Irreversible Universe as Time and Space with Multifractal Dimensions

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The theory of time and space with the multifractal dimensions (fractional dimensions (FD)) of time and space ($d_\alpha,\ \alpha = t, \mathbf{r}$) is proposed. The FD of time and space points are determined (using the principle of minimum of functionals of the FD) by means of the energy densities of the Lagrangians of physical fields. For describing the behavior of the functions defined on multifractal sets the generalization of the fractional Riemann-Liouville derivatives $D_t^{d(t)}$ is introduced. The orders of the fractional differentiation depends on time and coordinates and are equal to the FD. For $d_t = const$ the generalized fractional derivatives (GFD) are reduced to ordinary fractional Riemann-Liouville integral functionals and when d_t is close to integer value the GFD can be represented by means of the derivatives or the integrals of integer order. The Euler equations are defined on multifractal sets of time and space with help of the GFD and are obtained using the principle of minimum of the FD functionals. For time and space with multifractional dimensions the method is proposed to investigate the equations of theoretical physics are generalized by means of the GFD. The generalized Newton's equations are considered as an example and it is shown that these equations coincide with the equations of classical limit of general relativity if $d_t \to 1$.
Some of remarks about the existence of repulsive gravitation are made and discussed. The possibility of the geometrization of all known physical fields and forces is demonstrated in the frame of the multifractal theory of time and space. The relative motions in the space with multifractal time is considered for the "almost inertial" frames of reference, i.e. for the case when the time is almost homogeneous and is almost isotropic, the fractional dimensions of time are close to integer ($d_t = 1 + \varepsilon(\mathbf{r}, t)$, $|\varepsilon| \ll 1$). The absolute frames of reference are present in the multifractal space-time model and the infringement of conservation laws are exist due to openness of all physical systems and inhomogeneous of time (the latter though small because of the smallness of ε in our domain of Universe) in considered model. The total energy of a body moving with $v = c$ is described (it is a finite value) and modified Lorentz transformations for space with multifractal time are formulated. The total energy of a body is reduced to the known formula of special relativity (as the considered theory at a whole) in the case of transition to usual time with dimension equal to unity. Some consequences of the theory of the multifractal time and space for the energy of bodies moving with speed of light are considered. It is shown the necessity for the bodies moving with velocity $v = c$ to have the energy of order $E \sim E_0 10^3 t^{-1/2}$ where t is the time of the body acceleration, E_0 is a rest energy. It is shown the presence of boundary for applicability of the principle of equivalence of general relativity. The irreversible equation of quantum mechanics in the multifractal universe is formulated. The more detailed theory of physical phenomena in the model of universe with multifractal time and space was considered in papers [1]–[21].

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1 Introduction

The problem of the nature of space and time is one of the most interesting problems of modern physics. Are the space and the time of our universe continuous? Why is time irreversible? What dimensions do space and time have? How does the nature of time reflected in the equations of modern physics? There are different approaches (quantum gravity, irreversible thermodynamics, extended quantum mechanics, synergistic and others approaches) which provide us with different answers to these questions.

In this paper the hypothesis about the nature of time and space based on the ideas of the fractal geometry [22] is offered. We suppose that our universe is the multifractal sets of time and space are defined on their the carriers of measure (the latter plays role of large vacuum born our universe). The corresponding mathematical method is presented for describing this hypothesis. It is based on using, as the one of the main characteristics of time and space, the idea about existence of the FD of time and space and, in connection with it, on the generalizations of Riemann-Liouville fractional derivatives for the case of multifractal sets. So the method and the theory are developed to describe the dynamics of functions are defined on the multifractal sets of time and space with the FD. The theory propagates the mathematical methods of fractional differentiation on the multifractal universe and coincide with usual theories if universe is not multifractal (i.e. FD is equal unity).

2 What Dimensions Do Time and Space Have in Our Universe: Integer or Fractional?

Nobody knows what the nature of time and space is in re. A posteriori, at the present time there are no the new physical data which may give enough substantiation for fundamental changing the physical picture of describing of time and space which one exist now. On the first look it seems for these purposes are quite enough of several variants of general relativity or quantum gravity (Logunov [28], Einstein [23], Prigogine [31] and so on). For the fundamental changing of the science paradigm existed now it is necessary the appearing of the theory which may forecast some of the new experiments which will contradict to known science paradigm (for example the possibility of bodies existing moving with velocities which one exceed the speed of light).

In this paper we consider the new paradigm for our universe: the multifractal universe consists of real time field and real space field with multifractal dimensions. This paradigm permits (it will be shown) the existence of any velocities for moving bodies, predicts the new phenomena for bodies moving with speed of light and predicts the experiments that may it discover. Thus the consequences of new paradigm may be experimentally examined. So our tusk is to construct in this paper the multifractal theory of time and space based on fractal geometry. Following [1], we consider both time and space as only real material fields existing in universe and generating all others physical fields.

We assume that each of them (time and space) is consisted of a continuous, but not differentiable bound set of small elements. Let us suppose a continuity, but not a differentiability, of sets of small time intervals (from which the time is consisted) and small space intervals (from which the space is consisted). Let's consider, at first, the set of small time intervals S_t (for the set of small space intervals the reasons are similar).

Let time is defined on the multifractal set from such intervals (determined on the carrier of a measure $\mathcal{R}_t^n$ or $\mathcal{R}_t^{n'}$ where n' is not integer). The each of intervals of this set (further we use the approximation in which the description of the each multifractal interval of these sets is characterized by mean time moment t for these intervals and name the each of these intervals as "points") is characterized by global fractal dimension (FD) $d_t(\mathbf{r}(t), t)$ and for different intervals the FD are different (because the d_t has not only the time dependence, but the spatial coordinates dependencies too). For the multifractal sets S_t (or S_r) the each set is characterized by global the FD of this set and by local the FD of this set (the characteristics of local the FD of time and space sets in this paper we do not research). In this case the classical mathematical calculus or the fractional (say, Riemann – Liouville) differential calculus [25] can not be applied in order to describe a small changes of a continuous function of physical values $f(t)$ defined on time subsets S_t, because the fractional exponents depend on time and coordinates. Therefore, we have to introduce the new integral functionals (both the left-sided and the right-sided) which are suitable to describe the dynamics of functions defined on multifractal sets (see [5]). Actually, these functionals are simple and natural the generalization of the Riemann – Liouville fractional derivatives and integrals:

$$D_{a+,t}^{d} f(t) = \left(\frac{d}{dt}\right)^n \int_a^t \frac{f(t')dt'}{\Gamma(n - d(t'))(t - t')^{d(t') - n + 1}}, \tag{1}$$

$$D_{b-,t}^{d} f(t) = (-1)^n \left(\frac{d}{dt}\right)^n \int_t^b \frac{f(t')dt'}{\Gamma(n - d(t'))(t' - t)^{d(t') - n + 1}}, \tag{2}$$

where $\Gamma(x)$ is Euler's gamma function, and a and b are some constants in time and space coordinates which may be selected on the time axes from ($-\infty$ to ∞). In these definitions, as usually, $n = \{d\}+1$, where $\{d\}$ is the integer part of d if $d \geq 0$ (i.e. $n-1 \leq d < n$) and $n = 0$ for $d < 0$. The functions under the integral sign we will consider as the generalized functions defined on the base space of finite or Gelfand functions [26]. Similar expressions can be written down for the GFD of the functions $f(\mathbf{r}, t)$ with respect to spatial variables $\mathbf{r}$, with $f = f(\mathbf{r}, t)$. The latter is being defined on the elements of the set $S_{\mathbf{r}}$ whose dimensions are $d_{\mathbf{r}}$. For an arbitrary $f(t)$ it is useful to expand the generalized function $1/(t-t')^{\varepsilon(t')}$ under the integral sign in (1)-(2) into a power series in $\varepsilon(t')$ when $d = n + \varepsilon$, $\varepsilon \to +0$ and write

$$\begin{aligned} D_{a+,t}^{d} f(t) = \left(\frac{d}{dt}\right)^n \int_a^t & \frac{f(t')}{\Gamma(n - d(t'))(t - t')} \\ & \times \left(1 + \varepsilon(t') \ln(t - t') + ...\right) dt', \end{aligned} \tag{3}$$

$$\begin{aligned} D_{b-,t}^{d} f(t) = (-1)^n \left(\frac{d}{dt}\right)^n \int_t^b & \frac{f(t')}{\Gamma(n - d(t'))(t' - t)} \\ & \times \left(1 + \varepsilon(t') \ln(t' - t) + ...\right) dt'. \end{aligned} \tag{4}$$

Taking into account that all functions here are real functions and the generalized function $\frac{1}{t}$ may be defined as $1/t = P(1/t) \pm \pi i \delta(t)$ or $1/t = P(1/t) \pm \pi \delta(t)$ (see [24]) or may be expanded by any methods on imaginary plane with arbitrary coefficient (we must define the non existent value of integral!), the singular integrals here can be defined by means of the rule (the one of many methods,for example)

$$\int_0^t \frac{f(t')}{t-t'}dt' = af(t),$$
$$\frac{\partial}{\partial t}\int_0^t \frac{f(t-t')}{t'}dt' = \frac{\partial}{\partial t}(f(t-\tau_{mean})\ln t)|_0^t,$$
$$\frac{\partial}{\partial t}\int_0^t \frac{f(t-t')}{t'}dt' = \frac{f(0)}{t} + a\frac{\partial}{\partial t}f(t), \tag{5}$$

where a is a real regularization factor. The good agreement of (3)-(4) with exact values given by expressions (1)-(2) can be obtained for large time and $\dim\varepsilon\dim \ll 1$ by fitting of the value of a. Instead of usual integrals and usual partial derivatives, in frames of the multifractional time hypothesis it is necessary to use the GFD integral operators in order to describe the small alteration of physical functions and variables. These functionals are transformed to ordinary integrals and derivatives if space and time dimensions are taken as integer, and they coincide with the Riemann-Liouville fractional operators if $d_i = const$. If the fractional dimensions can be represented as $d_i = n + \varepsilon_i(\mathbf{r}(t), t)$, $|\varepsilon| \ll 1$, it is also possible to reduce the GFD to the ordinary derivatives of integer order. Here we show this only for the case when $d = 1 - \varepsilon < 1$

$$D_{0+}^{1+\varepsilon}f(t) = \frac{\partial}{\partial t}\int_0^t \frac{\varepsilon(\tau)f(\tau)d\tau}{\Gamma(1+\varepsilon(\tau))(t-\tau)^{1-\varepsilon(\tau)}} \approx \frac{\partial}{\partial t}\int_0^t \frac{\varepsilon(\tau)f(\tau)d\tau}{(t-\tau)^{1-\varepsilon(\tau)}}. \tag{6}$$

Though for $\varepsilon \neq 0$ the latter integral is well defined and is real-valued, if it is expanded in power series in ε that leads to the singular integrals like (5)

$$A = \int_0^t \frac{\varepsilon(t')f(t')}{t-t'}dt'.$$

For the regularization of this integral we will consider it as defined on the space of Gelfand finite base functions [26] and take the real part of the common regularization procedure (the one of determined by the rules of (5))

$$A = a\varepsilon(t)f(t). \tag{7}$$

Thus we obtain

$$D_{0+}^{1+\varepsilon}f(t) \approx \frac{\partial}{\partial t}f(t) + \frac{\partial}{\partial t}\left[a\varepsilon(\mathbf{r}(t),t)f(t)\right] + \frac{\varepsilon(\mathbf{r},0)f(0)}{t}, \tag{8}$$

or if we take into account the part of the integral with $P(\frac{1}{t})$ in sense of its main value

$$D_{0+}^{1+\varepsilon}f(t) \approx \frac{\partial}{\partial t}f(t) + \frac{\partial}{\partial t}\left[a\varepsilon(\mathbf{r}(t),t)f(t)\right] + \frac{\varepsilon(\mathbf{r},t)f(t)}{t}(*),$$

where a is a regularization parameter. For the sake of independence of the GFD from this constant it is useful in the following parts of paper to choose the factor β_i (on which ε depends linearly) is proportional to a^{-1}.

