On the Foundations of any Classical Theory of Spacetime

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A postulational basis for any non–quantum theory of spacetime is introduced. The postulates collect uniquely how space and time are measured. As a consequence, we show that one of the following four structures can be attached naturally to spacetime: (a) a Lorentzian metric $(-,+,+,+)$, as in Einstein's General Relativity, (b) a (positive-definite) Riemannian metric $(+,+,+,+)$, (c) a Leibnizian structure, which generalizes classic Newton's spacetime, or (d) an anti-Leibnizian structure, mathematically dual to the previous one.
Moreover, questions as the possible transitions among this structures (with a signature changing metric *à la* Hartle-Hawking), the variation of the speed of light or the compatibility with alternative theories to Einstein's, are also discussed.

1 Introduction

Physicists have Newton and Einstein's theories as non–quantum theories of spacetime. In Einstein's General Relativity, a Lorentzian manifold is assigned to spacetime. This manifold contains all information on "gravity" and some information on electromagnetism. Newton's theory is mathematically somewhat longer. Spacetime is a *scenario* where forces act. It is commonly accepted that Einstein's GR is the correct theory, and Newton's one can be regarded as a limit in some sense. However, Newton's theory is useful not only as a limit but also because, for the interpretation of Einstein's, "almost-Newtonian" concepts are used (see for example [14], [13]).

Einstein's success is impressive, and there are few alternative approaches commonly accepted. Some alternative variations are:

1. Ellis' et al. principle of "restricted covariance" [7], [18]. Its starting point is the following: physicists and astrophysicists almost always use *preferred coordinate systems* not merely to simplify the calculations but also to define quantities of physical interest.

2. Classical limit of Hartle-Hawking's "no boundary" proposal. Even though this proposal was introduced by quantum reasons [10], its classical limit is a *signature changing metric*, which has been widely studied recently (see for example [4],[8],[9],[11]).

3. Logunov's relativistic theory of gravity ([12] and references therein). This theory is *essentially different* to Einstein's General Relativity because Minkowski spacetime is considered as the background geometry of spacetime, and gravity is treated as an external force.

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In this talk we are going to wonder how any physical classical theory of spacetime is introduced, at least in the non-quantum case. Our concrete aims will be:

- To find a postulational basis for the measurements of time and space in *any* theory of spacetime, with simple postulates both, physically and mathematically clear.
- To deduce all the possible models of spacetime compatible with these postulates.

Surprisingly, all these models will be very few, and previous alternative approaches will have their natural place.

We will explain here the essential ideas of our approach (firstly introduced in [1]), emphasizing the physical and philosophical aspects rather than the mathematical details. For exhaustive mathematical treatment and further discussions we refer to [1], [2].

2 Previous standard approaches

In order to characterize how space and time are measured we can find two standard limit answers:

- (A) To assume the existence of *ideal instruments* (standard clocks, rigid rulers).
- (B) To assume the existence of *priviledged physical objects* (freely falling particles, light rays).

Answers based on (A) might be very attractive before the formulation of General Relativity, but GR implies a criticism to the idea of rigid bodies and, thus, something like a rigid ruler. Synge's approach [17] is a combination of previous two answers: standard clocks and particles are taken as primitive concepts.

Ehlers, Pirani, Schild's approach (in what follows *(EPS)*) [6], [5] is based enterely in the limit answer (B): *freely falling particles and light rays* are taken as primitive concepts. For many physicists, EPS has become the standard physics foundations of non-quantum spacetime. However, we can find the following limitations to EPS (or to any previous approach based on either (A) or (B) above):

1. *Target.* EPS axioms are *introduced as foundations of General Relativity*: GR is assumed to be the correct theory and these axioms justify how to work with it from a fundamental viewpoint. But it should be nice to have a set of postulates with no assumed physical theory *a priori*, that is, applicable not only to General Relativity but also to Newton theory, and even to any physically reasonable theory for space and time.
2. *Aesthetic.* The list of EPS axioms is long. Essentially these axioms:
   - (a) take events, light rays and freely falling particles as primitive concepts,
   - (b) postulate radar coordinate systems; in particular, one has charts, differentiable atlases and a manifold structure,
   - (c) ensure a good behaviour for a conformal structure, a projective structure and the compatibility between them, in addition to a chronological order,
   - (d) but these axioms are not enough to ensure the *compatibility of these structures with a metric* and, therefore, this compatibility is assumed additionally (as an "extraneous element of the theory" [5, pp. 36-37]).

