# Super Black Hole as Spinning Particle

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Non-perturbative approach to spinning particlelike solutions based on the Kerr-Newman black hole (BH) solution is considered. Non-trivial supergeneralization of the Kerr-Newman solution matched with complex structure of Kerr geometry is discussed. For parameters of spinning particles the BH horizons disappear and singularity is naked. We regularize this singularity, introducing a smooth superconducting baglike source which is built of the chiral and gauge fields (generalized Higgs model). We show that there appears a controlled by gravity phase transition to a new supersymmetric vacuum state near the core. A supersymmetric domain wall model for this phase transition is suggested which is based on the Witten $U(1) \times \tilde{U}(1)$ field model.

## 1 Introduction

The rotating Kerr-Newman black hole solution [1] is apparently the most suitable classical background for the models of spinning particles.

In 1969 Carter observed [2], that if three parameters of the Kerr-Newman solution are adopted to be ($\hbar$=c=1)

$$e^2 \approx 1/137, \quad m \approx 10^{-22}, \quad a \approx 10^{22}, \quad ma = 1/2, \tag{1}$$

then one obtains a model for the four parameters of the electron: charge $e$, mass $m$, spin $l$ and magnetic moment $ea$, and the gyromagnetic ratio is automatically the same as that of the Dirac electron. The first treatment of the source of the Kerr spinning particle was given by Israel [3] in the form of an infinitely thin disk spanned by the Kerr singular ring. Disk has the Compton size with radius $a = l/m = \frac{1}{2m}$. The Israel results where corrected by Hamity who showed that the disk is in a rigid relativistic rotation, and then Lòpez suggested a regularized model of the source in the form of a rotating ellipsoidal shell (bubble) covering the singular ring [4]. The structure of the electromagnetic field near the disk suggested superconducting properties of the material of the source, and there were obtained the analogue of the Kerr singular ring with the Nielsen-Olesen and Witten superconducting strings [5] and other stringy structures [6]. Since 1992 there has been considerable interest as to black holes in superstring theory, and the point of view appeared that some of black holes can be treated as elementary particles [7]. In particular, Sen [8] has obtained a generalization of the Kerr solution to low energy string theory, and it was shown [9] that near the Kerr singular ring the Kerr-Sen solution acquires a metric similar to the field around a heterotic string.

The treatment of super-Kerr-Newman geometry for modelling the spinning particles allows one to involve fermionic degrees of freedom in the most natural way. Supersymmetry and supergravity give also essential advantages by non-perturbative approach leading to cancelation of quantum divergences. In particular, the solutions saturating BPS-bound and retaining a part of supersymmetry may not receive quantum loop corrections. The simplest consistent Super-Kerr-Newman BH solution [10] was constructed on the base of the (broken) Ferrara-Nieuvenhuisen N=2 Einstein-Maxwell

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D=4 supergravity. Its structure is strongly matched with the complex structure of the Kerr geometry. Because of that we start the paper from a treatment of this structure and show that the super-Kerr geometry is constructed in full analogue with the Kerr complex structure. The resulting super-solution contains also singularity which is covered by BH horizon.

However, for the large angular momentum corresponding to spinning particles the Kerr horizons are absent, and there appears a naked singularity. It can be regularized being replaced by a matter source [11] built of the non-trivial chiral (Higgs) fields. In the sections 4-7 we consider non-perturbative approach to supergravity based on the solutions having the charged black hole form in external region while the BH singularity is replaced by a superconducting baglike core and described by chiral fields of a supersymmetric field model. The smooth (phase) transition to the core is provided by a supersymmetric domain wall model. The treatment of this phase transition can be done in a unique manner for rotating and non-rotating BH solutions and does not depend on specific twisting structure of Kerr geometry. Thus, corresponding part of the paper is practically independent of the sections 2 and 3 where the complex and super-structures of the Kerr geometry are considered.