It can be shown that for large t the exact expressions for the terms in (1)-(2) which are proportional to the ε are very close to the approximate expression given by the Eq.(8) (or Eq.$(*)$) provided by a special choice for parameter a was made and $t = t_0 + (t - t_0)$, $t - t_0 \ll t_0, a \sim \ln t \sim \ln t_0$, where t is the full time from the moment when big-bang had happened, t_0 is the mean time of the existence of universe and $t - t_0$ is the current time. It is necessary to pay attention on the latter addendum in the right-hand part of the equation (8) or Eq.$(*)$. It is very small and for current time is proportional to t_0^{-2}. It may be omitted often, but it describes the irreversibility of all physical equations and of all processes existing in our universe, including the arrow of time (see [1], [15], [6]). So this member has the principal importance only for philosophical questions for domain of universe where $\varepsilon \ll 1$.

3 The Equations of Physical Theories with the Multifractal Time and Space

The equations of the dynamics of physical fields, particles and so on can be obtained from the minimum principle of the fractional dimensions functionals. To do this, let us introduce the fractional dimensions functionals of space and time $F_\alpha(...|d_\alpha(\mathbf{r}))$, $\alpha = t, \mathbf{r}$. These functionals are quite similar to the free energy functionals, but now it is the fractional dimensions (FD) that plays the role of an order parameter (see also [1], [17]). Assume further that FD d_α is determined by the Lagrangian energy densities $L_{\alpha,i}, (i = 1, 2, ..., \alpha = t, \mathbf{r})$ of all fields $\psi_{\alpha,i}$ and $\Phi_{\alpha,i}$. The last describe the particles and the interactions in the point $(\mathbf{r}, t)$(the currents model of interactions of different fields is used and the dividing of the fields on particles fields and interactions fields is not essential)

$$d_\alpha = d_\alpha[L_{\alpha,i}(\mathbf{r}, t)]. \tag{9}$$

The equations which govern by the d_α behavior can be found by minimizing these functionals and it leads to the analogy of Euler's equations which one is written down in the terms of the GFD (the latter are defined in (1)-(2))

$$D^{d_\alpha}_{+,L_{\alpha,i}(x)} d_\alpha - D^{d_\alpha}_{-,x} D^{d_\alpha}_{+,L'_{\alpha,i}(x)} d_\alpha = 0 \tag{10}$$

Substitution in these equations the GFD (instead of usual derivatives) and specification of the selection for F of dependencies at d_α and selection the relations between d_α and L_α (the latter can corresponds to the well known quantum field theory Lagrangians) makes possible to write down the functional dependencies of $F[L]$ in the form (a, b, c are unknown functions of L or constants, L_0 is infinitely of large energy density of the measure carrier $\mathcal{R}^n$)

$$F(...|d_\alpha) = \int dL_\alpha \left\{ \frac{1}{2}[a(L_\alpha)\frac{\partial d_\alpha}{\partial L_\alpha}]^2 + \frac{b(L_\alpha)}{2}(L_\alpha - L_{\alpha,0})d_\alpha^2 + c(L_\alpha)d_\alpha \right\}, \tag{11}$$

or

$$F(...|d_\alpha) = \int d^4L_\alpha \left\{ \frac{1}{2}[a(L_\alpha)\frac{\partial d_\alpha}{\partial L_\alpha}]^2 + \frac{b(L_\alpha)}{2}(L_\alpha - L_{\alpha,0})d_\alpha^2 + \frac{1}{4}c(L_\alpha)d_\alpha^4 \right\}. \tag{12}$$

The next equations determine the value and functional dependencies of the fractional dimensions (these equations follow from comparing the variations of Eqs.(11)-(12 to zero) and read

$$\frac{\partial}{\partial L}\left(a(L)\frac{\partial d_{t,\alpha}}{\partial L}\right) + b(L)(L - L_0)d_\alpha + c(L)d_\alpha^2 = 0. \tag{13}$$

or

$$\frac{\partial}{\partial L_\alpha}\left(a(L_\alpha)\frac{\partial d_{t,\alpha}}{\partial L_\alpha}\right) + b(L_\alpha)(L_\alpha - L_{0,\alpha})d_\alpha^2 + c(L_\alpha)d_\alpha^4 = 0 \tag{14}$$

For non stationary processes it is necessary to add the time derivative of the d_α in the right-hand side of Eqs.(13)-(14). When we neglect by the diffusion of d_α processes in the space with energy densities given by the Lagrangians L, we can define the $L_\alpha - L_{\alpha,0} = \tilde{L}_\alpha \ll L_{\alpha,0}$ with $\tilde{L}_\alpha$ as the over vacuum energy density and for the simplest case the (13) yields ($\alpha = t$, $L_{t,i} \equiv L_i$)

$$d_t = \tilde{L}_t = 1 + \sum_i \beta_i L_i(t, \mathbf{r}, \Phi_i, \psi_i). \tag{15}$$

More complicated dependencies of d_α at $L_{\alpha,i}$ are considered in the [1]. Note that the relation (15) (and the similar expression for $d_\mathbf{r}$) does not contain any limitations on the value of $\beta_i L_i(t, \mathbf{r}, \Phi_i, \psi_i)$

unless the such limitations are not imposed on the corresponding Lagrangians, and therefore the d_t can reaches any whatever high or small values. The principle of minimum of the fractional dimensions functional consists in the requirement for the F_α variations to vanish under variation with respect to any field. In the theory of the multifractal universe, this principle produces, in his turn, the principle of energy minimum (for any type of the fractional dimensions dependencies of the Lagrangian energy densities). It allows to receive the Euler's-like equations with the generalized fractional derivatives for functions $f(y(x), y'(x))$ which one describe the behavior of physical value f that depends from the physical variables y and the generalized fractional derivatives $y' = D_{+,x}^{d_\alpha} f$. For this variation principle we have

$$\delta F_{t,y_i} \sim \delta d_{t,y_i} = 0, \tag{16}$$

$$\delta_{y_i} d_\alpha(f) = \delta_{y_i} L_{\alpha,i}(f) = 0, \qquad \alpha = \mathbf{r}, t, \tag{17}$$

$$D_{+,y_i(x)}^{d_\alpha} f - D_{-,x}^{d_\alpha} D_{+,y_i'(x)}^{d_\alpha} f = 0. \tag{18}$$

The boundary conditions have form

$$D_{+,y_i'(x)}^{d_\alpha} f \Big|_{x_0}^{x_1} = 0 \tag{19}$$

In these equations the variables x stand for either t or $\mathbf{r}$ (the latter takes into account the fractionality of spatial dimensions), $y_i = \{\Phi_i, \psi_i\}$, $(i = 1, 2, ...)$, $L_{\alpha,i}$ are the Lagrangian energy densities of fields and particles. Here the f can be of any mathematical nature (scalar, vector, tensor, spinor, etc.) and the modification of these equations for the functions f of more complicated mathematical structure does not encounter any principal difficulties. Any of the known in theoretical physics Lagrangians of fields and their sums may be used for the Lagrangians $L_{\alpha,i}$ and it is possible to take into account (as usual by the currents method) the interactions between different fields.

The Eqs.(18) allow to obtain the generalizations of all known the equations of physics (Newton's, Schrödinger's, Dirac's, Einstein's equations and etc.) and the similar equations for time and space with fractional space dimensions ($\alpha = \mathbf{r}$). Such the generalized equations extend application of the corresponding physical theories for cases when the time and the space are defined on the multifractal sets, i.e. these equations describe the dynamics of physical values in the time and the space with fractional dimensions. The Minkowski-like or the Riemann-like space-time with the fractional (fractal) dimensions for the case $d_\alpha \sim 1$ can be defined on the flat continuous Minkowski or Riemann space-time (i.e. in that case the measure carrier is Minkowski or Riemann space-time $\mathcal{R}^4$ or R^5). The chosen the carrier of measures may be selected and for more general cases when the values of d_α are arbitrary. The generalized equations of the fractal physical theories can be reduced to well known the equations of physical theories for small energy densities or, which is the same, for small forces ($d_t \to 1$) if we neglect by corrections arising due to fractionality of space and time dimensions (a number of examples from classical and quantum mechanics and general theory of relativity were considered in [1], [13]). For the statistical systems of many classical particles the GFD help to describe the influence of the fractal structures arising in systems on time behavior of distribution functions [17].

Now we make one remark. In the multifractal model of time and space is considered the all physical parameters and constants (such as an electron charge, Planck constant and so on) are functions of the FD of time and space (it is obvious from the dimensional point of view). For simplicity when we take it into consideration we do not write these dependencies in open form. As the algorithm of receiving of these parameters from the theory is unknown now, the only possibility now is to take these dependencies in mind, though for the large forces all these parameters may be very strong changed. For the FD close to topological value the latter dependencies may be described as good approach by the next form for parameter a: $a \to a(1+\varepsilon \ln a)$. We think the such corrections is not essential in that case (but we will take it in consideration in other papers) and do not change the main results.

