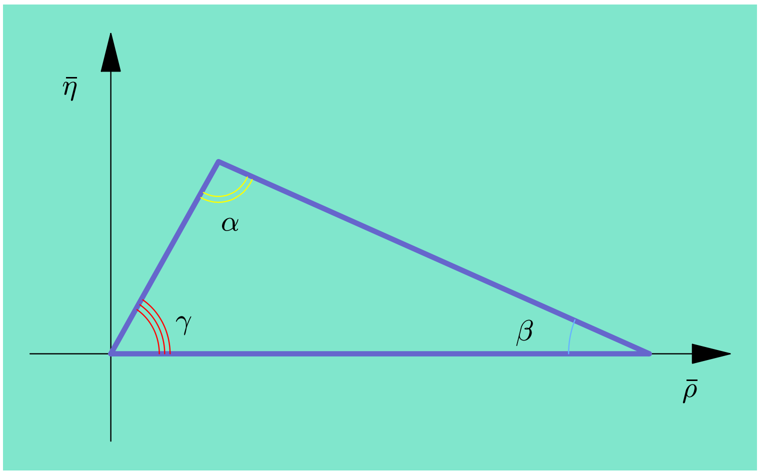

Fig. 2. The triangle.

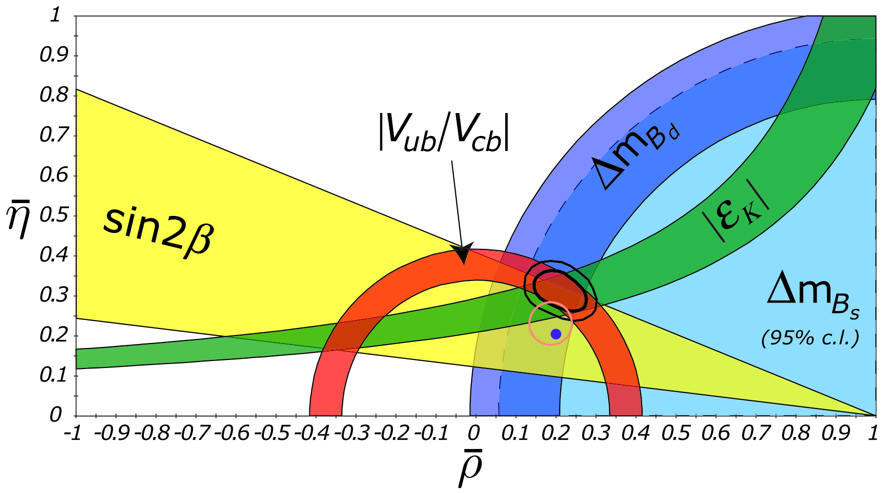

Fig. 3. The position of triangle vertex: the experimental data compared with the theoretical expectations at $\cos\tilde{\delta}_{\mathbb{Z}_3} = -\frac{5}{8} \Rightarrow \bullet$, $\cos\tilde{\delta} = -\frac{5}{8} \pm 0.045 \Rightarrow \bigcirc$. Characteristic values of quark masses at $\mu = m_Z$: $m_d \approx 4 - 4.5$ MeV, $m_u \approx 0.55\, m_d$, $m_s \approx 100 - 120$ MeV, $m_c \approx 0.65 - 0.67$ GeV, $m_b \approx 3 - 3.2$ GeV, $m_t \approx 181$ GeV.

2 Neutral B-mesons

In this section we attribute the general characteristics of quark interactions involving the CP-violating effects to the system of B-mesons. Making common remarks, we concentrate the attention to the neutral B-mesons. First, we describe how the forces breaking the CP parity manifest themselves in the dynamics of flavor content in the neutral B-mesons, i.e. in the static mass-width parameters, as well as in the time evolution of flavored states. Second, we show how the decay characteristics allow us to extract the CP-violating effects in the quark interactions. Third, we attend some problems in the theoretical interpretation of data as well as mention about the consideration of CPT violation in the framework of dissipative dynamics.