## 2 Real, complex and stringy structures of Kerr geometry

The Kerr-Newman solution can be represented in the Kerr-Schild form

$$g_{\mu\nu} = \eta_{\mu\nu} + 2h e^3_\mu e^3_\nu, \tag{2}$$

where $\eta_{\mu\nu}$ is metric of an auxiliary Minkowski space $\eta_{\mu\nu} = diag(-1,1,1,1)$, and $h$ is a scalar function. Vector field $e^3$ is null, $e^3_\mu e^{3\mu} = 0$, and tangent to principal null congruence $K$ of the Kerr geometry. The Kerr congruence $K$ is twisting i.e. corresponding to a vortex of a null radiation. [1] One of the main peculiarities of the Kerr geometry is singular ring (of radius $a$) representing a branch line of the Kerr space on the 'positive' $(r > 0)$ and 'negative'$(r < 0$ ) sheets which are divided by the disk $r = 0$ spanned by this ring. The Kerr singular ring is exhibited as a pole of the function

$$h(r,\theta) = \frac{mr - e^2/2}{r^2 + a^2\cos^2\theta},$$

where $r$ and $\theta$ are the Kerr oblate spheroidal coordinates.

The vortex of the Kerr congruence is in-going on the 'negative' sheet of space where $r < 0$, it crosses the disk $r = 0$ and turns into out-going one on the 'positive' sheet $r > 0$.

The simplest solution possessing the Kerr singular ring was obtained by Appel in 1887 (!) [12]. It can be considered as a Newton or a Coulomb analogue to the Kerr solution. On the real space-time the singular ring arises in the Coulomb solution $f = e/\tilde{r}$, where

$$\tilde{r} = \sqrt{(x - x_o)^2 + (y - y_o)^2 + (z - z_o)^2},$$

when the point-like source is shifted to a complex point of space $(x_o, y_o, z_o) \to (0, 0, ia)$. Radial distance $\tilde{r}$ is complex in this case and can be expressed in the oblate spheroidal coordinates $r$ and $\theta$ as

$$\tilde{r} = r + ia\cos\theta.$$

The source of Kerr-Newman solution, like the Appel solution, can be considered from complex point of view as a "particle" propagating along a *complex world-line* $x^\mu_0(\tau)$ in a complexification of the auxiliary Minkowski space-time $CM^4$ [6, 14] and parametrized by complex time $\tau$.

[1] The Kerr congruence is geodesic and shear free, it represents a bundle of twistors and can be described by the Kerr theorem [1, 13, 6, 14].

The objects described by the complex world-lines occupy an intermediate position between particle and string. Like the string they form the two-dimensional surfaces or the world-sheets in the space-time. It was shown [6, 14] that the complex Kerr source may be considered as a complex hyperbolic string which requires an orbifold-like structure of the world-sheet. In many respects this source is similar to the 'mysterious' $N = 2$ string of superstring theory, shedding a light on the puzzle of its physical interpretation. The second stringy structure of the Kerr geometry is the Kerr singular ring. In fact the both these stringy structures are different exhibitions of some membrane-like source. This source has a complex interpretation alongside with a real image in the form of a rotating bubble which will be discussed further.

Appearance of the twisting Kerr congruence may be understood as a track of the complex retarded-time construction. The null rays of the Kerr congruence are the tracks of null planes of the family of complex light cones emanated from the points of the complex world line [13, 14]. The complex light cone with the vertex at some point $x_0$ of the complex world line $x_0^\mu(\tau) \in CM^4$:

$$(x_\mu - x_{0\mu})(x^\mu - x_0^\mu) = 0, \tag{3}$$

can be split into two families of null planes: "left" planes

$$x_L = x_0(\tau) + \alpha e^1 + \beta e^3$$

spanned by null vectors $e^1(Y)$ and $e^3(Y, \tilde{Y})$, and "right" planes

$$x_R = x_0(\tau) + \alpha e^2 + \beta e^3,$$

spanned by null vectors $e^2$ and $e^3$. [2]

The Kerr congruence $K$ arises as the real slice of the family of the "left" null planes ($Y = const.$) of the complex light cones which vertices lie on the complex world line $x_0(\tau)$.

The subset of null rays of the Kerr congruence belonging to a coordinate surface $\theta = const.$ satisfies the retarded-time equation

$$r + ia\cos\theta = (t - \tau), \tag{4}$$

On the real slice of the Kerr geometry the coordinates $r, t$ and $\theta$ are real. It means that the complex time parameter $\tau = t_0 + i\sigma$, which is responsible for this family of null rays, is determined by $t_0 = t - r$, and $\sigma = -a\cos\theta$. Thus, only the cones lying on the strip $|\sigma| \le |a|$ have a real slice. One can also say that only this strip is "seen" from the real Kerr geometry and represents its source. Therefore, the ends of the resulting complex string are open. To satisfy the complex boundary conditions, a special orbifold-like structure of the worldsheet must be introduced [6, 14], which is closely connected with the above mentioned Kerr's twosheetedness. Next, one can note that the fixed value $\tau$ corresponds to the family of null rays with constant angle $\theta$ and angles $\phi \in S^1$. Thus, there is one more parameter on the worldsheet, and complex source represents really a membrane.