4 Generalized Newton's Equations

In this paragraph we write down the modified Newton's equations generated by the multifractal time fields in the presence only of gravitational forces

$$D_{-,t}^{d_t(r,t)} D_{+,t}^{d_t(r,t)} \mathbf{r}(t) = D_{+,r}^{d_r} \Phi_g(\mathbf{r}(t)), \tag{20}$$

$$D_{-,r}^{d_r} D_{+,r}^{d_r} \Phi_g(\mathbf{r}(t)) + \frac{b_g^2}{2} \Phi_g(\mathbf{r}(t)) = \kappa. \tag{21}$$

In the Eq.(21) the constant b_g^{-1} has order of size of universe and is introduced with purpose to extend the class of functions on which the generalized fractional derivatives concept is applicable. These equations do not present a closed system because of the fractionality of spatial dimensions. Therefore we approximate the fractional derivatives as $D_{+,\mathbf{r}}^{d_\mathbf{r}} \approx \nabla$, i.e. by usual space derivatives. Now we can determine the d_t for distances much larger than the gravitational radius r_0 (for the problem of a body motion in the field of spherical-symmetric gravitating center) as (see Eq. (15) and [1] for more details)

$$d_t \approx 1 + \beta_g \Phi_g. \tag{22}$$

We neglected by the fractional parts of spatial dimensions and by the contributions from the term with b_g^{-1}. Than take the β as $\beta_g = 2c^{-2}$) and using the energy conservation law (now this law is only approximate law though very good for $d_t \sim 1$, since our theory, and mathematical approaches have been used apply only to open systems) we obtain

$$\left[1 - \frac{2\gamma M}{c^2 r}\right] \left(\frac{\partial r(t)}{\partial t}\right)^2 + \left[1 - \frac{2\gamma M}{c^2 r}\right] r^2 \left(\frac{\partial \varphi(t)}{\partial t}\right)^2 - \frac{2mc^2}{r} = 2E. \tag{23}$$

Here we used the approximate relation between the generalized fractional derivative and usual integer-order derivative (8) with $a = 0.5$ and the notations corresponding to conventional description of the motion of mass m near gravitating center M. The value $a = 0.5$ follows from the regularization method is used and alters if we change the latter.

The Eq.(23) differs from the corresponding classical limit of the equations of general theory of relativity by the presence of the additional term in the first square bracket of the (23). This term describes velocity alteration during gyrating and is negligible in the case, when the perihelia gyrating of Mercury is calculated. If we neglect by it, then Eq.(23)is reduces to the corresponding classical limit of general relativity equation. For large energy densities (e.g., gravitational field at $r < r_0$) Eqs.(10), Eqs.(18), Eqs. (20)–(21) contain no divergencies [1] since the integral-differential operators of the generalized fractional differentiation will be reduced to the generalized fractional integrals (see (1)). Note, that if we choose for the fractional dimensions $d_\mathbf{r}$ in the GFD $D_{0+,\mathbf{r}}^{d_\mathbf{r}}$ Lagrangian dependencies in the form $L_{\mathbf{r},i} \approx L_{t,i}$, it yields in the equations (18) the additional factor equal 0.5, so the latter appears in the square brackets in (23). The latter additional factor can be compensated by fitting factor β_g.

5 The Fields are Originated by the Fractional Spatial Dimensions ("Temporal" or "vacuum" Fields)

If we want to take into account the fractional spatial dimensions ($d_x \neq 1,\ d_y \neq 1,\ d_z \neq 1$) in the Eqs.(10)-(19), we arrive to a new class of equations which describe the new physical fields (we name them as "temporal" fields or "vacuum" fields because it may be shown their relation to fields with negative pressure) generated by the space with fractional dimensions. These equations are quite similar to the corresponding equations which appear due to the fractionality of time dimensions and were considered earlier. In the Eqs.(10)–(19) we must select $x = \mathbf{r}, \alpha = \mathbf{r}$ and the fractal dimensions $d\mathbf{r}(t(\mathbf{r}), \mathbf{r})$ will obey Eq.(15) with t being replaced by $\mathbf{r}$. For example, for time $t(\mathbf{r}(t), \mathbf{r}))$, potentials $\Phi_g(t(\mathbf{r}), \mathbf{r})$ and $\Phi_e(t(\mathbf{r}), \mathbf{r})$ (the analogue of gravitational and electric fields) the equations are analogous to Newton's equations read (here the spatial coordinates play the role of time)

$$D_{-,\mathbf{r}}^{d_{\mathbf{r}}(\mathbf{r},t)} D_{+,\mathbf{r}}^{d_{\mathbf{r}}(\mathbf{r},t)} t(\mathbf{r}) = D_{+,t}^{d_t} \left(\Phi_g(t(\mathbf{r})) + e_r m_r^{-1} \Phi_e(t(\mathbf{r})) \right), \tag{24}$$

$$D_{-,t}^{d_t} D_{+,t}^{d_t} \Phi_g(t(\mathbf{r})) + \frac{b_{gt}^2}{2} \Phi_g(t(\mathbf{r})) = \kappa_r, \tag{25}$$

$$D_{-,t}^{d_t} D_{+,t}^{d_t} \Phi_e(t(\mathbf{r})) + \frac{b_{et}^2}{2} \Phi_e(t(\mathbf{r})) = e_r. \tag{26}$$

These equations should be solved together with the generalized Newton's equations (20)-(21) for $\mathbf{r}(t)$).

With the general algorithm proposed above, it is easy to obtain the generalized equations for any physical theory in the terms of the GFD. From this consideration it follows also that for every physical field originated by the time with the fractional dimensions there is the corresponding the field originated by the fractional dimensions of space. These new fields were referred to as "temporal" fields and they obey Eqs.(10)–(19) with $x = \mathbf{r},\ \alpha = \mathbf{r}$. Then the question arises: do these equations have any physical sense or can these new fields be discovered in certain experiments? Do these fields have relation to "vacuum fields" with negative pressure or may be they coincide with them? The problem now is open. It is useful to consider the next possibility. If the FD $L_{t,i}$ and $L_{\mathbf{r},i}$ can not be separated because of presence of strong interaction between the considered fields and because of the common time-space FD $d_{\mathbf{r},t}$ (for example, it may be case, when $L_{t,i} \approx L_{\mathbf{r},i}$ and the equality $d_{\mathbf{r},t} \approx (d_t + d_{\underline{\mathbf{r}}})$ is very rude approach), then no individual the new fields are generated, because it is impossible to separate them from the common complicated time-space physical fields. In that case there are exist only the new composed time-space fields in the fractional time and space. It seems that the such case corresponds to the FD with strong differences of their the FD from topological dimensions of time and space (and strong influence of the time FD on the space FD), i.e. the $1 + \varepsilon_{r,t} \simeq 1$ or more.

So, this is the case when the fractal dimensions of time and space $d_{t,\mathbf{r}}$ can not be divided on $d_t + d_{\mathbf{r}}$ because of the strong influences one on the other of the time FD and the space FD and the member are omitted which describes their interaction is not small. Thus, the time and space fractal sets can not be divided too. That case is very complicated for rigorous mathematical consideration and corresponds to the domain of universe with the essential fractionality of time space dimensions, because the FD of time and space $d_{\mathbf{r},t}$ are non divided and how in that case to define the GFD with respect to t and $\mathbf{r}$ is not clear (probably, it needs to introduce the new variables which one are consist from time and space variables). In the trivial case when the relation $d_{\mathbf{r},t} = d_t + d_{\mathbf{r}}$ is postulated there are no principle difficulties. We will consider only the case when $d_{\mathbf{r}} \ll d_t$. So the possible treatment of "temporal" fields as vacuum fields with negative pressure lay out of this particle though are very attractive. This point of view will be considered in the special paper connected with the picture of universe birth in the frame of big-bang and the multifractional dimensions of time and space.

6 Can Repulsive Gravitational Forces Exist in the Multifractal Universe?

In general theory of relativity there are no repulsive gravitational forces which one existence are possible without a change of Riemann space curvature (a metric tensor must be changed). In the frames of the multifractal time and space model, even when we can neglect by the fractional dimensions of space coordinates originated by Eqs.(24) the equation for $t(\mathbf{r})$ follows (for spherically-symmetric mass and electric charge distributions)

$$m_r \frac{d^2 t(\mathbf{r})}{d\mathbf{r}^2} = m_r \frac{d}{dt} \sum_i \Phi_i(t) \approx -\frac{m_r k_r}{t^2} \pm \frac{e^2}{ct^2} \tag{27}$$

with accuracy of the order of b^2. Here the m_r is the analogue of the ordinary mass m_t in the time space and corresponds to spatial inertia of time flow alteration with coordinate changing (may be it is possible that the m_r is proportional to ordinary mass up to a dimensional factor). Thus in the multifractal universe exists the new sort of masses: the masses born by the inhomogeneous of times flow. The Eq.(27) describes the change of time flow velocity from one space point to the other space point. The latter depends of the "temporal" forces (the new type of forces) and indicates that in the presence of physical fields time does not flow uniformly in different regions of space, i.e the time flow is irregular and heterogeneous (see also Chapter 5 in [1]). The equations like Eq.(27) introduced in the time space are connected with following from our model consequences (see Eqs.(10)–(19)) about certain the equivalence of time and space and so connected with possibility to describe properties of time (a real time field is generating all other physical fields except "temporal") by the methods used to describe the characteristics of space. Below we show that the usual gravitational field if it consider in presence of its the "temporal" analogous force yields way to the existence of gravitational repulsion. This repulsion is proportional to the third power of velocity. Indeed, the first term in the right-hand side of (27) is the analogous of gravity in the space of time (the "temporal" field). Newton's equations have form, if neglect by fractional corrections to dimensions and take into account both usual and "temporal" gravitation forces

$$\frac{d^2\mathbf{r}}{dt^2} = m^{-1}\mathbf{F}_{\mathbf{r}} + m_r^{-1}\mathbf{F}_t = -\frac{\gamma M}{\mathbf{r}^2} + \frac{k_r}{c^2 t^2}\left(\frac{d\mathbf{r}}{dt}\right)^3. \tag{28}$$

The criteria for the velocity which one divides the regions of attraction and repulsion reads

$$\left(\frac{d\mathbf{r}}{dt}\right)^3 = \left(\frac{\gamma M}{\mathbf{r}^2}\right)\left(\frac{c^2 t^2}{k_r}\right)^{-1}, k_r = \gamma_r M_r. \tag{29}$$

Here $\mathbf{r}(t)$ must also satisfy the Eqs.(20)-(21), γ_r and M_r analogies of γ and M for space of t. Introducing gravitational radius $r_0 = 2\gamma M c^{-2}$ and $t_0 = 2\gamma_r M_r$ (the latter is the "temporal" gravitational radius similar to the conventional radius r_0, M_r is induced by the temporal inhomogeneous center), we can rewrite the (29) as follows

$$\left|\frac{d\mathbf{r}}{dt}\right| = v = c\sqrt[3]{c\frac{t^2}{r^2}\frac{r_0}{t_0}}. \tag{30}$$

In the latter two relations we use designations: the r is the distance from a body with mass m to the gravitating center, the t is the time difference between points where a body and a gravitating center are situated, the $m_r = m/c$, $\kappa_r = \kappa_t/c$. If we admit that r_0 and t_0 are related to each other as $r_0 = \sigma c t_0$, $(\sigma < 1)$ then the necessary condition for the dominance of gravitation repulsion will be $rt^{-1} < \sigma c$. It is not clear whether this criteria is only a formal consequence of the theory or it has some relation to reality, so it is not clear now does gravitational repulsion exists in the multifractal universe? For $rt^{-1} \simeq v_{mean}$ and for $v \simeq v_{mean}$ there are small intervals of velocities less then c where criteria (30) is fulfilled (though the multifractal theory of time and space allows movements with velocities exceeding speed of light). Nevertheless it is undoubtedly, there are possibilities to introduce (though, may be, only formally) the dynamic gravitational forces of repulsion (as well as dynamical repulsive forces of any other nature, including nuclear) in the frames of the multifractal theory of time and space. We stress once more that the repulsion forces is introduced above have only dynamical nature, increases with increasing of velocities and originated by the inhomogeneous of time flow.