2.1 Eigen states

We define the phase of combined CP-inversion, so that $\mathsf{CP}|B^0\rangle \stackrel{\mathsf{def}}{=} |\bar{B}^0\rangle$, where B^0 is flavored state containing the $\bar{b}$ quark. Because of the mixing effects, the eigen states of mass operator do not coincide with the flavored states. So, we denote these states as

$$|B_\pm\rangle = p|B^0\rangle \pm q|\bar{B}^0\rangle,$$

while the mass matrix has the following general CPT-invariant form:

$$\boldsymbol{M} - \frac{i}{2}\boldsymbol{\Gamma} = \begin{pmatrix} M - \frac{i}{2}\Gamma & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{12}^* - \frac{i}{2}\Gamma_{12}^* & M - \frac{i}{2}\Gamma \end{pmatrix},$$

where $\boldsymbol{M}$ is a dispersive part of mass matrix, and $\boldsymbol{\Gamma}$ is an absorbtive one. Solving the equation

$$\left[\boldsymbol{M} - \frac{i}{2}\boldsymbol{\Gamma}\right]|B_\pm\rangle = \lambda_\pm|B_\pm\rangle,$$

we get the positions of poles in the complex plane

$$\lambda_\pm = \left(M - \frac{i}{2}\Gamma\right) \pm \frac{q}{p}\left(M_{12} - \frac{i}{2}\Gamma_{12}\right),$$

where

$$\frac{q}{p} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}.$$

In the SM we have got the nondiagonal mixing term

$$M_{12} = -\boxed{(V_{td}^* V_{tb})^2}\, G_F^2 M_W^2 M_B f_B^2\, \eta_B(\alpha_s)\, B_B\, S(m_t/M_W),$$

where the framed factor represents the dynamics of charged current mixing, $\eta_B(\alpha_s)$ includes the QCD corrections to the quark level diagrams, B_B gives the deviation for the hadronic matrix element of four-quark operator from the factrorized expression in terms of hadronic matrix elements for two-quark operators (the leptonic constant f_B). The function $S(m_t/M_W)$ is known and calculated at the quark level. The absorptive part of mixing is also known, and it is suppressed as

$$\left|\frac{\Gamma_{12}}{M_{12}}\right| \sim \mathcal{O}(m_b^2/m_t^2) \ll 1. \tag{6}$$

Alternative notations analogous to the K meson physics are also usually explored

$$|B_\pm\rangle = \frac{(1+\epsilon)|B^0\rangle \pm (1-\epsilon)|\bar{B}^0\rangle}{\sqrt{2(1+|\epsilon|^2)}},$$

with

$$\frac{1-\epsilon}{1+\epsilon} = \frac{q}{p}.$$

Due to the nonzero absorptive part, the absolute value of ratio representing the fractions of flavored states in the massive ones deviates from unity, so that to the subleading order of small parameter (see (6)) we get

$$\left|\frac{q}{p}\right|^2 = 1 + \left|\frac{\Gamma_{12}}{M_{12}}\right| \sin[arg(M_{12}) - arg(\Gamma_{12})] + \dots$$

We can point to the conditions, when the CP-violation effects are absent in the SM with three generations,

$$\left.\begin{array}{l} \left|\dfrac{q}{p}\right| = 1 \\[2ex] \mathfrak{Re}[\epsilon] = 0 \end{array}\right\} \Longrightarrow \text{no CP violation} \quad \Leftrightarrow \quad CP|B_\pm\rangle = \pm|B_\pm\rangle.$$

In that case the massive states are orthogonal to each other and have definite CP parities.