On the mathematical language we have: a fiber bundle of complex light cones over the complex world line $x_0(\tau) \in CM^4$, and a vector bundle of the "left" complex null planes over the base $\{\tau, Y\} \in C^2$. Real section of the "left" null planes are null rays of the Kerr congruence. The real Kerr geometry ($x \in M^4$) represents a local section of line bundle with the base $\{\tau, Y\}$ and the null rays $K$ as fibers. The null planes determine projections $\pi : x \to \{x_0(\tau), Y\} \to \{\tau, Y\}$ and $\pi^{-1} : \{\tau, Y\} \to \{\text{null ray of K}\}$.

[2] The Kerr null tetrad is given by one-forms: $e^1 = d\zeta - Y dv$, $e^2 = d\bar\zeta - \bar Y dv$, $e^3 = du + \tilde Y d\zeta + Y d\bar\zeta - Y\tilde Y dv$, $e^4 = dv + he^3$, where the Cartesian null coordinates $u, v, \zeta, \bar\zeta$ are used. Vector-form $e^3$ is real when $\tilde Y = \bar Y$. On real section $Y$ is the projective spinor coordinate $Y = e^{i\phi}\tan\frac{\theta}{2}$.

## 3 Super-Kerr-Newman geometry to broken N=2 supergravity

A supergeneralization of the Kerr-Newman solution can be obtained as a natural combination of the Kerr spinning particle and superparticle [10]. In fact, the complex structure of the Kerr geometry suggests the way of this supergeneralization.

The simplest consistent supergeneralization of Einstein gravity [15] represents an unification of the gravitational field $g_{ik} = e^a_i e_{ak}$, with a spin 3/2 Rarita-Schwinger field $\psi_i$ . The combined Lagrangian has the form

$$\mathcal{L}_{sg} = -eR/2k^2 - \frac{i}{2}\epsilon^{ijkl}\bar{\psi}_i\gamma_5\gamma_j\mathcal{D}_k\psi_l, \tag{5}$$

where $e = \det e_{ia}$; $\mathcal{D}_i = \partial_i + nonlin.terms$. Corresponding action $I = \int \mathcal{L}_{sg} d^4x$ is invariant under the local supersymmetry transformations with a Grassmann gauge parameter $\epsilon$:

$$\delta_\epsilon e^a_i = -ik\bar{\epsilon}\gamma^a\psi_i, \tag{6}$$

$$\delta_\epsilon \psi_i = -2/k\mathcal{D}_i\epsilon, \tag{7}$$

and all supergauge-related solutions are physically equivalent.

Note, that any exact solution of the Einstein gravity is indeed a trivial solution of supergravity field equations. The supergauge freedom allows one to turn any gravity solution into a form containing spin-3/2 field $\psi_\mu$ satisfying the supergravity field equations. Starting from an exact solution of Einstein gravity and using the supergauge freedom (6), (7), one can easily turn the gravity solution into a form containing spin-3/2 field $\psi_i$ satisfying the supergravity field equations. However, since this spin-3/2 field can be gauged away by the reverse transformation, such supersolutions have to be considered as *trivial*. The hint how to avoid this triviality problem follows from the complex structure of the Kerr geometry. In fact, from the complex point of view the Schwarzschild and Kerr geometries are equivalent and connected by a *trivial* complex shift.

The *non-trivial* twisting structure of the Kerr geometry arises as a result of the complex *shift of the real slice* concerning the center of the solution [13, 6]. Similarly, it is possible to turn a *trivial* super black hole solution into a *non-trivial*. The *trivial supershift* can be represented as a replacement of the complex world line by a superworldline $X_0^\mu(\tau) = x_0^\mu(\tau) - i\theta\sigma^\mu\bar{\zeta} + i\zeta\sigma^\mu\bar{\theta}$, parametrized by Grassmann coordinates $\zeta$, $\bar{\zeta}$, or as a corresponding coordinate replacement in the Kerr solution

$$x'^\mu = x^\mu + i\theta\sigma^\mu\bar{\zeta} - i\zeta\sigma^\mu\bar{\theta}; \qquad \theta' = \theta + \zeta, \quad \bar{\theta}' = \bar{\theta} + \bar{\zeta}. \tag{8}$$