Let us see one the special case of Eq.(27) when the right hand side of this equation is equal constant and we treat the time point t as an arbitrary point of real time field. How will change the time at this point with alteration of space coordinates? It is necessary to take into account the alterations of space coordinate $x(t)$ with time too (in simplest case the latter may be chosen as $x \sim v_0 t$ were v_0 is the velocity of expansion of space is defined by big-bang nature). Then after one integration and after substitution of the relation mentioned above for $x(t) \sim t$) we receive (for the case of one coordinate x:

$$\frac{\partial t}{\partial x} = H_t t,$$

where t is the difference of the life times of the objects of observation and the same for spectator, H_t is the analogy of Hubble constant (see in detail about it and connection H_t with Hubble constant in [12]). This relation is the analogue of known Hubble law for inhomogeneous time and gives the additional the redshift

$$\lambda_t = \frac{1 + \frac{t}{t_0}}{\sqrt{1 - (\frac{t}{t_0})^2}},$$

where t_0 is the time of our universe existence. The detailed considerations of this relation (it was received from other point of view) there are in [12] and was used in the mentioned paper for the explanation of Arp galaxies problem. The Hubble's law may be received from Newton's equation (20) by the same method if take into account the relation $t = v_0^{-1} x$. The latter is born by the expansion of time-space too. Some remarks about the problems of cosmic vacuum are born by the recent discoveries of the new class of super new stars with strange order of their luminosities [39].

The last may be interpreted as the increase of the speed of universe extensions that contradicts to standard universe model. The model of cosmic vacuum (with anti gravitation forces) improve it if introduce the necessary density of vacuum energy. The extension of universe with increasing velocity deduced by means of the researching of the luminosities of the very far super new stars may be of cause exist, but it seems not so convincing if the relation for $\frac{dt}{dx}$ takes place. Then the addition red shift are born by it appears (see [12]) and connection the red shift with the velocities of extension (Doppler's red shift) ceases to be ambiguous. Unfortunately, except for Doppler's red shift there are many other physical reasons resulting in red shift (the one of them is marked above, the other is the red shift of Logunov's the theory of gravitation [28] are born by the dependence of gravitation field from time and so on). So, it seems that the role of cosmical vacuum in modern astrophysical theories is realized and understand not completely or ambiguous. From the point of view of the multifractal theory of time and space the good candidate on the role of cosmical vacuum fields may be the mentioned above the "temporal" fields because relation between energy and pressure for them (see 28) have correct sign (it will be research in detail in the special paper).

7 Geometrization of All Physical Fields and Forces

The multifractal model of time and space allows to consider the fractional dimensions of time d_t and space $d_{\mathbf{r}}$ (or undivided the FD, i.e. $d_{t\mathbf{r}}$, though I think the last case has very little probability in our domain of the multifractal universe because of smallness of the fractional corrections to the topological dimensions of space and time) as the source of all physical fields (see the (15)) (including, in particular, the case when the flat (not fractal) Minkowski space-time $\mathcal{R}^4$ is chosen as a measure carrier). From this point of view all physical fields are the consequence of the appearance of the fractionality of the time and space dimensions. All physical fields and forces are existing in the considered model of the multifractal universe with the multifractal geometry of time and space as far as the real multifractal fields of time and space are exist. Thus in our model all physical fields are real as far as our model of real multifractal fields of time and space correctly predicts and describes the physical reality. But since all fields are determined in this model by the value of the fractal dimensions of time and space, all physical fields and forces appear as the geometrical characteristics of the fractal time and space (Eq.(15), Eqs. (10)-(19)). We see that in the multifractal model all physical fields is determined by the fractal dimensions of time and space and are arised as the geometrical characteristics of the fractal geometry (i.e. as the characteristics of the fractal or the fractional dimensions) of time and space. Therefore there is the complete geometrization of all physical fields based on the idea of the FD of time and space, the hypotheses about the minimum of the functional of the fractal dimensions and the GFD that is used in this model. Note that this geometrization is not determined by moving bodies along geodesic lines, because what the latter means in the multifractal geometry where there are no the rigorous trajectories in the time and space is not clear (in any case for me).

The origin of all physical fields is the consequence and the result of the appearance of the fractional dimensions of time and space. It is possible to say that the complete geometrization of all fields which take place in our model of multifractal time and space is the consequence of taking in consideration (and describing by the GFD operators) of the complicated structure of multifractal time and space as the multifractal sets of the multifractal subsets S_t and $S_{\mathbf{r}}$ with global and local FD. The fractional dimensions $d_{\mathbf{r}}$ of space leads to the new class of fields and forces (see Eqs. (10)-(19) with $\alpha = \mathbf{r}$). For the special case of integer-valued dimensions ($d_t = 1,\ d_{\mathbf{r}} = 3$) of the multifractal sets of time and space the sets S_t and $S_{\mathbf{r}}$ coincide with Minkowski's or Riemann's or some other measure carriers $\mathcal{R}^4$. From the Eq.(15) it follows then that neither particles nor fields can exist in the world with integer dimensions $d_t = 1, d_{\mathbf{r}} = 3$. Thus for example the four-dimensional Minkowski or Riemann worlds becomes an ideal physical vacuum (for the FD $d_\alpha > 1$ the exponent of R^n may has value $n > 4$). On this vacuum the multifractal sets of time and space (S_t and $S_{\mathbf{r}}$) are defined with their fractional dimensions. We pay attention that this vacuum is only non essential part of "the Vacuum" which born our universe, when "big-bang" is happened (more detailed see in [15]-[16]). The latter is generated all our multifractal universe with real time and space fields and all physical forces and particles.

Now the following question may be asked: what is the reason (or reasons) for appearance of the dependence of the fractional dimensions of time and space upon Lagrangian densities in the considered model of fractal theory of time and space? One of the simplest hypothesis that seems may to explain this fact is assumption that the appear of the fractional parts in the time and the space dimensions with the dependencies upon Lagrangian densities originate from certain deformations or strains in the spatial and time sets of our universe when big-bang is happened. Then, the measure carrier of our universe (universe is consisted of real time and space fields) caused by the influence of the real time field on the real space field and vise versa (we remember that generation of physical fields caused by deformations of complex manifolds defined in twistor space is well known [29]) born this deformations and strains. If we assume that the multifractal sets S_t and $S_{\mathbf{r}}$ are complex manifold (the complex parts of the dimensions of time and spatial points can be compacted and because it are concealed for observations), then the deformation (for example) of complex-valued sets S_t under the influence of spatial points sets $S_{\mathbf{r}}$ results in appearing of spatial energy densities in time dimensions (that is a generation of physical fields (see [29])). T

The fractional dimensions of space appear (under the influence of S_t sets deformations) and yield the new class of fields and forces. It can be shown also that for small forces (e.g., for gravity at distances much larger then gravitational radius) the generalized fractional derivatives (1)-(2) can be approximated by covariant derivatives in the effective Riemann space [1] and by the covariant derivatives of the space of the standard model of elementary particles theory [4] (with corrections which take into account the fields generating and characterize the openness of world at a whole [6],[9]). All this allows to speak about the natural insertion of the mathematical tools of GFD offered , at least for $\varepsilon \ll 1$, in the structure of all modern physical theories (note here, the theory of gravity as a theory of real gravitational fields with a spin 2 was presented in [28] and this theory is not only very serious competitor to general theory of relativity but in some aspects surpass the latter). Note also, that some of problems within the framework of the theory of the multifractal time and space (classical mechanics, non relativistic and relativistic quantum mechanics) were considered in [1]-[21].

The latter remark we make about the space fractional dimensions. The energy of any state in the world with the fractional space dimensions has imaginary (and vector-like) parts, so the "temporal fields" or "vacuum" fields must be probably not stationary. Interpretation of energy with imaginary parts for "temporal" fields needs in new understanding (though it may relate only to very small domain of space).

8 The Theory of "Almost Inertial" Systems (Can a Particle's Velocities Exceed Speed of Light in Empty Space with the Multifractional time?)

In this paragraph we consider some the questions of the classical relativistic theory of bodies moving in the multifractal universe and some the special question about restrictions of using in space with the multifractal time of the theory of special relativity (SR). The special relativity theory is the one of physical theories which are base of modern physics. It has well experimental foundation in the large area of reached velocities and energies. It is the working theory of modern physics and its results widely use in science and technique. Nevertheless, as any physical theory created by men, SR has the boundaries of applicability (only an inertial systems of reference, though some of physicists not agree with it). As far as we know the problem of energy boundaries SR for any moving body was not analyzed in detail. At the same time the tends of the energy of a moving body when it's velocity reaches the values of speed of light to infinity call doubts in the applicability of SR in this area of energies, as the occurrence of infinity in physical theories always testifies about their interior deficiencies.

As well known the special relativity theory is the theory of inertial systems and for such systems the answer to the question posed in the title of this paragraph is negative. In real nature an ideal inertial systems do not exist. It allows to raise the following problem: is it possible to develop the theories of systems are analogously to inertial systems (the "almost" inertial systems) which would include, as a special case, the special relativity, but at the same time would allow the motion of usual particles (not taxions) with any velocity and, in the same time, with real energy? Obviously, in order to invent such the theory it is necessary to refuse from the rigorous validity of the SR postulates: the homogeneous of space and time, the invariance of speed of light and the Galilean's invariance principle.