2.2 Time evolution

After the production of flavored states at the moment $t = 0$: $|B^0(0)\rangle = |B^0\rangle$ and $|\bar{B}^0(0)\rangle = |\bar{B}^0\rangle$, the evolution of flavor contents takes place, so that it can be expressed in terms of eigen-values of mass operator, and we obtain

$$\begin{aligned} |B^0(t)\rangle &= g_+(t)|B^0\rangle + \frac{q}{p}\, g_-(t)|\bar{B}^0\rangle, \\ |\bar{B}^0(t)\rangle &= g_+(t)|\bar{B}^0\rangle + \frac{p}{q}\, g_-(t)|B^0\rangle, \end{aligned}$$

where

$$g_\pm(t) = \frac{1}{2}\left(e^{-i\lambda_+ t} \pm e^{-i\lambda_- t}\right),$$

and

$$|g_\pm(t)|^2 = \frac{e^{-\Gamma t}}{2}\left[\cosh\left(\frac{\Delta\Gamma}{2}\, t\right) \pm \cos(\Delta m\, t)\right],$$

with

$$\Delta\Gamma = |\Gamma_+ - \Gamma_-|, \qquad \Delta m = |M_+ - M_-|.$$

We see that almost coherent oscillations of $B^0 \leftrightarrow \bar{B}^0$ untill decay take place, and the deviation is given by the non-unit absolute value of factor $|q/p|$.

If both decays of $B^0 - \bar{B}^0$ system are tagged by their b-flavor contents, then putting $\Delta\Gamma t \to 0$ we have got for the tagged events

$$|g_\pm(t)|^2\, \frac{2}{e^{-\Gamma t}} = [1 \pm \cos(\Delta m\, t)],$$

which is measured experimentally (see Fig.4).

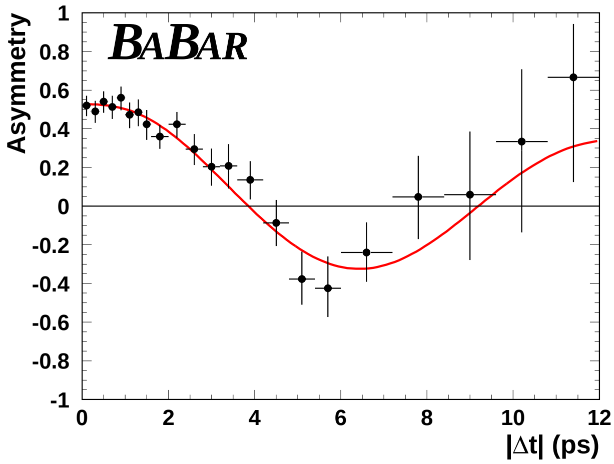

Fig. 4. The BaBar data on the oscillation of tagged events. The asymmetry is defined as $(|g_+(t)|^2 - |g_-(t)|^2)/(|g_+(t)|^2 + |g_-(t)|^2)$, while the deviation of amplitude from the unity is due to so-called dilution factor of mistagging.

2.3 Decays & CP-violation

The main feature of dynamics under consideration is that in order to observe the CP violation we have to involve the interference.

One usually isolates three classes of processes under interest:

- **Indirect CP-Violation**: $\left|\dfrac{q}{p}\right| \neq 1$ is enough to measure the CP-violation.

- **Direct CP-Violation I**: Asymmetry in flavored channel decay amplitudes with
$|A(f)| \neq |A^{CP}(f^{CP})|$ or different weak phases in two decays.
- **Direct CP-Violation II**: Decays to CP eigenstates:
  - $\boxed{\text{interference}}$ of oscillations with decay amplitudes even at

$$\left|\frac{q}{p}\right| = \tilde{q} = 1 \quad \text{and} \quad |A(f)| = |A^{CP}(f^{CP})|.$$

These classes correspond to the following decay processes:

1. Wrong-flavor decays, for example, the wrong lepton-sign widths give

$$a_{SL}^{CP} = \frac{\Gamma(\bar{B} \to l^+ X) - \Gamma(B \to l^- X)}{\Gamma(\bar{B} \to l^+ X) + \Gamma(B \to l^- X)} = \frac{1 - \tilde{q}^4}{1 + \tilde{q}^4}.$$

2. The asymmetry in decays to flavored channels (charged B_q mesons, too) occurs, when the weak CP-violating phases of amplitudes interfere with the terms of CP-even phases of strong interaction.