   This seems too close to the geometrical model (the Lorentzian metric) that will be obtained finally.

3. *"Methodic doubt."* he sure about the inexistence of a hidden force? Is not the priviledged role of light rays (and, thus, electromagnetism) rather surprising for the construction of a theory of spacetime? Why other forces do not appear, or appear in a different way? Is really essential for spacetime the existence of (non-accelerated) photons? Beyond the "primitive concepts", which live in spacetime, is there not a more elemental possibility of measuring space and time?

However, remark that any previous alternative approach seems to have at least these limitations. For example, in principle ideal instruments are also postulated for a concrete (Newton, Einstein) theory of spacetime.

Finally, in order to compare with our postulates, notice that, from EPS axioms: (i) light clocks can be constructed, and (ii) it is possible to proceed without rigid bodies, but ideal rulers can be understood as approximations in tangent space.

3 Our approach: postulates

Our starting point is that spacetime is a "set of events", with no structure a priori, but *which can be measured.* And we claim that the following postulates will be satisfied then.

3.1 First postulate: Spacetime and observers

It seems clear that, in order to measure or describe spacetime: (i) we have to use four coordinates, and (ii) we distinguish one of them (the *temporal* coordinate) from the other three (*spatial* coordinates). Our first postulate is just the mathematical translation of these facts.

Postulate 1, P1: Spacetime *is a (connected) differentiable* 4–manifold*, M, where each* observer *O take* a coordinate system $(U, \Phi) = (t, x^1, x^2, x^3)$ *(makes an observation) by using some type of measurement instruments.*

Responsible of the coordinate system (U, Φ).

The first coordinate t is called the temporal coordinate *and the other three* (x^1, x^2, x^3) *the* spatial coordinates.

Of course, one could argue against P1 that perhaps there are extra dimensions, as in many current physical theories. Nevertheless, these theories must explain why we do not perceive these extra dimensions directly, and the operational validity of our postulate remains.

Once P1 is accepted, we can give the following definitions, which describe associated infinitesimal concepts for any observer. If an observer O takes coordinates around $p \in U$:

- Tangent vector $\partial_t|_p \in T_pM$ is the *(instantaneous) temporal unit* of O at p. This vector spans the *temporal axis* of O at p. The one form $dt|_p$ is observer's *clock* at p.

- Tangent vector $\partial_{x^i}|_p \in T_pM$ is the *(instantaneous) i–th spatial unit* $i \in \{1, 2, 3\}$ of O at p. Each spatial unit spans the corresponding *spatial axis* of O at p. Spatial axis span observer's *rest space* at p.

The basis B_p of T_pM given by such timelike and spatial units, $B_p = (\partial_t|_p,\ \partial_{x^1}|_p,\ \partial_{x^2}|_p, \partial_{x^3}|_p)$, will be called the *(instantaneous) observer at p.*

Remark. For P1, the specific method to take coordinates is not relevant. But previous language suggest that O tries to use the "best instruments" he can find in order to take coordinates.

Formal summary. All this can be summarized, from a strictly mathematical viewpoint, as follows:

- Spacetime = (connected) four manifold M.
- Observer (or, more properly, "observation") O around p = coordinate chart (t, x^1, x^2, x^3).
- (Instantaneous) observer at p: basis of T_pM.

Each observer around p yields naturally the instantaneous observer at p $B_p = (\partial_t|_p, \partial_{x^1}|_p, \partial_{x^2}|_p, \partial_{x^3}|_p)$. Reciprocally, each observer at p comes from at least one observer around p. In what follows, the only relevant properties for any observer around p are those which concerns the corresponding instantaneous observer at p.