In this paragraph is presented the example of such theory based on the concepts of the time and space with fractional dimensions (FD)(the latter theory is developed in the theory of multifractal time and space [1]-[21]). The purpose of this part of paper is to present the theory of almost inertial systems in the space with the multifractal time (the fractional corrections to space dimensions are omitted). This theory has sense only in the case when our universe consists from the real fields of the multifractal time and space (let us suppose that which one it consists) i.e. of the time and the space with the fractional dimensions. It will be shown that in such universe the motions of bodies with arbitrary velocities (including the velocities more then speed of light) are possible and there are no energy infinity at $v = c$.

At the same time if the energy of a body is reached as a result of it's motion (not a collision between particles) are smaller than $E_0(a_0(t))^{-0.5}$ (or $\simeq E_0 10^3 sec^{0.5}\ t^{-1/2}$ on the surface of earth) where t is the time of acceleration of a particle up to such energies, E_0 is a rest energy, a_0 is defined by the FD of time) all results of the theory coincide with the results of SR and thus do not contradict known experiments. The differences between our theory and SR appear only if the energy of moving body exceeds or equal the energy $\sim E_0 10^3\ sec^{1/2} t^{-1/2}$. The theory is based, but as a good approach only, on the principle of the constancy of speed of light, on the invariance of modified Galilean and Lorentz transformations laws. This theory is not a generalization of SR, because any SR generalizations are no need in the domain of validity of SR (an inertial systems of reference). This theory describes the relative movements only in the almost inertial systems because in the multifractal universe the inertial systems are not exist, and, thus, the theory does not contradicts SR and coincide with it for the case of inertial systems (topological universe). For constructing the theory it is necessary to refuse from rigorous realization of SR postulates: homogeneous of space and time, constancy of speed of light, Galilean relativity principle. In an inhomogeneous space and time, if the inhomogeneous are small, the motion of bodies will be almost inertial, and velocity of light is almost stationary value.

This paragraph contains the example of the theory based on the time and space with the fractional dimensions (FD). The theory use the ideas of fractal geometry for description of time and space characteristics (the theory of multifractal time and space is considered in the above paragraphs and in [1]-[21]. The consideration is bounded by the case when the values FD are a little bit distinguished from the integer dimensions. In the multifractal universe time and space are real inhomogeneous fields so all systems of reference are absolute systems of reference (in our domain of universe where fractional corrections to time dimension are very small, the principle of Galileo and other principle mentioned above are very good approach). In the fractal theory the speed of light is almost independent from the velocity of lights sources and the motions of particles with arbitrary velocities are permissible. For example, on the surface of earth the differences of speed of light value under the change of moving direction v by $-v$ consist $\sim 2v/c 10^{-6} t$ where t is the time of light moving in the interferometer. The theory does not includes the singularity at $v = c$ but almost coincides with SR, for velocities which are less than speed of light. The theory coincides with special relativity after transition to inertial systems (if we neglect by the fractional additives to dimensions of time) or almost coincides (the differences are negligible) for velocities $v < c$. The movement of bodies with velocities large than the speed of light is accompanied by a number of physical effects which one can be found experimentally (these effects is considered in the separate papers in more detail [3], [7]).

In this paragraph the relative motion of bodies in space with multifractal time (the fractional dimensions of time is near to integer $d_t = 1+\varepsilon(\mathbf{r}, t)$, $|\varepsilon| \ll 1$) is considered. For the "almost" inertial frames of reference the time is almost homogeneous and almost isotropic. In the space with the multifractal time dimensions the some remarkable characteristics of the theory of almost inertial systems exist. We enumerate some of them now: in such space an absolute frames of reference are present and the violations of conservation laws (though, small because of the smallness of ε) due to openness of all physical systems and the inhomogeneous of the time are existing; the total energy of a body which moves with the speed $v = c$ is finite and modified Lorentz transformations can be formulated (these modifications essential only in the domain of velocities near $v = c$); the formula for total energy (and the theory at a whole) is reduced to the known formula of SR in the case of transition to the usual time with the dimension equal to unity.

We begin our consideration using the mentioned above SR principles as a good approach. In an inhomogeneous space and time, if the inhomogeneous are small enough, any motion will be close to that one in homogeneous space ("almost" inertial), but the velocity of light can alter slightly, being thus "almost" constant, because in the multifractal world there are no constant physical values (as the GFD with respect to any constant value is not equal zero). The next main assumptions lay in the base of the theory: only almost inertial systems are considering (i.e the values of the multifractional dimensions (FD) of time are near to the integer dimension of time), the light velocity is almost invariant; the invariance with respect to modified Lorentz's transformations and Galileo's relativity are good approach.

9 The Model of Multifractal Time and Space

Following the previous paragraphs of this paper, we will consider both time and space as the only material fields are constructing our universe and are generating all other physical fields always. We will construct the model of the multifractal time and space on the example of the multifractal model of time. Let's consider the time axes as the set of Planck's time intervals (i.e. as the set of subsets $S_i(t_i)$). Every Planck interval has his topological dimension equal to unit (as time axes at a whole). Let inside the each interval (the last is designed by s_i for the time moment t_i) the time moments are characterized by inhomogeneous distributions and the subsets s_i of times intervals may be characterized as a good approach by multifractal dimensions, i.e. s_i are multifractal sets. Then there are local fractal dimensions and global fractal dimensions for each of set s_i. Let's designate the global fractal dimensions of set s_i as $d(t_i)$. So the time axes at a whole consists now from the multifractional subsets $s_i(t_i)$ and is the multifractal set. Each of the s_i corresponds to time interval t_i (let's remind that this time intervals may be less then Planck time interval) with global dimensions $d_i(t_i)$.

Naturally that every physical value (such as an energy, a momentum, coordinates of time or space and so on) must be described now by non differentiated functions, such as the Weierstrass function, Kox curves or any the functions are defined on the fractal or the multifractal sets. So the differentiation and the integration of them with the help of the ordinary methods of calculus is impossible. It is necessary at first to use the generalization of the fractional Riemann-Liouville derivatives and integrals. At the second there are needs in the simplification of the mathematical side of the fractal time model (of cause the same must be done for the fractal space model) The simplification consists in the replacement of the multifractal set S which one is consisting of the multifractal sets of time intervals $s_i(t_i)$ by the lattice model: each of the multifractal set $s_i(t_i)$ we replace by the one value t_i which one belongs to the set s_i. So we will have the lattice (discrete) time model with unknown and non considered nature of intervals of time which one lesser or the same order then Planck time interval sizes.

Now we replace the time values from the time set s_i by only one the value of time, for example, by the middle time t'_i for this interval. The time axes at a whole will consists in this approach of the countable set of t'_i. As Planck intervals are very small it is a good approximation when the second step will consider the latter distribution of the t'_i as a continuously distribution of the "points" t'_i where each "point" t'_i corresponds to the set of Planck interval $s_i(t_i)$ with global dimensions $d_i(t_i)$. So after the transition to continuous distribution we keep again uncountable the set S. The principle difference in such approach from the usual the treatment of the time axes is: for every time moment point the t there are the global dimensions $d_t(\mathbf{r}, t)$ of this point, which one are born by corresponding the multifractal set $s_i(t_i) \sim s(t_i)$. So again the time axes at a whole is the multifractal set S, as to every point the t of it there are global dimensions of subsets $s(t)$ which one for set S (the whole time axes) may be treated as the local dimensions $d(t)$. Only difference between the exact description and the simplified one is: the multifractal sets $s_i(t_i)$ are represented only by the set of points with their fractal dimensions, i.e. the intrinsic structure of the multifractal sets s_i are neglected. Such assumption is of cause very rude as it neglect by the physical nature of the inside distribution of time instants into the intervals t_i of the sets s_i which one are playing the main role for the nature and structure of elementary particles (see [4]). In this case the classical mathematical calculus or fractional (say, Riemann - Liouville) calculus [25] can not be applied for describing the small changes of the continuous function of physical values $f(t)$ (the last is defined on the time subsets s_t), because the fractional exponents depend on coordinates and time.

Therefore, we have to introduce the integral functionals (both the left-sided and the right-sided) which are suitable in order to describe the dynamics of functions defined on multifractal sets and determined by the formulas (1)–(2) of this paper. The multifractal model of time allows, as will be shown below, to consider the divergence of energy for masses moving with the speed of light in the SR theory, as the result of the requirement of rigorous validity, rather than approximate fulfillment, the laws pointed out in the beginning of above paragraphs in the presence of physical fields. So the treatment of real time field is expanding: time is the multifractal set and every time point has his own characteristics $d(t)$. It is natural to include in the dependence of the multifractional dimensions $d(t)$ the dependencies of space coordinates $\mathbf{r}$,(it was done in designation earlier), so the $d_t are\ d(t) \to d_t(\mathbf{r}, t)$. The multifractal structure of space may be introduced by the analogous way. Let's designate the space multifractal dimension in the space point $\mathbf{r}$ as $d_r(\mathbf{r}, \mathbf{t})$. Then in the multifractal model of time and space there are two the multifractional dimensions: dimensions for time $d_t(\mathbf{r}, t)$ and dimensions for space $d_r(t(\mathbf{r}), \mathbf{r})$.

For further development of the theory we will use the generalization of the Riemann-Liouville fractional calculus on the multifractal sets. So the multifractal model of time and space will describe the open Universe as the even the Riemann-Liouville derivatives with respect to constant are not equal to zero. Thus, the every physical value in such universe is continuously changes and all conservation laws are fulfilled as good approximation only. The model of open universe allows to consider many of difficult physical problems with new light.

10 The Principle of the Velocity of Light Invariance

Because of inhomogeneous of time in our multifractal model, the speed of light, just as in general relativity theory, depends on the potentials of physical fields which define the fractal dimensions of time $d_t(\mathbf{r}(t), t)$ (see (6)). If the fractal dimensions $d_t(\mathbf{r}(t), t)$ is close enough to unity ($d_t(r(t), t) = 1+\varepsilon$, $|\varepsilon| << 1$), the difference of the speed of light in the moving frame (with velocity v along x axis) and the fixed frame of reference will be small. In the systems which move with respect to each other with almost constant velocity (stationary velocities do not exist in any mathematical theory based on the definitions of GFD (1)-(2)) the speed of light can not be taken as a fundamental constant. In the multifractal time theory the principle of the speed of light invariance can be considered only as approximate principle. But if the ε is small, it allows to consider the nonlinear coordinates transformations from the fixed frame to the moving frame (the last replacing the transformations of Galileo in inhomogeneous time and space), as the transformations close to linear (weakly nonlinear) transformations and, thus, makes it possible to preserve the all conservation laws (and so on all of the invariant of Minkowski space) as approximate laws. Then the way of reasoning and argumentation accepted in SR theory (see for example, [30]) can also remain valid, though as good approach.