Assuming that coordinates $x^\mu$ before the supershift were the usual c-number coordinates, one sees that coordinates acquire nilpotent Grassmann contributions after supertranslations. Therefore, there appears a natural splitting of the space-time coordinates on the c-number 'body'-part and a nilpotent part - the so called 'soul'. The 'body' subspace of superspace, or B-slice, is a submanifold where the nilpotent part is equal to zero, and it is a natural analogue to the real slice of a complex space.

Reproducing the real slice procedure of the Kerr geometry in superspace, one has to use the replacements:

- complex world line $\to$ superworldline,
- complex light cone $\to$ superlightcone,
- real slice $\to$ body slice.

Performing the body-slice procedure to superlightcone constraints

$$s^2 = [x_\mu - X_{0\mu}(\tau)][x^\mu - X_0^\mu(\tau)] = 0, \tag{9}$$

one selects the body and nilpotent parts of this equation and obtains three equations. The first one is the discussed above real slice condition of the complex Kerr geometry claiming that complex light cones can reach the real slice. The nilpotent part of (9) yields two B-slice conditions

$$[x^\mu - x_0^\mu(\tau)](\theta\sigma_\mu\bar{\zeta} - \zeta\sigma_\mu\bar{\theta}) = 0; \tag{10}$$

$$(\theta\sigma\bar{\zeta} - \zeta\sigma\bar{\theta})^2 = 0. \tag{11}$$

These equations can be resolved by representing the complex light cone equation via the commuting two-component spinors $\Psi$ and $\tilde{\Psi}$: $x_\mu = x_{0\mu} + \Psi\sigma_\mu\tilde{\Psi}$. "Right" (or "left") null planes of the complex light cone can be obtained, keeping $\Psi$ constant and varying $\tilde{\Psi}$ (or keeping $\tilde{\Psi}$ constant and varying $\Psi$.) As a result we obtain the equations $\bar{\Psi}\bar{\theta} = 0, \quad \bar{\Psi}\bar{\zeta} = 0$, which in turn are conditions of proportionality of the commuting spinors $\bar{\Psi}(x)$ determining the null ray the Kerr congruence and anticommuting spinors $\bar{\theta}$ and $\bar{\zeta}$, these conditions providing the left null superplanes of the supercones to reach B-slice. It also leads to $\bar{\theta}\bar{\theta} = \bar{\zeta}\bar{\zeta} = 0$, and equation (11) is satisfied automatically.

Thus, as a consequence of the B-slice and superlightcone constraints we obtain a non-linear submanifold of superspace $\theta = \theta(x), \quad \bar{\theta} = \bar{\theta}(x)$, which is a section of corresponding line bundle of superspace. Similarly to the complex Kerr source, this local section determines the supersource which is "seen" from the real Kerr geometry and is consequently responsible for formation the super-Kerr geometry. At this stage the super-Kerr geometry is trivial copy of Kerr geometry since the Rarita-Schwinger field is absent. However, by addition the fermionic field $\lambda = \{\zeta^\alpha(x), \bar{\zeta}^{\dot{\alpha}}(x)\}$ as a matter source, we obtain the model of non-linear realization of broken supersymmetry introduced by Volkov and Akulov [18, 19] and considered in N=1 supergravity by Deser and Zumino [16]. Similarly to the Higgs mechanism of the usual gauge theories, this Grassmann source field represents the Goldstone fermion which can be "eaten" by appropriate local supergauge transformation (6), (7) with a corresponding redefinition of the tetrad and the appearance the spin-3/2 field. Complex character of supertranslations in the Kerr case and presence of charge demand to use in this scheme the N=2 supergravity [17]. As consequence of this the initial supersymmetry is broken, and supergauge freedom is lost. Nevertheless, there is a residual supersymmetry based on free Grassmann parameters $\theta^1, \quad \bar{\theta}^1$.

We omit here details, referring to [10] and mentioning only that in the resulting exact solution the torsion and Grassmann contributions to tetrad cancel, and metric retains the exact Kerr-Newman form. However, there appear the extra wave fermionic fields on the bosonic Kerr-Newman background propagating along the Kerr congruence and concentrating near the Kerr singularity (traveling waves). Solution contains also an extra axial singularity which is coupled topologically with singular ring, threading it.