3.2 Second postulate: standard observers

This postulate will be the more original point in our approach and, so, we will discuss it in more depth. A priori, we do not know if there exist ideal instruments, priviledged particles or even any geometrical structure in spacetime. But if, at any case, a classical theory for spacetime is possible, then the following two facts will occur:

1. Given an event p, among all the observers around p then the theory will predict the existence of *standard observers for this theory* (at least infinitesimally as an idealized limit around p).

   For example: inertial or freely falling observers, or observers which use ideal instruments, or some observer which is priviledged because of any reason. (Of course, this is compatible with the general covariance of GR).

2. Given two such standard observers $O, \tilde{O}$ around p, the measurements of time (and, independently, of space) by them *cannot priviledge* one of the observers.

   For example: assume that O measures the interval of time between two events, and $\tilde{O}$ finds a "time dilation" comparing with his own measurement. Then, *O will also find a "time dilation"* when compares his measurements with analogous events measured by $\tilde{O}$.

   Or if O measured a sort of "absolute time" then $\tilde{O}$ will measure the absolute time too.

This will be our second postulate, which can be understood in a plain language as a principle of "restricted democracy"[1]:
$*$ at each p, there will be a set S_p of priviledged observers,
$*$ but these observers are not priviledged among them.

The mathematical expression of this intuitive ideas is the following.

Postulate 2, P2. *For each $p \in M$ there exists a (non-empty) set, S_p, of distinguished observers around p,* compatible with a fundamental system of units at p *(or simply, compatible).*
This means: $\forall O, \tilde{O} \in S_p$, with $O \equiv (U, \Phi) = (t, x^1, x^2, x^3)$ and $\tilde{O} \equiv (\tilde{U}, \tilde{\Phi}) = (\tilde{t}, \tilde{x}^1, \tilde{x}^2, \tilde{x}^3)$:

$$\partial_t \tilde{t}|_p = \partial_{\tilde{t}} t|_p \text{ and } \partial_{x^j} \tilde{x}^i|_p = \partial_{\tilde{x}^i} x^j|_p, \ \forall i, j \in \{1, 2, 3\}.$$

Such observers will be called standard observers around p.

Discussion. P2 says two essential things:

[1]We are in debt with Prof. Pirogov, who suggested us this name, much more appropriate than the names "aristocracy" or even "oligarchy" previously used by us.

1. There exists a Physics way to find the set S_p of distinguished observers at each $p \in M$ (think in observers which are: inertial, freely falling, absolute in some sense...)

   Nevertheless, P2 *does NOT say how* to find these observers: this is an experimental problem to be solved for the concrete theory of spacetime.

2. These standard observers in S_p must satisfy just one condition: given two such observers O, $\tilde{O}$, the comparison between their temporal (resp. spatial) coordinates at p cannot priviledge any of them. In fact:

   (a) $\partial_t \tilde{t}|_p = \partial_{\tilde{t}} t|_p$ means: *the $\tilde{O}$ time, measured with the O clock, goes by as the O time, observed by the $\tilde{O}$ clock.* For us, this is just the only sensible mathematical translation of the assertion: "O and $\tilde{O}$ measure at p using the same fundamental or abstract unit of time".

   For example, if there exists a sort of "absolute time" then probably standard observers should measure it and $\partial_t \tilde{t}|_p = 1 = \partial_{\tilde{t}} t|_p$. Otherwise, the observation of time by $\tilde{O}$ may present some "time dilation" $\partial_t \tilde{t}|_p$ with respect to the observation by O; then, O will present an equal time dilation under the analogous situation.

   (b) $\partial_{x^j} \tilde{x}^i|_p = \partial_{\tilde{x}^i} x^j|_p$ means: *the i–th spatial unit of $\tilde{O}$, measured with the j–th ruler of O, is identical to the j–th spatial unit of O, observed with the i–th ruler of $\tilde{O}$.* Again, for us this is the only sensible mathematical translation of the assertion: "O and $\tilde{O}$ measure at p using the same fundamental or abstract unit of space".