Designating the coordinates in the moving and the fixed frames of reference through x' and x, accordingly, we write down

$$\begin{aligned} ,x' &= \alpha(t,x)[x - v(x,t)t(x(t),t] \\ x &= \alpha'(t,x)[x' + v'(x'(t'),t'),\, t'(x'(t')t')]. \end{aligned} \tag{31}$$

In the (31) $\alpha \neq \alpha'$ and velocities v' and v (as well as t and t') are not equal (it follows from the inhomogeneous of multifractal time). Now we place the clocks in origins of both frames of reference and let the light signal be emitted in the moment, when origins of the fixed and the moving frames coincide in space and time at the instant $t' = t = 0$ and in the points $x' = x = 0$. The propagation of light in the moving and the fixed frames of reference is then determined by equations

$$x' = c't' \;\; x = ct. \tag{32}$$

These equations characterize the propagation of light in both of the frames of reference at each moment. Due to time inhomogeneous $c' \neq c$, but since $|\varepsilon << 1|$ the difference between velocities of light in two frames of reference will be small. For this case we can neglect by the difference between α' and α and, for different frames of reference we may write the expressions for the velocities of light, using the Eq.(8) to define velocity (denote $f(t) = x$, $dx/dt = c_0$). Thus we obtain

$$c = D_{+,t}^{1+\varepsilon} x = c_0(1-\varepsilon) - \frac{d\varepsilon}{dt}x, \tag{33}$$

$$c' = D_{+,t}^{1+\varepsilon'} x' = c_0(1-\varepsilon) - \frac{d\varepsilon}{dt}x', \tag{34}$$

$$c_1 = c_0(1-\varepsilon) + \frac{d\varepsilon}{dt}x', \tag{35}$$

$$c_1' = c_0(1-\varepsilon) + \frac{d\varepsilon}{dt}x. \tag{36}$$

The equalities (35) and (36) appear in our model of multifractal time as the result of fact, that in this model all frames of reference are the absolute frames of reference (because of the real character of the time field) and the speed of light depends on the state of frames: if the frame of reference is a moving or a fixed one, if the object under consideration in this frame moves or not. So the signs ($+$ or $-$) before the last members in the right hand parts of the equations (33)-(34) and (35)-(36) are different if the frames of reference move in reality at the different directions. This dependence disappears only when $\varepsilon = 0$ (i.e. the systems become rigorous inertial). Before substitution the relations (32) in the equalities (31) - (36) (with $\alpha' \approx \alpha$) it is necessary to find out how $d\varepsilon/dt$ depends on α. Using for this purpose the Eq.(15) we obtain:

$$\frac{d\varepsilon}{dt} = \frac{\partial\varepsilon}{\partial t} + \frac{d\varepsilon}{d\mathbf{r}}\mathbf{v} \approx -\sum_i \beta_i(\mathbf{F}_i\mathbf{v} - \frac{\partial L_i}{\partial t}), \tag{37}$$

where $\mathbf{F}_i = dL_i/d\mathbf{r}$. Since the forces for moving frames of reference are proportional to α, we get (for the case when L_i have no explicit dependencies of time variable)

$$\frac{d\varepsilon}{dt} \approx -\sum_i \beta_i \mathbf{F}_{0i}\mathbf{v}\alpha, \tag{38}$$

where F_{0i} are corresponding the forces at zero velocity. Now the equations (33) - (36) necessary to multiply on the corresponding times t, t', t_1, t_1'. We yield expressions

$$c't' = c_0 t\left[1 + \frac{v\sum_i \beta_i F_{0i}}{c_0}\alpha^2 ct(1-\frac{v}{c})\right], \tag{39}$$

$$ct = c_0 t'\left[1 + \frac{v\sum_i \beta_i F_{0i}}{c_0}\alpha^2 ct(1+\frac{v}{c})\right], \tag{40}$$

$$c_1' t_1' = c_0 t_1 \left[1 - \frac{v \sum_i \beta_i F_{0i}}{c_0} \alpha^2 ct(1 - \frac{v}{c}) \right], \tag{41}$$

$$c_1 t_1 = c_0 t_1' \left[1 - \frac{v \sum_i \beta_i F_{0i}}{c_0} \alpha^2 ct(1 + \frac{v}{c}) \right]. \tag{42}$$

Since in our model all motions and the frames of reference are absolute the frames of reference, the times t_1 and t_1' correspond to cases, when the moving and the fixed frames of reference exchange by their roles — the moving of one frame becomes the fixed and vice versa. These times coincide only when $\varepsilon = 0$. The times in square brackets, as well as the velocities, are taken be equal, because the terms which contain them are small as compared to unity. The principle of the invariance of the velocity of light for transition between the moving and the fixed frames of reference in multifractal time model is approximate (it is quite natural for multifractal universe, because the frames of reference are the absolute frames of reference). The relations (39)-(42) take the form (if take into account the Eq.(32)

$$c't' = c\alpha t(1 - \frac{v}{c}), \quad c_1' t_1' = c\alpha t_1(1 - \frac{v}{c}), \tag{43}$$

$$ct = c\alpha t'(1 + \frac{v}{c}), \quad c_1 t_1 = c\alpha t'(1 + \frac{v}{c}). \tag{44}$$

Once again we note, that the four equations for $c_1' t_1'$ and $c_1 t_1$, instead of the two equations in SR, appear as the consequence of the absolute character of motion and the frames of reference in the model of the multifractal time. In the right-hand side of (43)-(44) the dependence of velocity of light on the fractal dimensions of time is not taken into account (just as in equations (39)-(42)). Actually, this dependence leads to pretty unwieldy expressions. But if we retain only the terms which depend on $\beta = \sqrt{|1 - v^2/c^2|}$ or a_0 and neglect by non essential terms (the latter contain the products βa_0), utilizing (39) - (42) after the multiplication of the four equalities (43) - (44) we receive the following equation for calculating α (it satisfies by all four equations):

$$4a_0^4 \beta^4 \alpha^8 - 4a_0^2 \alpha^4 + 1 = \beta^4 \alpha^4 + 4a_0^4 \beta^4 \alpha^8, \tag{45}$$

where

$$\beta = \sqrt{\left| 1 - \frac{v^2}{c^2} \right|} \tag{46}$$

$$a_0 = \sum_i \beta_i F_{0i} \frac{v}{c} ct, \tag{47}$$

From (45) follows

$$\alpha_1 \equiv \beta^{*^{-1}} = \frac{1}{\sqrt[4]{\beta^4 + 4a_0^2}}. \tag{48}$$

Solutions $\alpha_{2,3,4}$ are given by $\alpha_2 = -\alpha_1$, $\alpha_{3,4} = \pm i\alpha$. Applicability of above obtained results is restricted by the requirement $|\varepsilon| \ll 1$

11 Lorentz Transformations and the Transformations of Length and Time in the Multifractal Time Model

Lorentz transformations, as well as the transformations of coordinate frames of reference, in the multifractal model of time are nonlinear because of the dependence of the fractional dimensions of time $d_t(\mathbf{r}, t)$ from coordinates and time. Since nonlinear corrections to Lorentz's transformations are very small because the $\varepsilon \ll 1$, we shall take into account only the corrections which eliminate the singularity at the value of velocity $v = c$. It replace the factor β^{-1} in Lorentz transformations by the modified factor $\alpha = 1/\beta^*$ which is defined by the Eq.(48). The Lorentz transformation rules (for the motion along the x axis) then take the form

$$x' = \frac{1}{\beta^*}(x - vt), \quad t' = \frac{1}{\beta^*}(t - x\frac{v}{c^2}). \tag{49}$$

In the equations (47) and the (48) the velocities v and c weakly depend on x and t and their contributions to the singular terms are small. Hence, we can neglect by these dependencies. The transformations from the fixed system to the moving system are almost orthogonal (for $\varepsilon \ll 1$) and the squares of almost the four-dimensional vectors of almost Minkowski space are changing under coordinates transformations very slightly (i.e. they are almost invariant). Then it is possible to neglect by these correction terms of order $O(\varepsilon, \dot{\varepsilon})$. These terms are very small too. From the Eqs. (47)-(48) the possibility of bodies motion (with nonzero rest masses) with arbitrary velocities follows. We neglect now by the corrections of order $O(\varepsilon, \dot{\varepsilon})$ in nonsingular terms. Then a momentum and an energy of a body with a nonzero rest mass in the frame of reference which is moving along the x axis ($E_0 = m_0c^2$) are equal

$$p = \frac{1}{\beta^*}m_0 v = \frac{m_0 v}{\sqrt[4]{\beta^4 + 4a_0^2}}, \quad E = E_0\sqrt{\frac{v^2c^{-2}}{\sqrt{\beta^4 + 4a_0^2}} + 1}. \tag{50}$$

The energy of such a body reaches its maximal value at $v = c$ and is equal then $E_{v=c} \approx E_0/\sqrt{2\alpha_0}$. When $v \to \infty$, the energy is finite an tends to $E_0\sqrt{2}$. For $v \leq c$ the total energy of a body is represented by expression

$$E \cong \frac{E_0}{\sqrt[4]{\beta^4 + 4a_0^2}} = mc^2, \quad m = \frac{m_0}{\beta^*}. \tag{51}$$

For $v \geq c$, total energy, defined by (24), is given by

$$m = \beta^{*-1}m_0\sqrt{1 + \beta^{*2} + \sqrt{\beta^{*4} - 4a_0^2}}. \tag{52}$$

If we take into account only the gravitational field of earth and neglect by the influences of all other fields, the parameter $a_0(t)$ can be estimated as $a_0 = r_0r^{-3}x_Ect$, where r_0 is the gravitational radius of earth, r is the distance from earth surface to its center ($\varepsilon = 0.5\beta_g\Phi_g$, $\beta_g = 2c^{-2}$, $x_E \sim r$, $v = c$). For the energy maximum we get $E_{max} \sim E_0 \cdot 10^3 t^{-0.5} sec^{0.5}$.
The shortening of lengths and time intervals in the frames of references are moving in the model of the multifractal time also have several peculiarities. Let l and t be the length and the time intervals in a fixed frame of reference. In a moving frame

$$l' = \beta^* l, \quad t' = \beta^* t. \tag{53}$$

Thus, there exists the maximal shortening of length when a body velocity equals to speed of light. With further the increasing of velocity (if it is possible to fulfill some the requirements for a motion in this region with constant velocity without radiating), the length of a body begins to grow and at the infinitely large velocity is also infinite. The slowing-down of time, from the point of view of observer in the fixed frame (the maximal shortening equals to $t' = t\sqrt{2a_0}$) is replaced, with the further increase of velocity beyond the speed of light, by the acceleration of time passing ($t \to 0$ when $v \to \infty$).