# 4 Regular sources for the rotating and non-rotating black hole solutions of the Kerr-Schild class

One of the approaches to regularization of the particlelike BH solutions is based on the old idea of the replacement of singularity by a "semiclosed world", internal space-time of a constant curvature (M. Markov, 1965; I. Dymnikova [20]). The known Dirac classical electron model, as well as the bag models could also be related to this class by the assumption that regularization is provided by a flat core region.

We consider development of these models leading to a non-perturbative soliton-like solution to supergravity and assuming that the external field is the Kerr-Newman black hole solution, and the core is described by a domain wall bubble based on the chiral fields of a supersymmetric field model.

The Kerr-Schild class of metrics $g_{\mu\nu} = \eta_{\mu\nu} + 2hK_\mu K_\nu$ allows one to consider the above regularization for the rotating and nonrotating, charged and uncharged BH's in unique manner [11]. It allows one to describe the external BH field and the internal (A)dS region, as well as a smooth interpolating region between them, without especial matching conditions, by using one smooth function, $f(r)$, of the Kerr radial coordinate $r$. In this case the scalar function $h$ has the form [3]

$$h = f(r)/(r^2 + a^2 \cos^2\theta). \tag{12}$$

In particular, for the Kerr-Newman BH solution

$$f(r) = f_{ext}(r) = mr - e^2/2, \tag{13}$$

where $m$ and $e$ are the total mass and charge. The transfer to nonrotating case occurs by $a = 0$, when the Kerr congruence turns into a twist-free "hedgehog" configuration, and $r, \theta$ are usual spherical coordinates. It is important that function $f(r)$ is not affected by this transfer that allows one to simplify treatment concentrating on the $a = 0$ case.

By $a = 0$, the regularizing core region of a constant curvature can be described by $f = f_{int}(r) = \alpha r^4$, where $\alpha = \Lambda/6$, $\Lambda$ is cosmological constant, and energy density in core is [4] $\rho = \frac{3}{4}\alpha/\pi$ .

The smooth matching of the internal and external metrics can be provided by any smooth function $f(r)$ interpolating between $f_{int}$ and $f_{KN}$. When the function $f(r)$ does not strongly deviate from these branches, the position $r_0$ of the phase transition can be estimated as a point of intersection of the plots $f_{int}$ and $f_{ext}$,

$$\frac{4}{3}\pi\rho r_0^4 = mr_0 - e^2/2. \tag{14}$$

Analysis of the stress-energy tensor for this metric shows [11] that for charged sources there appears a thin intermediate shell at $r = r_0$ with a strong tangential stress that is typical for a domain wall structure. Dividing this equation on $r_0$ one can recognize here the *mass balance equation*

$$m = M_{int}(r_0) + M_{em}(r_0), \tag{15}$$

where $m$ is total mass, $M_{int}(r_0)$ is ADM mass of core and $M_{em}(r_0) = e^2/2r_0$ is ADM mass of the external e.m. field. It should be mentioned, that gravitational field is extremely small at $r_0$, especially as $r_0$ is much more of gravitational radius [5] ( $r_0/m \sim 10^{42}$). Nevertheless, eq. (15) shows that *phase transition is controlled by gravity, but non-locally!* Note, that $M_{int}$ can be either positive (that corresponds to dS interior) or negative ( AdS interior ). As we shall see, supergravity suggests AdS vacua inside the bubble.

As consequence of this treatment we obtain also some demands to the supergravity field model for corresponding matter source:

i - It has to provide a phase transition between internal and external vacua.

ii - External vacuum has to be (super)-Kerr-Newman black hole solution with *long range* electromagnetic field and zero cosmological constant.

iii - Internal vacuum has to be (A)dS space with superconducting properties.

These demands are very restrictive and are not satisfied in the known solitonlike bag, domain wall and bubble models. Main contradiction is connected with demands ii) and iii) since in the most of models external electromagnetic field is short range. An exclusion is the $U(I) \times \tilde{U}(I)$ field model which was used by Witten to describe the cosmic superconducting strings [21]. We use a supersymmetric generalization of this field model for description the superconducting baglike configuration.

[3]For $a \neq 0$ the Kerr coordinates $r$ and $\theta$ are oblate spheroidal ones.