   For example, assume that the rest spaces of O and $\tilde{O}$ coincide. The equality between spatial derivatives is equivalent to the following fact: *If O declares that his spatial units are an orthonormal basis, then he must admit that so are the spatial units of $\tilde{O}$* (and vice-versa). So, both observers can agree when they measure the length of a vector in the common rest space: if $v = \sum_i a^i \partial_{x^i}\,|_p = \sum_j \tilde{a}^j \partial_{\tilde{x}^j}\,|_p$ the common number $(\sum_i (a^i)^2)^{1/2} = (\sum_j (\tilde{a}^j)^2)^{1/2}$ is the "length of v" for both observers.

   When the rest spaces do not coincide, we are imposing the minimum symmetry assumption.

Remark. In particular, P2 is satisfied by Einstein' GR (standard observer at p= orthonormal basis of tangent space at p) and Newton's theory (standard = inertial), as well as by any any physical theory such that: (i) 1+3 coordinates are needed and (ii) a final agreement between the "best observers" (those using the final "ideal instruments" for the theory) is possible.

Note. From a mathematical viewpoint, it is convenient to assume that S_p is *maximal*, in the sense that no bigger set of compatible observers S_p^α contains S_p (or, more exactly, that S_p is the intersection of all the maximal sets of compatible observers containing it). If S_p is not maximal then it determines univocally a new subset of such compatible observers $S_p^* \supset S_p$. All the elements in S_p^* can be regarded as physically equivalent to those in S_p. From a purely formal viewpoint, S_p^* can be regarded as the system of units shared by standard observers at p.

3.3 Third postulate: differentiability

Our last postulate is just a technical assumption on differentiability.

Postulate 3, P3. The mathematical objects assigned to each event $p \in M$ by using previous two postulates vary smoothly with p.

That is, the transition between the structures assigned to spacetime by standard observers at different points of the manifold will be *differentiable* (C^1) in the natural sense. We will use this postulate in Section 5, however, the mathematically precise "natural" sense of this postulate can be formulated rigorously a priori in terms of the bundle of linear frames [1].

Because of the problems in the gap between macroscopic and microscopic physics, one can discuss if P3 expresses a true property of spacetime or just a property of our measurements of spacetime.

3.4 An additional postulate: temporal orientability

Even though in our approach just previous three postulates are sufficient, the following one may be instructive.

Additional Postulate (Temporal Orientation). *For any two standard observers $O, \tilde{O} \in S_p$, necessarily: $\partial_t \tilde{t}|_p > 0$.*

Of course, this postulate is completely intuitive: it expresses just that the temporal coordinate $\tilde{t}$ of standard observer $\tilde{O}$ will increase with respect to the temporal coordinate t of any other standard observer O. However, it is not strictly necessary in our approach; so, when we use it, this will be said explicitly.

4 Mathematical development

Now, we will study the mathematical implications of our three postulates. We will be very concise and summarize it in the following three points.

4.1 Properties of the groups $O^{(k)}(4, \mathbf{R})$

Let $O^{(k)}(4, \mathbf{R})$, $k \in \mathbf{R}, k \neq 0$, be the group of 4×4 real matrices with preserve the matrix

$$I_3^{(k)} = \begin{pmatrix} k & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

by congruence, that is:

$$A^t \, I_3^{(k)} \, A = I_3^{(k)} \tag{4.1}$$

(^{t} denotes transpose). This definition is naturally extendible to $k = 0$, but in this case one has to assume $\det^2 A = 1$ in addition to (4.1). Moreover, taking inverses in (4.1), this equality is equivalent to:

$$A^{-1} \, I_3^{(1/k)} \, (A^t)^{-1} = I_3^{(1/k)}. \tag{4.2}$$

Now, putting $k = \omega (\equiv \pm\infty)$ we can define naturally $O^{\omega}(4, \mathbf{R})$ as the group of matrices satisfying

$$A^{-1} \, I_3^{(0)} \, (A^t)^{-1} = I_3^{(0)} \quad \text{and} \quad det^2 A = 1.$$

