Chaos as the True Source of the Irreversibility of Time

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Summary

Several definitions of time are presented in the first part: freedom seems to be an essential element of the notion of time. The second part begins with some classical examples of irreversibility, that have some surprising relationships. The paradox of reversible physical laws associated with irreversible phenomena is related to the existence of systems of chaotic character with a large number of parameters. A simplified model allows to understand and to solve the paradox.

Хаос как действительная причина необратимости времени

В первой части будут представлены и обсуждены несколько разных определений времени: свобода представляется существенным элементом идеи времени. Вторая часть начинается с обсуждения классических примеров необратимости, которые обнаруживают удивительное родство. Парадокс обратимых физических законов, связанных с необратимыми феноменами, соотносится с существованием систем хаотического характера с большим числом параметров. Упрощенная модель позволит понять и разрешить этот парадокс.

Le chaos source véritable de l'irréversibilité du temps

Différentes définitions du temps sont présentées et discutées dans la première partie : la liberté semble un élément essentiel de la notion de temps. La deuxième partie commence par des exemples classiques d'irréversibilité, lesquels présentent des liaisons surprenantes. Le paradoxe des lois physiques associées à des phénomènes physiques irréversibles est lié à l'existence de systèmes de caractère chaotique avec un très grand nombre de paramètres. Un modèle simplifié permet de comprendre et de résoudre le paradoxe.

Introduction

Many phenomena of the day-to-day life are, or at least seem, irreversible: we remember the past and ignore the future, we are growing old, the heat always goes from hot bodies to cold ones and the sugar disappears into the coffee... However the known physical laws are reversible.

This paradox has led to many researches and many controversies at the limit of philosophy. The classical answer are incompatible with each other and neglect or underestimate the importance of chaos that is certainly the essential reason of existence of physical irreversibility.

Preliminary notice: numeration system

In this paper with many very large and very small numbers, we will use the numeration "by figures and sizes" with the letter p for "positive power of ten" and the letter n for "negative power of ten".

Hence for instance : Avogadro number = $6.02 \times 10^{23} = 6.02 \text{ p23}$;

Proton rest mass = $1.673 \times 10^{-27} = 1.673 \text{ n}27 \text{ kg}$.

Notice that the "figures", that is here 6.02 and 1.673, are always between one and ten. This give an unambiguous definition to the "sizes", here p23 and n27. The size is the main element of very large and very small quantities, it is very often their only known element.

First part. The time

Let us consider for a moment an old Babylonian tablet of the upper period, it shows for instance a mathematical demonstration or the computation of the square root of two with six digits...

What move us? These scientific results so small at our scale? No it is the time that move us... the fact that these stammerings of human intelligence occurred 4000 years ago.

Time is something very mysterious, it is extremely subjective with one meaning per profession and almost one meaning per people.

We find for instance:

Table 1. The time of cosmologists (in the standard model)

Time	Events	
0	Big bang	
n43 second	Planck's wall	
n35 to n32 s	Inflation	
n6 to n4 s	Hadronic era (protons, neutrons)	
n4 s to 1 s	Leptonic era (electrons, photons, neutrinos)	
0.5 s	Escape of neutrinos	
100 s	First atomic nucleus, primordial nucleo-synthesis	
300 000 years	Escape of photons	
	Formation of atoms (H, He,)	
	Quasars, formation of galaxies	
4 p9 years	Successive generations of stars	
p10 years	Birth of the solar system	
1.5 p10 years	Today	

Table 2. The time of geologists

Approximated dates (millions of years)	Events, eras and	d periods	
4600	Pre-Cambrian time	Formation of Earth Pre-Cambrian time	
500 440 400	Cambrian Ordovician Silurian		Paleozoic era
350 270	Devonian Carboniferous Permian		Primary
	Triassic Jurassic Cretaceous	Triassic Jurassic	
55 38 25 5.1	Paleocene Eocene Oligocene Miocene Pliocene	Tertiary	Cenozoic era
0.01	Pleistocene Holocene	Quaternary	

It is well known that the time of cosmologists is a mixture of space and time, but this is also true (in another way) for the time of geologists. For instance the "cretaceous-tertiary transition" is given by the disappearance of dinosaurs, disappearance that has perhaps required tens of thousand of years to go from India to Mexico.

It must be notice that geologists recognize the periods of sedimentary strata by the presence of various types of fossils. Very few fossils exist in the pre-cambrian strata and at the beginning of the Cambrian period occurred literally an "explosion of life".