3. The time-dependent asymmetry in decays of neutral B mesons to CP eigenstates represents the normalized difference of events in decays of states with definite b-flavor at zero time, which is determined by tagging the flavor in the decay of associated neutral B meson. So, it has the form

$$a^{CP}(t) = \Im m \left[\frac{q}{p}\,\frac{A^{CP}}{A}\right] \sin[\Delta m\, t], \qquad A = A(B \to f). \tag{7}$$

Following the items 2 and 3, one needs quite a definite understanding of decay amplitude structure by isolating various dynamical factors such as the phases of weak and strong interaction terms.

The CP-odd phases of amplitudes are classified by the effective weak lagrangian of heavy quark decays.

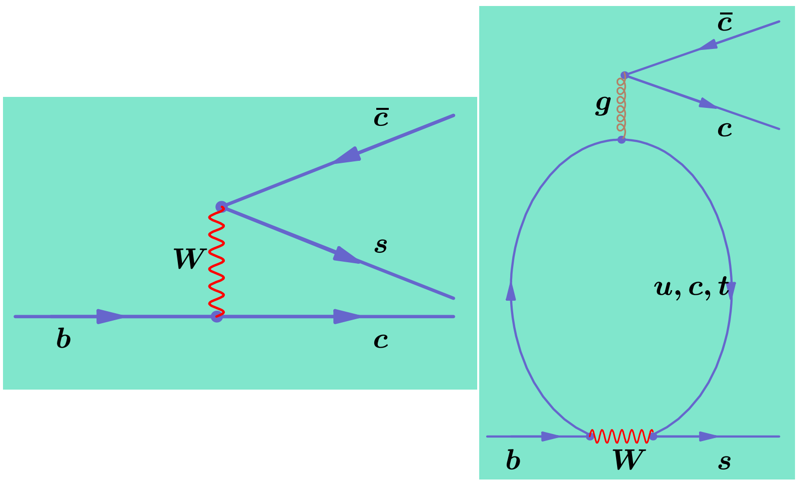

Fig. 5. The diagrams representing various weak phase structures in the effective four-fermion weak lagrangian of quarks, i.e. the tree diagram and the penguin. The gluon corrections, which do not change these phases, are not shown, but they are known to two-loop order in QCD coupling α_s [9].

The corresponding diagrams in the decays of $b \to c\bar{c}s$ as they contribute to the effective weak lagrangian of quarks are shown in Fig. 5. Some other gluon corrections are also calculable (see review in [9]). Various weak CP-violating terms are arranged in powers of small parameter, the sine of Cabibbo angle λ, and presented in Table 2.

As an example we point to the so-called Golden plated mode $J/\Psi + K_S$. In this case the amplitudes are determined theoretically with extremely low uncertainty (below 1%), since the corrections to the amplitudes are suppressed by the sine of Cabibbo angle squared.

Table 2. The quark processes in decays of neutral B-mesons and their arrangment in λ with the indication of appropriate angle in the unitarity triangle. The symbol * means that the angle would be extracted if the competitive term of correction would be small.

Current	Leading term	Correction	B_d decay mode	weak angle
$b \to c\bar{c}s$	$V_{cb}V^*_{cs} = A\lambda^2$ tree + penguin (c-t)	$V_{ub}V^*_{us} = A\lambda^4(\rho - i\eta)$ penguin (u-t)	$J/\Psi\, K_S$	β
$b \to s\bar{s}s$	$V_{cb}V^*_{cs} = A\lambda^2$ penguin only (c-t)	$V_{ub}V^*_{us} = A\lambda^4(\rho - i\eta)$ penguin (u-t)	$\phi\, K_S$	β
$b \to c\bar{c}d$	$V_{cb}V^*_{cd} = -A\lambda^3$ tree + penguin (c-u)	$V_{tb}V^*_{ts} = A\lambda^3(1-\rho+i\eta)$ penguin (t-u)	$D^+\, D^-$	$^\star\beta$
$b \to c\bar{u}d$	$V_{cb}V^*_{ud} = A\lambda^2$ tree	0	$D^0\, \pi^0(\rho^0)$ $\hookrightarrow$ CP eigen state	β
$b \to u\bar{u}d$	$V_{ub}V^*_{ud} = A\lambda^3(\rho - i\eta)$ tree + penguin (u-c)	$V_{tb}V^*_{ts} = A\lambda^3(1-\rho+i\eta)$ penguin (t-c)	$\pi\pi$; $\rho\pi$	$^\star\alpha$