The rule for velocities transformation retains its form, but β is replaced by β^*

$$u_x = \frac{u'_x + v}{1 + \frac{u'_x v}{c^2}}, \; u_y = \frac{u'_y\beta^*}{1 + \frac{u'_y v}{c^2}}, \; u_z = \frac{u'_z\beta^*}{1 + \frac{u'_z v}{c^2}}. \tag{54}$$

Since now there is no law which prohibits the movements with velocities greater than that one of light, the velocities in (53)-(54) can also exceed the speed of light. The electrodynamics of moving media in the model of multifractal time can be obtained, in most cases, by substitution $\beta \to \beta^*$.

We remark the analogue between the behavior of bodies with the velocities exceed speed of light with the behavior of taxions. The essential differences are: the bodies of our theory have real masses in the superluminal domain of velocities (the taxions have imaginary masses); the momentum and energy of bodies in the multifractal theory have no infinity values at $v = c$, so there are no restrictions for any values of velocities and there are no impenetrable boundary between bodies with $v \leq c$ and superluminal bodies. Thus, any usual elementary particle may moves with the more then speed of light velocity.

12 Newton's Equation for Relativistic Particle

It is necessary to change Newtonian mechanics in order to bring it into accordance with the principles of the multifractional universe. The algorithm presented for this purpose in above mentioned paragraphs allow to do it. Newton's equation for a particles with relativistic velocities $v \leq c$ in the multifractal universe has the form

$$\frac{\partial}{\partial t}(mv) = \frac{\partial}{\partial t}\left(\frac{m_0 v}{\sqrt[4]{(1 - \frac{v^2}{c^2})^2 + 4a_0^2}}\right) = eE_e. \tag{55}$$

This equation may be a good approach and for $v \geq c$ and differs from exact equation (in the latter it is necessary to use other representation for m) only by the energy in the limit case $v \to \infty$ on constant value. For $v \geq c$ the mass m in the equation (55) is determined by Eq.(52). If $E_e = eVl^{-1} = \textit{constant}$ the above equation yields for $v = c$ (in the idealized the variant of the theory without radiation) a possibility to calculate the minimum of the time t_0. The time t_0 corresponds to the velocity $v = c$. A particles are accelerated by constant electrical field with the electric strange E_e may receive the velocity that is equal c for the time t_0:

$$t_0 = \sqrt[3]{\frac{\beta_{e0}^2 l^2}{2c^2\omega\varepsilon^3}}. \tag{56}$$

In this formula the ω is the frequency of the electromagnetic field E_e with amplitude V which accelerating the particles, l is the distance between of accelerating electrodes. When the particles move between the electrodes, the value of E_e may consider equal to V. The t_0 is defined by equation (56) yields for accelerating particle the value of the time which is necessary in order to receive by the particle the velocity equal $v = c$. The maximum energy at $v = c$ may be written now (if we select the $t = t_0$ that is only a methodically example) as

$$\begin{aligned}
&E_{max} = E_0 \sqrt[3]{\frac{2c}{\beta_{e0} l\omega}},\\
&\varepsilon = \beta_{e0}\frac{eV}{E_0},\\
&\beta_{e0} = V_0\psi^*(\mathbf{r}, t)\psi(\mathbf{r}, t),
\end{aligned}$$

where β_{e0} is the probability to find the particle in the point $\mathbf{r}, t)$, the value of electric strange E_e and the representation of the l and the ω by their values are determined by the construction of the accelerators and the conditions of their work regimes. After the rude estimation for accelerated proton of these parameters for the maximum energy at $v = c$ receive: $E_{max} \sim E_0(10 - 10^3)$. It seems, many of the particles are falling on earth from cosmos (if they got their energies more then the mentioned above by means of gravitational acceleration) must radiate and transforming its kinetic energy into the energy of electro-magnetic waves with the frequency near $\nu \sim E_{max}h^{-1}$. Stress, the theory presented here (though it is the theory of open systems [27], as our universe is defined on the carrier of measure and continually interacts with it) does not contradicts SR because it describes only almost inertial systems and for inertial systems (the dimension of time is topological) coincides with SR.

The main results this paragraph are: a) a bodies may move with arbitrary velocities without appearance of energy infinity and imaginary masses (the particles with imaginary masses may exist but these particles are not taxions); b) the energy of a moving body has maximum for the velocity $v = c$ (the latter depends of the time of acceleration; c) the experimental verification of main the results of theory is exists: the movement of bodies with the velocities exceeding the speed of light is accompanied by new physical effects which can be discovered experimentally (these effects were considered in the papers [2], [3], [7], [10],[11] in more details); d) the values of β_{e0} are not determined by this phenomenological variant of the multifractal theory and must be determined by other theory or by empirical methods. For verification of the theory, it is necessary to receive (if use the rude estimations are considered above) the particles with the kinetic energies $E \sim E_0(10 - 10^3)$.

It is very interesting problem to find possibilities for organizing the works that may discover the superluminal protons with such energies and change our viewing on the nature of time and energy.

13 Has the Principle of Equivalence of General Relativity His Boundary?

In this paragraph we consider the problem of the applicability of the general relativity principle of equivalence. It is well known that the principle of equivalence (i.e. the independence of body acceleration given by the gravitational field from it mass) is the base of general theory of relativity. Is this principle rigorous or only a good approach? In the multifractal universe (non-closed system) all conservation laws and principle are not rigorous and are only the good approaches for domain of universe where physical forces are small and so the fractional corrections to the dimensions of time and space are small too (the mentioned laws and principle were rigorous for closed systems). Now we demonstrate the approximate character of this principle by calculating corrections to gravitational force in the space with the fractional dimensions of time for the case of classical limit (Newton's equation, see the (20). Let the body with mass m moves in the gravitational field of the body with mass M on the distance r ($r >> r_0$ where r_0 is the gravitational radius of body with mass M) and let $M >> m$. Then it is possible to consider these bodies as points. Now we more precisely calculate the corrections to the time dimensions in the (20). At the point r, where is a center of point mass m, there are two contributions in the FD of d_t: the one is caused by the field of body with mass M, the another is caused by the mean gravitational field of body with mass m and mean radius r_m. The latter is $\frac{am\gamma}{r_m}$ where a is constant defined by distribution of masses within body with mass m. In that case the Eq.(20) reads ($r_0 = \frac{2\gamma M}{c^2}$)

$$[1 - \frac{r_0}{r}(1 + \frac{amr}{Mr_m})]\frac{d^2}{dt^2}\mathbf{r} = \nabla\frac{\gamma M}{r} = \mathbf{F_g}. \tag{57}$$

This equation may be rewritten (if take into account that $\frac{r_0}{r} << 1$) as

$$\frac{d^2}{dt^2}\mathbf{r} \approx \mathbf{F_g}[1 + \frac{r_0}{r}(1 + \frac{amr}{Mr_m})]. \tag{58}$$

The right-hand part of the Eq.(58) describes the gravitation force with the correction of general relativity and the extra correction because of violation of the principle of equivalence (the member with dependence of mass m). It yields for the effect of Mercury perihelia rotation the additional correction near $0.004a$ of the corrections of general relativity and results to the full coincidence of the experimental data and theoretical calculations. Thus, in the multifractal universe there is the dependence of the gravitational acceleration from the mass of accelerated body. That dependence is very small in the considered case, but has the principal role and show the limitation of the principle of equivalence. It is natural for the multifractal universe as all systems of reference in it are absolute systems. So any the equivalence of them is treated as approximation (very good in the domains with almost topological dimensions of time and space).

14 Irreversible Quantum Mechanics of Multifractal Universe

In the last years main contributions in the problem of irreversibility of quantum mechanics was made by works of Prigogine and his collaborates [31]-[38]. Our contribution in this problem is not contradicts their results and only propagate them on the domain of universe with the multifractal time and space dimensions. The equations of quantum mechanics in the multifractal universe were for the first time considered in [1]-[3],[12] (the very detailed considerations are in [20], [21]). In the approaches of the relation Eq.(15) (if we used its approach) the quantum equation for electron in the electric potential field Φ reads (more general non relativistic quantum mechanics equations is in [20]):

$$i\hbar D^{d_t}\psi \approx i\hbar\frac{\partial}{\partial t}\psi(\mathbf{r},t) + i\hbar\frac{\partial}{\partial t}[\varepsilon\psi(\mathbf{r},t)] + i\hbar\frac{\varepsilon}{t}\psi(\mathbf{r},t) = \frac{-\hbar^2}{2m}\Delta\psi + e\Phi\psi, \quad \varepsilon = -\frac{e^2V_0\psi^*\psi}{rmc^2}, \tag{59}$$

where t is the time which begins at the point of reference when big-bang was happened ($t = t_0 + t_1$, t_0 is the mean time of universe existence, t_1 is current time), V_0-is the dimensional constant (with dimension of sm^3) which is defined by the selections of physical tusk. On the left hand side of the Eq.(59) there are two extra members. One of them (with the derivative respect to time) yields the corrections to energy specter of electron born by the fractional dimensions of time and transforms the equation into nonlinear equation (because of the dependence of ε of $\psi^*\psi$).

For Newton's equation when the analogies member has no probability sense, the correction from this member yields the modified results of general relativity (see [19] or the above paragraph). In the fractal quantum mechanics this member, though it is very small, transforms the equation into the nonlinear equation with cubic nonlinearity and so transforms the rigorous superposition principle of quantum mechanics into a good approximation principle. For a hydrogen atom the qualitative corrections from FD born by protons field were found in the [1]-[2]. In the cited papers, for simplicity, the probabilistic character of ε born by the probabilistic character of quantum mechanics was not taken into account. The more detailed calculations of the corrections born by taking into account the probabilistic character of ε in the fractal quantum mechanics for hydrogen atom were considered in [21]. It yields nonessential corrections for Lamb-Rietherford splitting $\sim 10^{-6}MGz$ for the sizes of electron about $10^{-16}sm$. The other extra member in Eq.(59) describes the irreversibility of quantum mechanics with the flow of time and so small that may be (almost always) omitted. This member has important role for the describing of the origin of irreversibility in the multifractal universe.

The latter is distinguished from the origins of the irreversibility considered by I. Prigogine and his collaborates and lay the "outside" of the concrete characteristics of dynamical systems. The irreversibility of all physical fenomena in our theory is the universe characteristics born by the multifractional nature of time. The Eq.(59) is the example of the equations of non relativistic nonlinear irreversible quantum mechanics in the multifractal universe. The generalization of Dirac equations were considered in [20].

15 Conclusions and Overlook

In this paper the new theory of time and space is formulated: the multifractal theory of time and space or the physical theory of the multifractal universe. Let us now enumerate some of main results are connected with considered in the paper the examples of physical phenomena in the frame of the multifractal theory of time and space and also some of main results that are the consequences of the multifractal theory and considered in papers [1]-[21].