[4]In this case, as shows (12), gravitational singularity is regularized also by $a \neq 0$.

[5]In particular, if interior is flat ($\rho = 0$ ) $r_0 = e^2/2m$ -'classical electromagnetic radius'.

## 5 Supersymmetric superconducting bag model

The model contains two Higgs sectors: A and B. The chiral field of sector A, $\phi(r)$ forms a structure similar to "lumps", Q-balls and the other known non-topological solitons [6]. However, potential is specific here and determined by a supersymmetric domain wall model. The gauge field of this sector, $A_\mu$, acquires mass from the field $\phi(r)$ in the core and forms the long range electromagnetic field $F_{\mu\nu}$ in external region.

Sector B is 'dual' in the sense that chiral field of sector B, $\sigma(r)$, describes a cavity in superconductor, superconducting bag confining the gauge field $B_\mu$ ( $F_{B\mu\nu} = B_{\mu,\nu} - B_{\nu,\mu}$). Besides, there are hints in flavor of the dual type of superconductivity for sector B.

Supersymmetric version of the Witten field model (suggested by J. Morris [23]) has effective Lagrangian of the form

$$L = -2(D^\mu\phi)\overline{(D_\mu\phi)} - 2(\tilde{D}^\mu\sigma)\overline{(\tilde{D}_\mu\sigma)} - \partial^\mu Z\partial_\mu\bar{Z} \\ -\frac{1}{4}F^{\mu\nu}F_{\mu\nu} - \frac{1}{4}F_B^{\mu\nu}F_{B\mu\nu} - V(\sigma,\phi,Z), \tag{16}$$

where $D_\mu = \Delta_\mu + ieA_\mu$, $\tilde{D}_\mu = \Delta_\mu + igB_\mu$. The potential $V$ is determined through the superpotential $W$ as

$$V = \sum_{i=1}^{5}|\partial_i W|^2, \tag{17}$$

and the superpotential $W(\Phi^i)$ is a holomorphic function of the fife complex chiral fields $\Phi^i = \{Z,\phi,\bar{\phi},\sigma,\bar{\sigma}\}$,

$$W = \lambda Z(\sigma\bar{\sigma} - \eta^2) + (cZ + m)\phi\bar{\phi}. \tag{18}$$

In the effective Lagrangian the "bar" is identified with complex conjugation, so there are really only three independent scalar fields, and the "new" (neutral) fields Z provides the synchronization of the phase transition. The supersymmetric vacuum states corresponding to the lowest value of the potential are determined by the conditions

$$\partial_i W = 0; \tag{19}$$

and yield $V = 0$. These equations lead to two supersymmetric vacuum states:

$$I) \qquad Z = 0; \quad \phi = 0; \quad |\sigma| = \eta; \quad W = 0, \tag{20}$$

we set it for external vacuum; and

$$II) \qquad Z = -m/c; \quad \sigma = 0; \quad |\phi| = \eta\sqrt{\lambda/c}; \quad W = \lambda m\eta^2/c, \tag{21}$$

we set it as a state inside the bag.

The treatment of the gauge field $A_\mu$ and $B_\mu$ in $B$-sector is similar in many respects because of the symmetry between $A$ and $B$ sectors allowing one to consider the state $\Sigma = \eta$ in outer region as superconducting one [7] in respect to the gauge field $B_\mu$. Field $B_\mu$ acquires the mass $m_B = g\eta$ in outer region, and the $\tilde{U}(I)$ gauge symmetry is broken, which provides confinement of the $B_\mu$ field inside the bag. The bag can also be filled by quantum excitations of fermionic, or non Abelian fields.

[6] See for example [22].

[7] The version of dual superconductivity in B-sector seems the most interesting.

One can check the phase transition in the planar wall approximation (neglecting the gauge fields). It can be shown that it is a BPS-saturated domain wall solution interpolating between supersymmetric vacua I) and II). Using the Bogomol'nyi transformation one can represent the energy density as follows

$$\rho = T_{00} = \frac{1}{2}\delta_{ij}[(\Phi^i{}_{,z})(\Phi^j{}_{,z}) + (\frac{\partial W}{\partial \Phi^i})(\frac{\partial W}{\partial \Phi^j})] \tag{22}$$

$$= \frac{1}{2}\delta_{ij}[\Phi^i{}_{,z} + \frac{\partial W}{\partial \Phi^j}][\Phi^j{}_{,z} + \frac{\partial W}{\partial \Phi^i}] - \frac{\partial W}{\partial \Phi^i}\Phi^i{}_{,z}\,, \tag{23}$$

where the last term is full derivative. Then, integrating over the wall depth $z$ one obtains for the surface energy density of the wall

$$\epsilon = \int_0^\infty \rho dz = \frac{1}{2}\int \Sigma_i(\Phi^i{}_{,z} + \frac{\partial W}{\partial \Phi^i})^2 dz + W(0) - W(\infty). \tag{24}$$