Summing up, identifying naturally $]-\infty, \infty[\cup \{\omega = \pm\infty\}$ with the circle S^1, we have defined a group of matrices $O^{(k)}(4, \mathbf{R})$ for any $k \in S^1$. For $k = 1$ (resp. $k = -1$) this group is the orthonormal group (resp. Lorentz group), and for $k \in]0, \infty[$ (resp. $k \in]-\infty, 0[$) $O^{(k)}(4, \mathbf{R})$ is conjugate to the orthonormal (resp. Lorentz) group. We will call $O^{(\omega)}(4, \mathbf{R})$ the *Leibnizian group* and $O^{(0)}(4, \mathbf{R})$ the *anti-Leibnizian group*.

Remark. Each $O^{(k)}(4,\mathbf{R})$ is a subgroup of the group of regular 4×4 matrices $Gl(4,\mathbf{R})$ and if $k\neq k'$:

$$\{\pm 1\}\times O(3,\mathbf{R}) = O^{(k)}(4,\mathbf{R})\cap O^{(k')}(4,\mathbf{R}) \; (=\cap_{\tilde{k}\in S^1} O^{(\tilde{k})}(4,\mathbf{R})). \tag{4.3}$$

4.2 Mathematical implications of P2

Let $O\equiv B_p = (\partial_t|_p, \partial_{x^1}|_p, \partial_{x^2}|_p, \partial_{x^3}|_p)$, $\tilde{O}\equiv \tilde{B}_p = (\partial_{\tilde{t}}|_p, \partial_{\tilde{x}^1}|_p, \partial_{\tilde{x}^2}|_p, \partial_{\tilde{x}^3}|_p)$ be two (instantaneous) standard observers at p, with the transition matrix:

$$A = \left(\begin{array}{c|c} \partial_t \tilde{t}|_p & \partial_{x^j}\tilde{t}|_p \\ \hline \partial_t \tilde{x}^i|_p & \partial_{x^j}\tilde{x}^i|_p \end{array}\right).$$

P2 says exactly:

$$A = \left(\begin{array}{c|c} \mathbf{a}_{00} & a_h \\ \hline a_v^t & \hat{\mathbf{A}} \end{array}\right) \qquad \Longrightarrow \qquad A^{-1} = \left(\begin{array}{c|c} \mathbf{a}_{00} & \tilde{a}_h \\ \hline \tilde{a}_v^t & \hat{\mathbf{A}}^t \end{array}\right),$$

where the common value $\mathbf{a_{00}}$ is any real mumber, $\hat{\mathbf{A}}$ is a 3×3 submatrix with transpose $\hat{\mathbf{A}}^t$, and $a_h, a_v, \tilde{a}_h, \tilde{a}_v$ are four triplets of real numbers. Moreover, if the additional postulate of Temporal Orientation holds then $\mathbf{a_{00}} > 0$. Working algebraically with the expression of A^{-1} is not difficult to prove:

Theorem. *Let S_p be a set of standard observers at $p\in M$.*

If the Postulate of Temporal Orientation holds then there exist a non-positive k ($k\in[\omega,0]$) such that

$$A\in O^{(k)}(4,\mathbf{R})$$

for any transition matrix A between standard observers in S_p.

Moreover, either k is unique or $A\in\{\pm 1\}\times O(3,\mathbf{R})$.

Remarks. (1) The number $c=\sqrt{|k|}$ admits the natural interpretation of *supremum of the relative velocitites between standard observers.* We can call c "*speed of light*" but, under our approach, this is just a name (we have not used electromagnetism!)