Since then, the differences are essentially given by the mass extinctions (Table 3). If sometimes species seem to appear here or there, as mammals at the beginning of tertiary, this is mainly because before theses periods these species were confined by their enemies to few varieties and remote areas.

Table 3. The mass extinction record

periods	Percentage of disappearing species (among the remaining species just before the extinction).	
	80%	
Triassic		
	45%	
Jurassic		
	20%	
Cretaceous		
	70%	
Paleocene		
Locone	50%	
Oligocene	3070	
rnocene		
	Triassic	

The true motor of the evolution is not yet known and even if Darwinism is true it is insufficient. It explains very well a small evolution (change of scale, apparition of a color) or a regressive evolution (loss of an organ), but the billions and billions of mutations of geologic times are, by far, not numerous enough to explain a progressive evolution such as the gain of a new organ.

Notice two opposite effects: the time of cosmologists seems to run very fast at the beginning with many events occurring in a very small period, and it progressively slows down. This is the contrary for geologic times with seven eighths occurring in pre-Cambrian times and with always smaller and smaller eras.

This acceleration is partly an effect of perspective: recent events and recent fossils have a larger chance to let remains in our time, but it is also partly a real effect that is even more visible in the time of prehistorians and archeologists (Table 4).

The below paleolithic scale of table 4, in which glacial ages have German names and periods have mostly French names, has been defined for Europe. The different paleolithic scales (Europe, Africa, Middle-East, Far-East, America) have still uncertain and approximate correspondences.

Notice Adam and Eve at the beginning of middle paleolithic: these names have been given by the prehistorians to the first men that have buried their deads... the awakening of human consciousness.

Table 4. The time of prehistorians and archeologists

4.1. Paleolithic age (hunters, cavern artists).

Dates (years BC)	Men	GLACIAL AGES	Periods	
3 millions				
	Australopithecus (Homo habilis)			
1 million	,			
500 000	Pithecanthropus	GÜNZ		Lower Paleolithic
	Sinanthropus		Abbevillian	Lower raicontine
	(Homo erectus)	MINDEL	Clactonian	
		RISS	Chellean Premousterian	

100 000	(Adam and Eve) Neanderthal	WÜRM(1)	Mousterian Pontinian Altmühlian Jabroudian	Middle Paleolithic
35 000	First Amerindians	WÜRM(2)	Perigourdian Aurignacian	Unner Poloolithia
0.000	Cro-Magnon (Homo sapiens)	WÜRM(3)	Solutrian Magdalenian Azilian	Upper Paleolithic

9 000

4.2. Neolithic age (shepherds, farmers, builders).

Dates (BC)	Domestic ani	imals and cereals	Events
9000	Dogs Bulls and cows		Villages with round houses
8000	Horses		
7000	Goats Sheeps	Corn	Villages with square houses Jericho
6000	Pigs	Maize (US corn)	Baked clay Catal Hoyük (Turkey)
5000		Rice	Megaliths
3300			
4.3. Bronz	e age	End of Prehistory /	
4.4. Iron a			/ Beginning of History

The acceleration encountered in geological and archeological times is even stronger in historical times (Antiquity, Middle Age, Modern times) and events seems to appear at an always faster rate.

This obvious property of lived time is in strong contrast with the regular times defined and used by the scientists :

The time of mechanicians: $\mathbf{F} = \mathbf{m} d^2 \mathbf{r} / dt^2$ The time of quantum physicists: $\Delta \mathbf{E}$. $\Delta t \ge \mathbf{h}$

The time of biologists: cycles, rhythms, yearly hibernations, prolonged hibernations.

Similarly we find the times of astronomers, seamen, aviators, clock-makers, musicians etc...

The scientific definition of the time

Historically the scientific definition of the time is a creation of astronomers.

Up to the eighteenth century the "time", or preferably the civil time used in ordinary life, was determined by the daily motion of the Sun: It was midday, literally the middle of the day, when the Sun was at its highest position in the sky and then in the direction of either South or North.

Unfortunately this "local true solar time" is very irregular for two main reasons: the Earth has a large inclination, about 23.5°, and its orbit is an ellipse and not a circle. It was then difficult to built clocks able to follow this time (even if to-day this irregularity would be no more an obstacle) and a more regular time was defined: the "local mean solar time" related to the daily revolution of stars with its periodicity of 23h 56mn 4.099 66s.