The minimum of energy is achieved when the first-order Bogomol'nyi equations $\Phi^i{}_{,z} + \frac{\partial W}{\partial \Phi^i} = 0$ are satisfied, or in terms of $Z, \Phi, \Sigma$

$$Z' = -\lambda(\Sigma^2 - \eta^2) - c\Phi^2, \tag{25}$$

$$\Sigma' = -\lambda Z \Sigma, \tag{26}$$

$$\Phi' = -(cZ + m)\Phi. \tag{27}$$

Its value is given by $\epsilon = W(0) - W(\infty) = \lambda m \eta^2/c$. Therefore, this domain wall is BPS-saturated solution. One can see that the field $Z$, which appears only in the supersymmetric version of the model, plays an essential role for formation of the phase transition.

The structure of stress-energy tensor contains the typical for domain walls tangential stress. The non-zero components of the stress-energy tensor have the form

$$T_{00} = -T_{xx} = -T_{yy} = \frac{1}{2}[\delta_{ij}(\Phi^i{}_{,z})(\Phi^j{}_{,z}) + V]; \tag{28}$$

$$T_{zz} = \frac{1}{2}[\delta_{ij}(\Phi^i{}_{,z})(\Phi^j{}_{,z}) - V] = 0. \tag{29}$$

## 6 Stabilization of spherical domain wall by charge

The energy of an uncharged bubble forming from the BPS domain wall is

$$E_{0bubble} = E_{wall} = 4\pi \int_0^\infty \rho r^2 dr \approx 4\pi r_0^2 \epsilon. \tag{30}$$

However, the Tolman mass $M = \int dx^3 \sqrt{-g}(-T_0^0 + T_1^1 + T_2^2 + T_3^3)$, taking into account tangential stress of the wall, is negative

$$M_{Tolm.bubble} = -E_{wall} \approx -4\pi r_0^2 \epsilon. \tag{31}$$

It shows that the uncharged bubbles are unstable and form the time-dependent states [24].

Charged bubbles have extra contribution caused by the energy and mass of the external electromagnetic field

$$E_{e.m.} = M_{e.m.} = \frac{e^2}{2r_0}, \tag{32}$$

and contribution to mass caused by gravitational field of the external electromagnetic field ( determined by Tolman relation for the external e.m. field)

$$M_{grav.e.m.} = E_{e.m.} = \frac{e^2}{2r_0}. \tag{33}$$

As a result the total energy for charged bubble is

$$E_{tot.bubble} = E_{wall} + E_{e.m.} = 4\pi r_0^2 \epsilon + \frac{e^2}{2r_0}, \tag{34}$$

and the total mass will be

$$M_{tot.bubble} = M_{0bubble} + M_{e.m.} + M_{grav.e.m.} = -E_{wall} + 2E_{e.m.} = -4\pi r_0^2 \epsilon + \frac{e^2}{r_0}. \tag{35}$$

Minimum of the total energy is achieved by

$$r_0 = (\frac{e^2}{16\pi\epsilon_{min}})^{1/3}, \tag{36}$$

which yields the following expressions for total mass and energy of the stationary state

$$M^*_{tot} = E^*_{tot} = \frac{3e^2}{4r_0}. \tag{37}$$

One sees that the resulting total mass of charged bubble is positive, however, due to negative contribution of $M_{0bubble}$ it can be lower than BPS energy bound of the domain wall forming this bubble. This is a remarkable property of the bubble models, existence of the 'ultra-extreme' states [24] gives a hope to overcome BPS bound and get the ratio $m^2 \ll e^2$ which is necessary for particle-like models.