(2) If the Postulate of Temporal Orientation does not hold, the results are essentially equal, but now perhaps $k>0$. The only non-trivial difference is the existence of a "residual case" such that not all the transition matrices belong to a group $O^{(k)}(4,\mathbf{R})$. Nevertheless, in this case at most four observers $O^{(1)},\ldots,O^{(4)}$ and four constants $0<k_1<\ldots<k_4$ can be chosen such that the transition matrix A of any other observer O with some of the $O^{(i)}$ belongs to the group $O^{(k_i)}(4,\mathbf{R})$. This residual case is scarcely representative, and it will not be taken into account in what follows; however, it can be characterized completely, see [1][2].

Conclusion. Given a set of standard observers at p, S_p, then one of the two following possibilities must be taken into account:

1. There exist a unique $k\in S^1$ such that $A\in O^{(k)}(4,\mathbf{R})$ for any transition matrix A between standard observers in S_p.

2. Any such matrix A satisfies $A\in\{\pm 1\}\times O(3,\mathbf{R}) = \cap_k O^{(k)}(4,\mathbf{R})$.

[2] Essentially, the only modifications in what follows would be: (1) in Subsection 4.3, one must take into account the possibility that, when $k>0$ then it must be replaced by (at most) four $0<k_1<\ldots<k_4$ and four Euclidean products can be assigned, and (2) in Section 5, perhaps the Riemannian metric splits in (at most) four Riemnnian metrics, locally.

4.3 Mathematical structures preserved by a group $O^{(k)}(4, \mathrm{R})$

Recall that at, up to now, we have at each $p \in M$: (1) The set of (instantaneous) standard observers at p, that is, a set of basis of T_pM (each one $B_p = (\partial_t|_p, \partial_1|_p, \partial_2|_p, \partial_3|_p)$), and (2) for some $k \in S^1$, necessarily $A \in O^{(k)}(4, \mathbf{R})$, whenever A is any transition matrix between two such standard observers. Depending of the value of k, some of the following cases will occur:

1. Case $k < 0$ ($k \in \mathbf{R}$). Then a *Lorentzian scalar product* g_p can be defined on T_pM just declaring:

$$(g_p(\partial_\mu|_p, \partial_\nu|_p))_{\mu,\nu} = I_3^{(k)} \qquad \mu, \nu = t, 1, 2, 3.$$

   (Each base B_p is orthonormal up to the normalization of the first vector).

2. Case $k = \omega (= \pm\infty)$. For any given basis B_p, consider the dual basis $B_p^* = (dt_p, dx_p^1, dx_p^2, dx_p^3)$. Then:

   (a) dt_p is equal for any dual basis B_p^*, up to a sign.

   (b) An Euclidean scalar product h_p can be defined on Kernel dt_p ($= \mathrm{Span}\{\partial_1|_p, \partial_2|_p, \partial_3|_p\}$) just declaring:

$$(h_p(\partial_i|_p, \partial_j|_p))_{i,j} = I_3 \qquad i, j = 1, 2, 3.$$

   (I_3 denotes the identity matrix, thus, $(\partial_1|_p, \partial_2|_p, \partial_3|_p)$ is an orthonormal basis for h_p).

   Therefore, neglecting the question relative to the sign[3] of dt_p, a *Leibnizian (vector) structure* is obtained on T_pM, that is: a non-null one form $dt_p \neq 0$ plus an Euclidean metric h_p on its kernel.

3. Case $k = 0$. This case is mathematically completely analogous to the previous one, but "dual":

   (a) $\partial_t|_p$ is equal for any basis B_p, up to a sign.

   (b) An Euclidean scalar product h_p^* can be defined on Kernel $\partial_t|_p$ ($= \mathrm{Span}\{dx_p^1, dx_p^2, dx_p^3\}$) just declaring:

$$\left(h_p^*(dx_p^i, dx_p^j)\right)_{i,j} = I_3 \qquad i, j = 1, 2, 3.$$

   ((dx_p^1, dx_p^2, dx_p^3) is an orthonormal basis for h_p^*). Now, an *anti-Leibnizian structure* is obtained on T_pM, that is: a non-null vector $\partial_t|_p \neq 0$ plus an Euclidean metric h_p^* on its kernel.