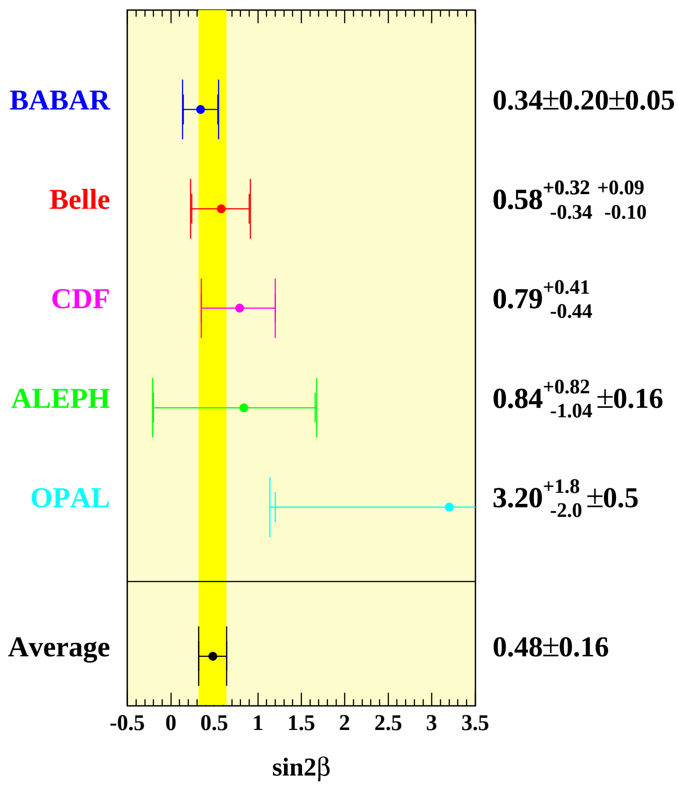

Fig. 6. The data on $\sin 2\beta$ and the world-average value.

Following the equation for the asymmetry, we can determine the quantities entering (7) and their weak CP-odd phases:

$$\begin{aligned} \frac{q}{p} &\Longrightarrow \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}, \\ \frac{A_B^{CP}}{A_B} &\Longrightarrow \frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*}, \\ K_S &\Longrightarrow \frac{V_{cd}^* V_{cd}}{V_{cd} V_{cd}^*}, \end{aligned}$$

so that we get

$$a_{J/\Psi K_S}^{CP}(t) = \sin[2\beta]\,\sin[\Delta m\, t].$$

This quantaty is observed in various experiments, while the Belle and BaBar are tending to improve the measurement of angle β. The summary of current data on $\sin 2\beta$ is presented in Fig.6.

2.4 Some problems

In contrast to the golden plated mode the other decay channels for B mesons cannot be analyzed with a high accuracy of theoretical description. In this section, first, we point to some difficulties in the extraction of weak angles α, γ from the data, and second, we mention a general possibility of violation in the invariance of interactions under the complete CPT conjugation.

Mode $B \to \pi\pi$

In this decay the ratio of amplitudes for the CP conjugated initial states with definite flavor is not well determined theoretically because of significant contribution by the penguin with the different weak phase. The magnitude of this penguin depends on the strong dynamics of light quarks bound inside the mesons, which is described with large numerical uncertainties or even model-dependent. The problem is connected to the evaluation of hadronic matrix elements for the quark currents:

- the factorization & nonfactorizable effects in the evaluation of four quark operators,
- unknown relative strong phases between the contributions possessing different weak CP-odd phases,
- the isotopic symmetry could help, but $\pi^0\pi^0$ channel is difficult to measure,
- one has to use dynamical, theoretical predictions, that leads to uncertainties, model-dependent formfactors.