The difference "true solar time" minus "mean solar time" has an annual periodicity and varies 14mn 18s (February 11) and + 16mn 24s (November 3).

However with this local system each city has its own local time and with the development of railways the confusion become so great that many countries decide to adopt as civil time the mean solar time of their capital. For instance the civil time of Netherlands was twenty minutes earlier than that of Greenwich.

The mean solar time of the Greenwich observatory, near London, became the civil time of all Britain in the 1880's under the name of GCT, Greenwich civil time, later called GMT, Greenwich mean time.

In the large countries, USA, Canada and especially Russia, a unique time was not considered as possible (although today China, India and also continental Europe from Spain to Poland have a unique hour) and the time-zone system was slowly organized.

The global time-zone system with its 24 zones was defined at the "international conference on hour" (Paris 1913). France was already at the GMT since March 9, 1911.

In 1925 the GMT was renamed "universal time" (UT or TU, temps universel, or WZ, Weltzeit).

The progress of time measurements have led to the following subdivisions.

TU zero or TU0 is the original GMT defined by the daily revolution of stars as seen from the Greenwich observatory.

TU1 is a more regular time taking account of the small motions of the poles with respect to the Earth surface. It can be defined as an averaged mean time for many suitable observatories that would be regularly spaced on Earth.

TU2 takes account of the main yearly variations of Earth rotation. These variations are related to the largest meteorological phenomena such as the monsoon.

TU3 takes account of the main lunar and solar effects either direct (attraction, modification of the vertical) or indirect (tides).

These four TU have only small differences:

$$|TU1 - TU0| < 30$$
 milliseconds
 $|TU2 - TU1| < 60$ milliseconds
 $|TU3 - TU2| < 4$ milliseconds

The discovery of random variations of Earth rotation and of its very slow decrease related to tidal frictions has ruined the validity of Universal Time for a regular scale of time.

In 1955 the unit of time, the second, became defined by the motions of planets ("second of ephemeris") but this difficult and inconvenient definition was forsaken in 1967 for the accurate and convenient second defined by the atomic frequency standards... And thus the rule of astronomers over the definition of time has come to an end. Their place is now taken by the physicists.

A sufficiently large series of atomic clocks now gives the extremely regular and accurate "international atomic time" written TAI (*temps atomique international*). Its full parallelism with the ephemeris time given by the motion of planets has been verified over thirty years with a relative accuracy of n10 (that is 10^{1}).

The ephemeris time is now called TDT (temps dynamique terrestre, terrestrial dynamic time). It is henceforth defined by:

$$TDT = TAI + 32.184$$
 seconds

the difference being there for historical reasons.

Our ordinary civil time cannot be based only on the TAI, it needs a relation to the Sun. We arrive thus to the "universal time coordinated" (UTC), or in a better English the coordinated universal time (but CUT is not a good sigle).

The UTC is standard since January the first, 1972; it is defined by the following:

- A) The difference (TAI UTC) is always an integer number of seconds.
- B) The difference | TU1 UTC | is always smaller than 0.9 seconds.
- C) From time to time the director of the *Bureau international de l'heure* (**BIH**, also known as **IERS**: International Earth Rotation Service) decides to add an intercalate second in order to take account of the irregularities of the TU1, that is the irregularities of the Earth rotation.
- D) These supplementary seconds always fall at the end of a semester: the last minute of time U T C has then 61 seconds.
- E) The difference (TAI UTC) was 10 seconds during the first semester of 1972 and, up to May 2001, twenty two seconds have been added at the following dates.

June 30	December 31	June 30	December 31
1972	1972	1982	
	1973	1983	
	1974	1985	1987
	1975		1989
	1976		1990
	1977	1992	
	1978	1993	
	1979	1994	1995
1981		1997	1998

Hence the difference (TAI – UTC) was 32 seconds in the year 2000.

The universal time coordinated UTC has thus a mixed statute. It has the extremely regular unit of time of the international atomic time, T A I, but is also related to the irregular Earth rotation and can thus, for instance, be used for the determination of longitudes.

For very accurate determinations the publications of the *Bureau international de l'heure* allows to know the difference (UTC – TU1) at any time with an accuracy of one millisecond.