# 7 Peculiarities of the rotating model and role of supergravity

For the rotating Kerr case we have the basic relation $J = ma$, and for $J \sim 1/2$ one can see that parameter $a \sim 1/m$ has the Compton size. Coordinate $r$ takes the oblate spheroidal form, and the matter is foliated on the rotating ellipsoidal layers [8]. Curvature of space is concentrated in equatorial plane, near the former singular ring, forming a stringlike tube [9].

In supergravity, for strong fields there is also an extra contribution to stress-energy tensor leading to negative cosmological constant $\Lambda = -3k^4 e^{k^2 K}|W|^2$ which can yield AdS space-time for the bag interior.

The considered here supersymmetric model is more complicated than the traditionally used domain wall models [24], and it demonstrates some new properties. One of the peculiarities of this model is the presence of gauge fields which, as it was shown in thin wall approximation, allow one to stabilize bubble to a finite size. Second peculiarity is the presence of a few chiral superfields that can give a nontrivial sense to Kähler metric $K^{i\bar{j}}$ of the supergravity field models. One can expect that extra degrees of freedom of the Kähler metric can play essential role for formation of the bent (spherical or ellipsoidal) domain wall configurations. In this case there appears a singularity in the Kähler potential, and involving the axion and dilaton fields (coming from low energy string theory)

[8] It can be established since the stress-energy tensor is diagonalized in the locally corotating coordinate system [11].

[9] For the parameters of electron the phase transition region represents an oblate rotating disk of Compton size and thickness $\sim e^2/2m$.

can be necessary to suppress its influence. Therefore, some extra internal structure ("stringy or dilatonic core") can appear for the bent domain walls on the Plankian scale. This second core has to be placed inside of the large Compton scale region connected with the above domain wall structure and the chiral (Higgs) fields. It should be noted that the typical superstring BH solutions do not contain the Higgs fields and are usually regular if only they are magnetically charged. On the other hand, there was mentioned close similarity between N=2 black holes, N=4 black holes of superstring vacua and domain wall solutions to N=1 supergravity [25, 24] (in particular they have similar causal structure and a common classification in terms of the dilaton coupling parameter). These facts suggest existence of non-perturbative regular solutions to supergravity having a *hybrid* form in which a phase transition changes character of theory near the core: from electrically charged external solution of the Einstein-Maxwell (super)gravity to an internal solution described by a dual magnetically charged black hole solution to superstring theory.

We would also like to note, that the connected with superconductivity chiral fields acquire a nontrivial geometrical interpretation in the Seiberg–Witten theory and in the Landau – Ginzburg theory where the $N = 2$ chiral superfields refer to the moduli of the internal Calabi–Yau spaces [26]. In the case of a few chiral fields it gives an interesting link to higher dimensions with an alternative look on the problem of compactification.

# 8 Conclusion

The treatment shows that:

- super-Kerr geometry displays very rich complex, stringy and super-structures and represents one of the most adequate backgrounds in non-perturbative approach to the structure of spinning particles;
- regularization of the Kerr singularity can be achieved by a phase transition to some new vacuum state described by supersymmetric domain wall model;
- in spite of the extreme smallness of the local gravitational field, supergravity can control the position of phase transition at large distances;
- core of the Kerr spinning particle has the shape of oblate rotating disk (of the Compton size), and one can expect a sensitivity of differential sections for polarized spinning particles, depending on the direction of polarization.

It should also be mentioned that the parameters of elementary particles are very far from the typical extreme values of black hole parameters, ($m \ll e$), and quantum corrections can be high, leading to quantum excitations of the fields inside the bag (circular traveling waves) as well as of the bag boundary. It has to lead to a nonstationary Kerr background [13, 27] that represents a still unsolved problem.

The model suggests that the formation of spin can be connected with a non-trivial rotating vacuum state forming the disklike bag. Since gravitational field is extremely small, its influence on the geodesic motion of the scattering particles has to be negligible besides very thin region (string) near the border of the Kerr disk where the strong fields are concentrated. Vacuum state has relativistic boost in this region. The trapped partons have also to be relativistically boosted, reproducing Zitterbewegung similarly to some old models of spin built of the lightlike circular currents and traveling waives (H. Hönl, A. Schild and others [28]).

Thus, interaction by particle collisions can occur only by direct contact with this thin string or with partons inside the bag, either via the Coulomb excitation (including the case of very soft photons). Apparently, one can expect also excitation of the vacuum state of the bag (Higgs fields) in the deep inelastic processes.

We would like to thank Organizing Committee for kind invitation to give this talk and for financial support.

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