The analysis of above problems was carefully performed in [10]. A characteristic picture following from such the calculations is shown in Figs. 7 and 8. The restrictions presented in Fig. 7 are separated in two classes. The first is the shaded region as descrined in Section 2. The second class is the analysis of $\pi\pi$ and $K\pi$ modes in [10], that is shown as 1, 2 and 3 sigma regions as well as the dots giving both the factorization-based result and the model dependent evaluation of nonfactorizable effects. Fig. 8 shows the experimental bounds on the various ratios of decay widths under consideration (horizontal bands) in comparison with the factorization result (dashed curve) as well as the reasonable variation of form factors (shaded regions) versus the weak angle γ [10].

Mode $B \to \rho(\pi\pi)\pi$

For this mode in comparison with the $\pi\pi$-channel, the additional problem is the presence of strong resonances in two-pion states. This involves a large uncertainty because of increase of unknown parameters such as the strong phases and absolute values of various amplitudes. However, one can explore the advantage of Dalitz plot analysis:

- a hope to extract all of phases and amplitudes [11],
- to recognize uncertainties of reconstruction:
  - many resonances,
  - nonresonant contribution.

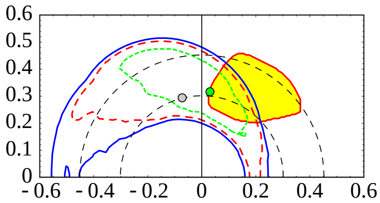

Fig. 7. The analysis of restrictions following from $B \to \pi\pi$ and $B \to K\pi$ decays [10]. The explanations are given in the text.

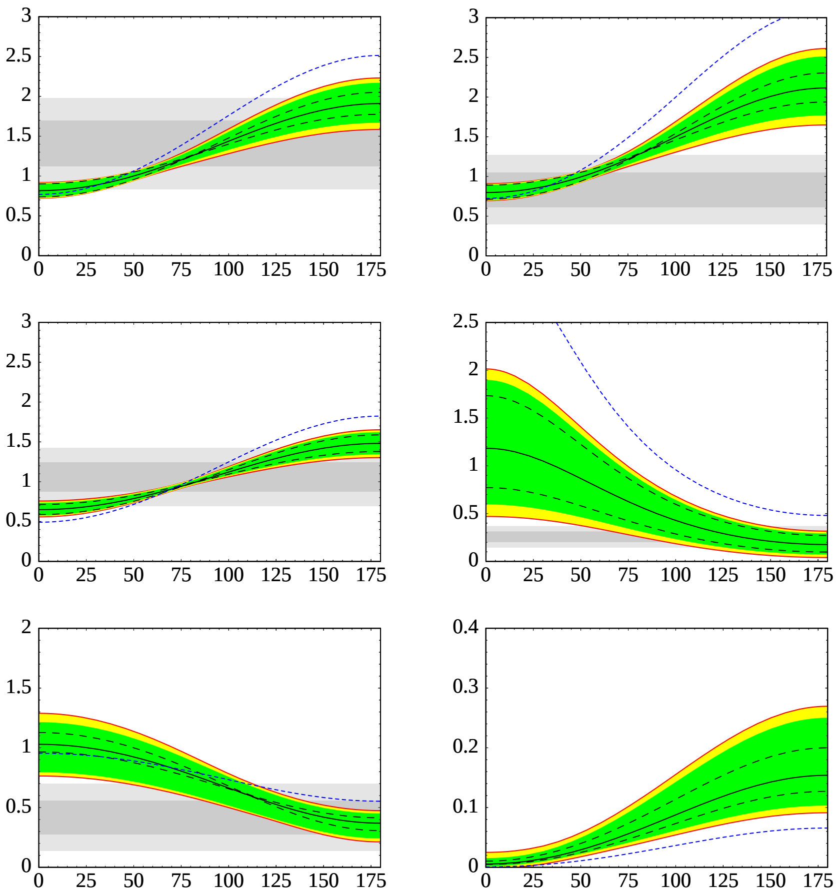

Fig. 8. The comparison of experimental bounds with the theoretical predictions on $B \to \pi\pi$ and $B \to K\pi$ decays [10]. The explanations are given in the text.