The UTC remains always very near to the old TU1 and GMT, it plays now the role formerly played by the GMT, for instance :

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Moscow time = UTC + 3 hours
US Eastern Standard Time (E.S.T.) = UTC - 5 hours
US Eastern Daylight savings Time (E.D.T.) = UTC - 4 hours
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Today the GMT has only an historical interest. It has been abandoned since the thirties for successively the TU1, the TU2 and finally, in 1972, for the much more regular UTC.

The international abbreviation of UTC is the following from the largest to the smallest unit:

June 25, 2001 at 7 hours 42 minutes 3.26 seconds UTC = 2001-06-25-07-42-03.26 UTC

The corresponding day is of course 2001-06-25 and the corresponding hour is 2002-06-25-07 UTC.

Notice finally that the astronomers have defined a theoretical and fictitious much more regular time that takes account of the relativistic effects and is used in the study of the motions of the Solar System: The "barycentric time" B.T. (Consider an atomic clock very far from the Sun and without velocity with respect to the center of mass, i.e. the barycenter, of the Solar System). The barycentric time is, in the average, a little "faster" than the International Atomic Time, in spite of their same unit of time, but their relative difference is very small, only 1.55 n8, that is about 0.5 second per year.

A few philosophy

Time is of course one of the most cherished subject of philosophers and they have noticed that in front of the wide variety of the "times" of all professions there is at least a time that is common to all people : the psychological time.

"Waiting half an hour for Dad when he is going to arrive at the station... It's very long! Playing half an hour before sleeping...It's very short! I have to go to bed already! It's unfair!". My granddaughter Eléonor was only five years old when she has said these words (in French of course) and thus a five years old child already know that the psychological time is not uniform. Furthermore it has the unpleasant but universal property of being slow during unpleasant periods and fast during pleasant ones.

Unfortunately my granddaughter died in a car accident a few weeks later and this is the strongest and most painful evidence of the irreversibility of time...

From Saint Augustine to Ferdinand Gonseth the propositions of philosophers about the time are famous: "When I am alone I believe that I know what time is...But as soon as someone put me the question I don't know it anymore". "Present is a door through which future is hurl down into past". "Past don't exist because it exists no more, future don't exist because it don't yet exist and present is continuously disappearing".

However I contest that kind of symmetry that some philosophers do between past and future. Past seems to me much more concrete than future. Notice that this corresponds to the popular feeling: in most languages future and conditional are almost alike... While if we look at some outer galaxy we will see it as it was millions of

year ago. If we discover a well oriented mirror on a planet near Alpha Centaurus (distance 4 light-years) we will see the Earth as it was eight years ago.

Let us consider an astronomical study such as the evolution of the Earth-Moon system under the influence of known forces (gravitation, tidal forces, etc.). The average Earth-Moon distance is slowly increasing, about three centimeters per year today, it was much smaller 3 billions of years ago, it will be 10 or 20% larger in 3 billions of years.

However the past let traces (in geology, in magnetism, in the present state of the Moon, etc.) and our hypotheses on the past are then partly testable, while our hypotheses on the future are pure intellectual constructions. They could be completely upset either by some unexpected phenomenon (passage of a star in the vicinity of the Solar System) or because of the neglect of some apparently extremely small phenomena that will increase very much in the future.

If we have such a conscience of the present time it is certainly because we are **free**. Of course no scientific experiment can prove that we are really free, or not, and we must admit freedom as an axiom, as one of the unprovable axioms of geometry or arithmetic: "There is a source of freedom in each human being..." With this axiom our world is much more understandable than with the opposite axiom and the present is the instant in which we exercise our freedom.

It is possible to find *a contrario* proofs of this proposition in the state of sleep in which we lack both present and freedom, or in the situation of a severely guarded prisoner that is forbidden to work, to read, to write, to sing...He forsakes the present and dreams about the past in which he was free and about the future in which he will be liberated.

Second part. The paradox of time

The second principle of thermodynamics is the pillar of physical irreversibility. However it is a "principle" and not a "law"; this linguistic subtlety allow us to write that all physical laws can be written in a reversible form, a form in which past and future are symmetrical.

This leads to well known reversible properties. For instance if, in the Solar System, we reverse the velocities of all planets and satellites, the orbits will remain the same and they will be described in the other direction.

However this beautiful symmetry contradicts many phenomena of ordinary life. We remember the past and ignore the future, the heat always go from hot bodies to cold ones, we always goes from childhood to old age, etc.