4. Case $k > 0$ ($k \in \mathbf{R}$). Analogous to the first case but, now, g_p is *Euclidean.* This case cannot hold if the Postulate of Temporal Orientation holds.

5 Models of spacetime

In the last section, using P2 (and, implicitly, P1) at each event $p \in M$, we have assigned a mathematical structure on the corresponding tangent space T_pM (Lorentzian, Leibnizian, anti-Leibnizian or Euclidean); this structure depends on the value of the parameter $k (= k(p))$. Now, P3 says that this mathematical structure must vary differentiably from one point p to another one. Then, all the possible mathematical models of spacetime (compatible with our postulates) are obtained.

[3]This can be done canonically either working locally or considering the corresponding two-fold covering. Of course, if the Postulate of Temporal Orientation holds then the indetermination for the sign disappears.

1. *Points with $k(p) < 0$.* A Lorentzian metric (as in the the classical model of GR) is obtained. Recall that we also obtain the real function $c(p) = \sqrt{|k(p)|} > 0$ which says how the "speed of light" varies.

2. *Points with $k(p) = \omega$.* A Leibnizian structure is obtained i.e.: a non–vanishing 1-form Ω (*absolute clock*) plus a Riemannian metric h on the vectorial fiber bundle Kernel Ω. Recall:

   (a) When Ω is exact, $\Omega = dt$ then function t (unique up to a constant) is the *absolute time*[4]. Each slice $t = t_0$ (constant) is then the *absolute space* at time t_0.

   (b) In this case, when the Riemannian metric h on Kernel Ω is *flat* then we can say that *the absolute space is Euclidean* (locally).

   (c) Nevertheless, there is no a canonical "Levi-Civita connection" for a Leibnizian structure (this justify the name "Leibnizian", and is fully characterized in [2]).

   (d) At any case, Leibnizian structures generalize classical Newton's model of spacetime (see [2] for a detailed study).

3. *Points with $k(p) = 0$.* An anti–Leibnizian structure is obtained, i.e.: a non–vanishing vector field Z (*ether field*) plus a Riemannian metric h^* on the vectorial fiber bundle Kernel Z. Recall:

   (a) Mathematically this case is completely analogous (*dual*) to the previous case.

   (b) Physically, all the standard observers are... at relative rest!

4. *Points with $k(p) > 0$.* A Riemannian metric plus the real function $k(p) > 0$ are obtained.

Note. In previous classification, each $k(p)$ is assumed to be unique. Notice that, otherwise, all the transition matrices between standard observers at p belong to $\{\pm 1\} \times O(3, \mathbf{R})$, that is, $k(p)$ can be chosen as any value of S^1. But P3 implies that, if this happens at some p, then it will happen at any $p \in M$[5]. Thus, if $k(p)$ is not unique at some p then all previous structures can be assigned to spacetime, and TM admits a global splitting (this would be a model of spacetime with an absolute clock, absolute movement respect to the ether, etc.)

Remarks. (1) The possibility of transition among the four structures is possible, and controlled by function $k(p)$.

(2) The transition of the metric g between Lorentzian and Riemannian is carried out through regions where either (i) the metric degenerates (that is, $k = 0$), or (ii) the dual metric g^* on cotangent space T^*M canonically associated to g, degenerates ($k = \omega$).

(3) Additional structures to the metric have appeared: (a) in the non-degenerate regions, function $k(p)$, (b) in the degenerate regions, either the one form Ω or the vector field Z.

6 Conclusions

We can summarize our conclusions as follows.

[4]Of course, at each event p we have $\Omega_p = dt_p$, and the kernel of this 1-form is the "infinitesimal absolute space at p". But the equality $\Omega = dt$ does not hold necessarily even in a neighborhood of p; so, we must assume additionally that Ω is closed in order to find a local universal time. Moreover, Ω must be exact to find a (global) absolute time, with the corresponding absolute space (non-infinitesimal).