The main postulates of the theory (new paradigm) are:

1.Time and space are real material physical fields with the fractional dimensions constituting our universe.

2. The fractional parts of time dimensions, defining all known physical fields and forces describe their behavior (as the consequences of it, all physical fields are geometrizationed in the frame of multifractal geometry and they are characteristics of the multifractal time).

3. The fractional parts of time and space dimensions define the Lagrangian energies densities of all physical fields.

4. In the multifractal universe the principle of minimum for the functional of fractional dimensions is valid.

5. Our universe appears in the multifractal model of universe as the manifestation of characteristics of the real fields of the multifractal time and space.

6. The multifractal model of time and space is natural generalization of known physical ideas and propagate them on multifractal sets of time and space. The latter are defined on the measure carriers (the spaces like R^n) and the measure carriers play the role of vacuum that born universe.

7. We postulate the existence of the multifractal material fields of space and time and treat the “Vacuum” as the topological or multifractal sets (the carrier of measure) with very complicated structure and fractional or complexes values of dimensions n defined on sets $\mathcal{R}^n$. These sets are the measure carriers for the sets of the multifractal time and space sets constituting our universe (more detailed discussion about problem of vacuum in the multifractal theory see [15],[16]). The fractional nature of the time dimensions leads then to appearing of space-time energy densities $L(\mathbf{r}(t), t)$ in the each point of time-space, i.e. it generates all known fields and forces. The fractional nature of space dimensions yields the new time-space energy densities $L(t(\mathbf{r}), \mathbf{r})$ and the new class of fields (named as “temporal” or “cosmical vacuum” fields). The roles of d_t and $d_\mathbf{r}$ in distorting (accordingly space and time) dimensions are relative and my be interchanged. Apparently, one can consider the “united” dimensions $d_{t,\mathbf{r}}$ — the dimensions of undivided on the time and the space the multifractal continuum in which the time coordinates and the space coordinates are related with each other by the relations like the those one for Minkowski space (i.e. not using the approximate relation utilized in this paper): $d_t t, \mathbf{r} = d_t + d_\mathbf{r}$). In this case one would have to calculate the generalized fractional derivatives with respect to t and $\mathbf{r}$ on the same complicated Lagrangians, and the new “temporal” fields may not be separated. As the nature of time is the analogies of the nature of space (but not the same), then time and space always bound by relations that are very like (only more complicated) as in Minkowski world (see [1]-[2]). For the universe with topological dimensions this the relation coincide with the relation of Minkowski flat space-time.

Thus the model of the multifractal time and space considered in the paper offers the new look (both in mathematical and philosophical senses) on the properties of space and time and their description, on the nature of all fields generated by the fractional time and space. This yields way to many interesting results and conclusions (detailed discussion of several physical problems can be found in [1]-[21]). Now we restrict ourselves only by the brief enumerating of the most important ones as the conclusion of this paper:

(a) The multifractal model does not contradict any of existing physical theories. Moreover, it is reduced to them when potentials and fields are small enough. The theory yields new predictions (as an example - a freedom of divergencies in the multifractal theory) for strong fields when the fractional additives to topological dimensions are large. The question about the applicability of the proposed relations between fractal dimensions and Lagrangian densities may be treated as a first approach (see (10)-(13)) and more rigorous relations are the other solutions of the equation for the FD.

(b) Time and space are unique material fields which constituting our material universe. In such universe there are an absolute frames of reference (all systems of references are absolute because they bound with inhomogeneous time and space) and all conservation laws of modern physics are only a good approximations valid for the fields and forces of low energy density (and as consequence, the small differences of d_t compare with unit). The small violation of the conservation laws are consequences of the irreversibility of all physical processes since the universe is an open system defined on certain measure carrier and interacting with it continually. All absolute systems of reference become a relative frames of references only when we may neglect by the fractional additives (the latter makes time homogeneous) to time dimension . The smallness of the fractional corrections to the value of the time dimensions in many cases (i.e. on the Earth's surface it is about $(d_t - 1) \sim 10^{-9}$) makes possible to neglect by it and use the conventional models of the physics of closed systems for cases of real life.

(c) The fractal model of time and space allows to consider all fields and forces of the real world as the result of geometrization of time and space (may be more convenient the term "a fractalization" of time and space) in terms of fractal geometry. The appearance of the fractional dimensions of time and space yields birth to all fields and the forces which one exist in universe. The model introduces the new class of physical fields, which is originated by the fractional nature of dimensions of space. It yields appearance of the new fields (the "themporal" fields and forces) which one are analogies of all known physical fields and forces and may be good candidate for describing the cosmical vacuum fields with negative pressure. These fields are generated by fractional dimensions of space. Thus the presented model of time and space allows to consider all physical forces and fields as the result of appearance of the fractional dimensions of time and space. The fractal theory is the theory that included all physical forces in uniform theory in the frame of fractal geometry. Stress once more: the model allows to consider all fields and forces of the world as the result of the existence of fractal geometrization and the FD of time and space born all of them. It allows to receive from general principle the equations of all physical theories (that is impossible in general relativity but possible in Logunov's theory gravitation). It is the non integer dimensions of time and space that produce all observable fields. The new class of fields comes naturally into consideration, originating solely from the fractional nature of space dimensions and with the equations similar to those of the usual fields. So the presented model of space and time is the theory that allows to consider all of physical fields and forces in terms of a unique fractal geometrical approach.

(d) Basing on the multifractal model of time and space, it is possible to develop the theory of bodies moving in almost inertial systems [4], [6]-[7]. This theory coincide with special theory of relativity when we neglect by the fractional corrections to the time dimensions. In such the "most inertial" frames of reference the motion of particles with any velocities becomes possible.

(e) Very natural conclusion may be made (but with great care) on the base of considered the fractal theory of time and space: all the theories of modern physics are valid only for weak fields and forces, i.e. in the domain of FD with almost integer values (i.e. with the fractional additives to dimensions compare with zero (near to zero)). This conclusion depends on the fact that only the first members of expanding of Lagrangians (included in the functionals of FD) in a power series in the functions of different fields are used (this problem are common for modern theoretical physics, but in the fractal theory it is especially obvious).

(f) The problem of the proper forms selection of the deformation that would define appearing of fractional dimensions also remains to be solved. So far there is no clear understanding now which the type of fractal dimensions we must use, d_t and $d_{\mathbf{r}}$ or $d_{t,\mathbf{r}}$.

(g) The theory of relative motions in the almost inertial systems based on the multifractal time theory and have considered in this paper yields the new method for describing of the physical characteristics (i.g. an energy, a momentum, a mass and so on) of moving bodies. The main results this theory are: the body movements with arbitrary velocities without appearance of infinitum of energies and imaginary masses is possible ; the maximum a body energy is existent if $v = c$; there is possibility for the experimental verification of main theoretical results; the theory coincides with SR after transition to inertial systems (if neglect by fractional additives to the dimensions of time) or practicably coincides (the differences are non-essential) for velocities $v < c$; the movement of bodies with the velocities which one exceed the speed of light is accompanied by the series of physical effects which can be discovered and experimentally verificated (these effects was considered in the separate papers ([3], [7]) in more details).
What is necessary for the verification of the theory of almost inertial systems? It is necessary to accelerate particles to the energy $\sim E_0\ (10^4 - 10^3)\ \sqrt{t}^{-1}$ were t is the time of acceleration of particle till $v = c$. Such energies may be achieved in near years. If particles with such energies will be received and our theory will be proved, the results of it will change our views at the nature of time and space, an energy and so on.
(g) The Lorentz transformations must be altered near velocities $v = c$ (the latter is consequence of the theory of almost inertial systems, see [13], [14]).

(h) The theory describes the multifractal universe as the open system, so there are no in the universe any of non altering values. All laws of modern physics are not rigorously fulfilled and are only very good approaches. They valid in the domains of space and time where the physical forces are small, i.e. the fractional corrections to the topological dimensions of time and space are small too (just in the such domain of universe we are livng). The multifractal theory of time and space has not infinitum. An information may be transferred in the multifractal universe with any speed and it does not contradicts to the physical laws of multifractal universe. So the new physics of super light velocities is appears with the new predicted physical effects. Probably, the part of them may be used for practical needs by mankind. The equations of classical and quantum mechanics (relativistic and non relativistic), electrodynamics, general theory of relativity are formulated in the multifractal time and space and may be used in the multifractal universe. All physical fields have a rest masses originated by the fractional dimensions structure of real time field.

(i) The laws of thermodynamics are consequences of multifractal nature of time and space for the domains of universe with state near the state of thermodynamically equilibrium.

(j) All equations of modern physical theories (classical and quantum mechanics, electrodynamics, Einstein's theory and so on) in the multifractal universe are irreversible.

(k) The arrow of time exists in the multifractal universe and the impossibility for changing the direction of time is the consequence of only energetically reasons. The nature of it lay in the multifractional dimensions of time and the irreversibility of the equations of physics.

(l) The theory predicts existence of the new fields with new physical nature (in particular, the fields with anti gravitational forces, the fields with imaginary masses) and predicts some new laws (in particular, the law, which is analogous to Hubble law, of the dependence of the velocity of time changing with coordinates at time flow in the inhomogeneous time flow). The new type of masses is introduced: the masses m_r which characterize the inertia of time alteration with the alteration of space coordinates in the inhomogeneous time flow and are the coefficients before the second derivative of the time $t(\mathbf{r})$ with respect to $\mathbf{r}$ in the equation defining the alteration of time with alteration of space coordinate in the inhomogeneous time field.

(m) Some new physical effects predicted for the superluminal physics of usual particles (not taxions).

(n) The multifractal theory of time and space (it is based on the principle of minimum for functional of the multifractional dimensions of time and space) use the generalized Riemann-Liouville fractional derivatives and integrals (GFD). The GFD are usual Riemann–Liouville functionals propagating on the multifractal sets.

(o) For time and space with integer dimensions all results of the theory coincide with known results of physics in topological spaces.

(p) In the multifractal universe any volume of space and time have very large amount of energy . This energy was got by the universe from the measure carries when big-bang is happened.

(q) All modern physical theories (including the string and super symmetry theories) may be rewritten by using the idea and the mathematical methods of the theory presented in this paper, i.e. the theory of time and space with the fractional dimensions. So the presented theory may be treated as the theory giving directions for the development of the future physics of the multifractal universe if our universe is the multifractal sets of time and space in (reality).

The author hopes that new ideas and mathematical tools presented in this paper will be a good first step on the way of the investigations of complicated the multifractal characteristics of time and space of our multifractal universe.

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