Besides these usual phenomena the physicists consider that the major irreversible phenomena are:

- A) The expansion of Universe.
- B) The black holes: light and matter fall into black holes, they never escape from them.
- C) The propagation of light: by diverging waves and never by converging waves.

There are some remarkable connections between these irreversible phenomena.

Let us consider the example given by Hubert Reeves (Ref. 1): A lake in a cold country, it freezes each autumn and thaws each spring.

Ice is much more organized than water, it is then during freezing that the entropy of the lake decreases. What happens then?

The lake cannot be considered as isolated and, during the long and starry night of November and December, it sends towards space a huge quantity of infra-red photons that carry away its heat and its entropy.

What happens to these photons? Because of the expansion of Universe, most of them will never arrive anywhere, they will wander forever in an always emptier space...

If Universe was static, the number of arriving photon would equal this of departing photon and the lake could not freeze.

There is however a question: if Universe is dense enough, it will not expand forever and in some tens of billions of years it will begin to contract. What will then happen? Some theoreticians think that then the time will reverse, we will escape from grave and return to childhood! This is really surprising and it is more probable that the black hole, so rare today, will then be very numerous. They will take the place of the expansion and most wandering photons will fall into a black hole and disappear forever. The lakes will continue to freeze each autumn...

But let us go back to the paradox between the reversible physical laws and the irreversible phenomena. Let us consider the following experiment: two vessels are full of gas. If we open the communication between these two vessels the Brownian motion will equalize the temperatures, the pressures and the compositions while the opposite evolution never appears.

However:

- A) The Brownian motion and the kinetic theory of gas are conservative and reversible, as conservative and reversible as the Celestial Mechanics itself.
- B) Henri Poincaré has demonstrated that, for bounded and conservative systems, almost all initial conditions lead to an infinite number of returns in the vicinity of initial conditions (Ref. 2). The mathematicians specify: in any vicinity of the initial conditions.

These returns in the vicinity of initial conditions are of course contradictory with the equalization of temperatures, pressures and compositions.

There are classical but unsatisfactory answers to this paradox:

A) "There exist perhaps some small, irreversible and dissipative hidden phenomena that forbid the application of Poincaré results..."

This rejection of a major symmetry of nature is not justified and our present knowledge is sufficient for the resolution of the observed contradiction.

B) "For a given phenomenon the notion of trajectory remains accurate for only its time of divergence that is about fifty or one hundred "Liapounov time" and much less than the Poincaré return time that has never been observed in this type of experiment..."

This answer is true but insufficient. The impossibility of accurate computations doesn't solve the contradiction.

C) "In principle Poincaré is right and for strictly isolated systems there indeed exists this mysterious correlation between initial and final conditions (after the Poincaré return time). But our systems are not strictly isolated and even very small perturbations destroy this correlation."

These "mysterious correlations" are imaginary and it is in a natural fashion that the system returns towards all states attainable from the initial conditions. The "very small perturbations" will not modify the order of magnitude of the Poincaré return time, even it is true that they can modify very much the evolution in a relatively short interval of time (a few "Liapounov times") and thus contribute to the disappearance of correlations.

The true answer is related to the notion of chaotic motions and "sensitivity to initial conditions". The evolution of a non chaotic system (a "regular system") remains in a very small part of phase space and has a natural reversibility, especially for periodic and quasi-periodic systems.

But let us consider a usual system with billions of parameters (the positions and velocities of the molecules of a small volume of gas...). That system is extremely sensible to initial conditions (a slight difference in the initial velocity of a molecule and its future is completely different...) and, because of the very large number of parameters, the Poincaré return time is much larger than the age of Universe.

In these conditions we are unable to follow the individual evolution of given molecules and we are accustomed to define averaged statistical elements (temperature, pressure, etc.). The evolution of these statistical elements can be deduced by statistical considerations with an excellent accuracy because of the very great number of elements (and because of the chaoticity itself that introduces randomness permanently, which allows to use successfully the laws of random and the laws of the great numbers)...But the average evolution of statistical elements is not conservative and reversible even if the system of interest is conservative and reversible.

This property is not a physical one, it is a pure mathematical property of averaged statistical elements and it is also the real reason of the second principle of thermodynamics and of the usual "arrow of time".

But how is it possible to reconcile the reversible laws of individual elements with the irreversible laws of averaged statistical elements? The reconciliation is in the difference between the average and the reality of these statistical elements. For system with a large number of parameters that difference is usually very small and inappreciable, but it can become large after a "sufficiently long time", for instance for the Poincaré return in the vicinity of initial conditions.