The results of simulations performed in the original paper by Snyder & Quinn [11] as well as the expected picture at BTeV and LHCb are shown in Fig. 9. An error of such the analysis in the determination of angle γ is expected on the level of less than 10°.

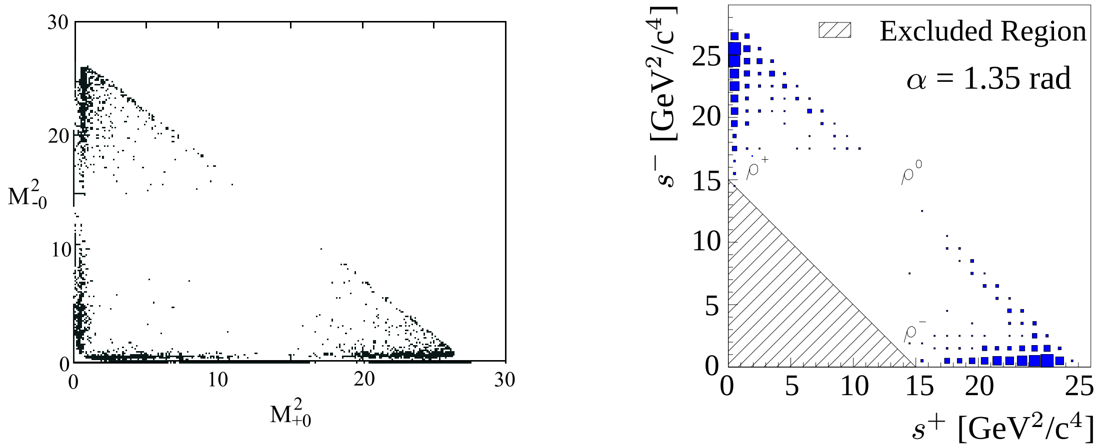

Fig. 9. The Dalitz plots in $B \to 3\pi$ decays simulated in [11] (left figure) & at BTeV, LHCb (right figure).

CPT-violation

Another problem is that the CP and T conjugations could be nonequivalent if the complete CPT inversion is violated in the interactions. This could happen, for example, if we deal with incomplete dynamics:

- extra dimensions could results in that 4D conversions do not lead to equivalent representations of extended 'Poincare' group, that implies at low energies with the effective four dimensional interactions the extended dynamics would cause the CPT-violation,
- a general scheme with a dissipative Hamiltonian resulting in description in terms of quantum dynamical semigroups was considered in [12].

This dissipative dynamics involves **6** additional parameters of correction L to the mass matrix of neutral B mesons under the conditions of conserving the positivity and entropy growth, so that

$$\boldsymbol{M} - \frac{i}{2}\boldsymbol{\Gamma} \quad \Longrightarrow \quad \boldsymbol{M} - \frac{i}{2}\boldsymbol{\Gamma} + L.$$

In that case the asymmetries of T-inverted amplitudes A are different from the CP-conjugated ones, while the asymmetry with CPT conversion is not equal to zero, if the diagonal elements of L do not coincide with each other:

$$A_T \neq A_{CP}, \quad A_{CPT} \neq 0 \quad \Longleftarrow \quad L_{11} \neq L_{22}.$$

One expects for rather strict limits on the **CPT**-violating parameters from current experiments [12].

3 Conclusion

To summarize we draw the following conclusions:

- Mass matrices of fermions govern the charged current mixings:

  - we observe the **charged fermion mass hierarchy**, and
  - the **mixing hierarchy**,

  which can be presented as a common property of Yukawa interactions in the form of democratic (almost equal) couplings to the scalar vacuum field.

- Small perturbations of generation democracy result in the following:
  - the **mixing angles** are related with the **mass ratios**,
  - the $\mathbb{Z}_3$ symmetry of vacuum leads to a definite CP-phase of mixing matrix,
  - the current knowledge of CKM matrix is in agreement with the mass relations and very close to the $\mathbb{Z}_3$ symmetry of vacuum.

- Golden plated mode brings the angle β of unitarity triangle.

- Strong theoretical & experimental efforts are challenged to extract other angles.

Some other aspects of CP-violation in B decays are discussed in reviews [13].

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