[5]Recall that dimension $O(3, \mathbf{R}) = 3 \neq 6 =$ dimension $O^{(k)}(4, \mathbf{R})$, and the dimension of the group determined by S_p should vary continuously with p.

About our approach:

1. We have introduced a *minimal postulational basis* for any non–quantum theory of spacetime.
2. This is *previous and simpler* than any other approach, as far as we know (in particular EPS).
3. It is applicable to Einstein's theory, to Newton's one and to less standard approaches, as the one by Ellis et al. or Logunov.

About our basic postulate $\boldsymbol{P2}$:

1. It ensures just the *possibility that, at the end, infinitesimal standard instruments can be constructed.* That is, at the end some observers (standard observers) will agree in their units to measure space and time, at least infinitesimally.
2. Once $P2$ is admitted, one has the experimental problem of finding the postulated standard observers (one has to choose or find the concrete theory of spacetime).

   At this level, axioms as those in EPS are useful. However, they can be *simplified or better understood* now. For example, the EPS condition of compatibility with the metric (the "extraneous element of the theory") becomes now completely natural: among our four structures, the other postulates in EPS are compatible only with a Lorentz metric.

About the deduced models of spacetime:

1. We obtain just four models, with the possible transitions among them.
2. The possibility of "variations of the speed of light" $c(p) = \sqrt{|k(p)|}$, also appears, but with a new interpretation (compare, for example, with [16]).
3. Signature changing metrics "à la Hartle-Hawking" also appears now naturally, collecting our four models. Moreover, we find *some new elements* for these metrics:
   - The degeneration in dual space of the metric, which seems as reasonable as the standard degeneration of the metric in tangent space[6], and generalizes Newton's spacetime.
   - In the degenerated regions, additional structures (absolute clock Ω, ether field Z), which preserve constant the dimension of the automorphism groups of the tangent space, $O^{(k)}(4, \mathbf{R})$.
   - Function $k(p)$, which controls not only the "speed of light" but also how the metric degenerate (compare, for example, with the discusion about absolute time in [11]).
4. On the other hand, Leibnizian structures has their own interest from both, the mathematical and the physical viewpoint [2].

Final remarks:

- When (Special or General) Relativity is explained, "gedanken experiments" are usually claimed. In these experiments, one sees that Newton's theory is not appropiate, and tries to discover the appropiate geometry of spacetime. Thus, in such gedanken experiments neither Newtonian nor Lorentzian geometries are imposed, but *our postulates* (including temporal orientation, of course) *are assumed implicitly for the "reasonable good" observers of the theory.* Our approach

[6]Only one of these two degenerations seems to have been considered in references on this topic; see at any case [4],[8], [9],[11].

shows that, then, it is not so surprising to arrive at a Lorentzian (or Lorentz-Minkowski) Geometry. In fact, one must arrive at a Lorentzian metric if: (a) our "speed of light" (supremum of relative speeds between standard observers) is identified with the speed of physical (electromagnetic) light in empty space, and (b) this speed must be a finite non-zero constant.

- Of course, it would be conceivable the existence of standard observers who use just some type of matter and forces (electromagnetism, gravity) and obtain a natural Lorentzian metric for spacetime (or any other of our structures), and the existence of a different set of standard observers who use a different type of matter or forces and obtain a *different* new Lorentzian metric for spacetime. Under this viewpoint, a proposal as Logunov's one [12] becomes natural.

- What about the quantum case? Of course, more radical proposals are possible in this case (take discrete cronons of time, quantize spacetime...) Nevertheless, recall that when physicists use currently Quantum Mechanics then either the interpretation of Copenhaguen is claimed, or path integrals are used; notice than even quantum non–locality is founded in local processes, measured locally [3]. That is, spacetime is a rather classical scenario for current Quantum Mechanics. Thus, at least from an operational viewpoint, our postulates seems again a minimum unavoidable and, therefore, our picture of spacetime as a manifold with a metric (signature changing, with additional structures) remains valid.

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