In reference 3 is presented a small application of these ideas: Two identical vessels contain a total number of one billion billion of molecules (that is p18 molecules, a rather small number only anyway since it is the number of molecules in 37 mm³ of ordinary air). The initial temperature is the same on both sides while the initial pressures are 1.4 bar and 0.6 bar (the initial share is 70%, 30%).

At the beginning of the experiment we open the communication between the two vessels and their rate of exchange is one million billion (i.e. p15) molecules per second.

In agreement with the nature of chaotic phenomena we can assume that each exchange is chosen at random with an equal probability for all molecules. That assumption is almost the "Boltzmann hypothesis": no correlation between successive variations.

The average evolution is simple: the temperature and the sum of the two pressures remain constant while the individual pressures converge exponentially towards one bar. The largest pressure falls to 1.3548 bar after one minute, to 1.1205 bar after ten minutes and to 1.0003 bar after one hour.

The standard deviation of the pressure remains forever very small: always less than a billionth of a bar, that is 0.0001 Pascal, and thus if we measure the pressure with the excellent accuracy of 0.005 Pascal (that is nevertheless 50 standard deviations) we will never meet a fluctuation with respect to the average evolution: the first fluctuation has a chance smaller than n500 (i.e. 10^{1}) to appear before fifty billion milleniums!

In these conditions the Poincaré return time is of course purely theoretical, but it can be computed: With the exception of the very small proportion of n200 of initial conditions, the Poincaré return time T at the initial pressures of 1.4 and 0.6 bar will verify:

 $T = 10^{R}$ milleniums; with: 35 735 000 089 859 491 < R < 35 735 000 089 859 696.

Thus the paradox of reversible laws associated with irreversible phenomena can be explained without "small hidden irreversibilities", "perfect isolation" and/or "hidden correlations". The main reasons of the physical irreversibilities and of the arrow of time is the chaotic character and the very large number of parameters of irreversible systems.

The Boltzmann hypothesis of "molecular chaos" gives an excellent approximation of chaotic phenomena and allows very accurate computations. The correlations will not increase slowly and insidiously after a very long time and we can almost write that the return of Poincaré occurs by chance which require such a large delay, much larger than the age of Universe, that the corresponding decrease of entropy never appears in our experiments.

Conclusions

The irreversibility of the second principle of thermodynamics agree fully with <u>our experiments</u> and <u>our measures</u> that are by far neither long nor numerous enough to lead to a contradiction.

If we meet so many phenomena with an increase of entropy, it is because desequilibriums are easy in our world. The poles are cold and the equator is hot, the smallest valley has a sunny side and a shady one... The fundamental reason is our existence in the middle of a giant stream of energy that arrives continuously from the burning Sun and escapes to the frozen space.

At each scale of nature (quantic, atomic, microscopic, ordinary, geographical, astronomical, cosmological) the chaotic motions destabilize the individual elements (position and velocity of a particle) but stabilize the corresponding mean statistical elements (pressure, temperature) that become the basic elements of the larger scale. Phenomena are thus nested in one another up to the astronomical and cosmological scales that use the notion of "center of mass of a celestial body" and study its motion without being disturbed by the inner motions and the streams of this body.

Thus in our world, in which we have now recognized so many large numbers and so many chaotic phenomena, the irreversibility of time - that is the existence of irreversible phenomena - is not contradictory with the reversibility of physical laws. We understand now how much these chaotic phenomena participate in the image of our physical world and how far we were from the truth when, only three decades ago, these chaotic phenomena were either ignored or considered as very exceptional and very singular phenomena described for the first time one century ago by the gigantic imagination of the great pioneer Henri Poincaré.

References

- 1. **H. Reeves.** "Patience dans l'azur" Editions du Seuil, collection "Science ouverte", page 186, 1984.
- 2. **H. Poincaré.** "Les méthodes nouvelles de la Mécanique Céleste" Dover Publication, New-York, Vol 3, pages 140-174. 1957.
- 3. **C. Marchal.** "Determinism and arrow of time. Henri Poincaré philosopher and scientist" 22^{nd} International Workshop on the Fundamental Problems of High Energy Physics Protvino Russia, June 23-25, 1999. Also ONERA-TP n° 1999-123.