(Non) Abelian Gauged Supergravities in Nine Dimensions

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We construct five massive deformations of the unique nine–dimensional $N = 2$ supergravity, each with two parameters. All of these deformations have a higher–dimensional origin via Scherk–Schwarz reduction and correspond to gauged supergravities. The gauge groups we encounter are $SO(2), SO(1,1)^+, \mathbb{R}, \mathbb{R}^+$ and the two–dimensional non–Abelian Lie group $C\mathbb{R}^1$, which consists of scalings and translations in one dimension.

We make a systematic search for two classes of vacuum solutions: maximally symmetric solutions with constant scalars and half-supersymmetric domain wall solutions. In the first category we find explicit solutions in the form of (non-supersymmetric) de Sitter space solutions. In the second category we find precisely the three classes of domain wall solutions that were given in an earlier work. We discuss which of the D=9 gauged supergravities can be considered as candidate low-energy limits of compactified superstring theory.

1 Introduction

The procedure of gauging a global symmetry includes the replacement of the ordinary derivative by a covariant derivative:

$$\partial_\mu \longrightarrow D_\mu = \partial_\mu + gA_\mu \,. \tag{1}$$

Here A_μ is the gauge field and g is the gauge coupling constant which acts as a deformation parameter of the ungauged theory. In the case of Einstein gravity with scalars one can consider as an independent deformation the addition of a scalar potential $V(\varphi)$:

$$R + (\partial\varphi)^2 \to R + (\partial\varphi)^2 + m^2 V(\varphi) \,. \tag{2}$$

In the supersymmetric case, i.e. the case of *gauged* supergravity, the two deformations are not independent. Supersymmetry relates the two deformation parameters:

$$g = m \,. \tag{3}$$

Due to the scalar potential the Minkowski spacetime is no longer a maximally supersymmetric vacuum solution of the gauged supergravity. Instead we will search for other vacuum solutions, like, e.g., non-supersymmetric de Sitter space solutions. A natural class of *half-supersymmetric* vacuum solutions that makes use of the scalar potential is the set of domain wall solutions. Recently, domain wall solutions of supergravity theories have attracted attention in view of their relevance for a supersymmetric Randall-Sundrum scenario [1, 2], the domain wall/QFT correspondence [3, 4] and applications to cosmology [5, 6]. In all these applications the properties of the domain walls play a crucial role and these properties are determined by the details of the scalar potential.

Motivated by this we studied general domain wall solutions in D=9 dimensions [7]. We took D=9 because on the one hand this case shares some of the complexities of the lower-dimensional cases, on the other hand the scalar potential for this case is simple enough to study the corresponding domain wall solutions in full detail. The supergravity theory we considered in [7] was obtained by a generalized Scherk-Schwarz (SS) reduction of D=10 IIB supergravity. This is not the most general possibility in D=9. In this talk we will present a systematic search for massive deformations of the unique D=9, N=2 supergravity theory. All deformations we find correspond to *gauged* supergravities. The hope is that the D=9 case will teach us something about the more complicated situation in D < 9 dimensions. The results presented in this talk are taken from [8].

2 Massive deformations of D=9, N=2 Supergravity

The field content of the unique $D = 9$, $N = 2$ massless supergravity theory is given by $(i = 1, 2)$

$$e_\mu{}^a, \phi, \varphi, \chi, A_\mu, A_\mu^{(i)}, B_{\mu\nu}^{(i)}, C_{\mu\nu\rho}, \psi_\mu, \lambda, \tilde{\lambda}\,. \tag{4}$$

The massless 9–dimensional theory has four global scaling symmetries, with parameters α, β, γ and δ, respectively. The scaling weights of all these symmetries are given in Table 1.

Table 1. *The scaling weights of the 9 dimensional supergravity fields.*

	$e_\mu{}^a$	e^ϕ	e^φ	χ	A_μ	$A_\mu^{(1)}$	$A_\mu^{(2)}$	$B_{\mu\nu}^{(1)}$	$B_{\mu\nu}^{(2)}$	$C_{\mu\nu\rho}$	ψ_μ	λ	$\tilde{\lambda}$
α	$\frac{9}{7}$	0	$\frac{6}{\sqrt{7}}$	0	3	0	0	3	3	3	$\frac{9}{14}$	$-\frac{9}{14}$	$-\frac{9}{14}$
β	0	$\frac{3}{4}$	$\frac{\sqrt{7}}{4}$	-$\frac{3}{4}$	$\frac{1}{2}$	$-\frac{3}{4}$	0	$-\frac{1}{4}$	$\frac{1}{2}$	$-\frac{1}{4}$	0	0	0
γ	0	-2	0	2	0	1	-1	1	-1	0	0	0	0
δ	$\frac{8}{7}$	0	$-\frac{4}{\sqrt{7}}$	0	0	2	2	2	2	4	$\frac{4}{7}$	$-\frac{4}{7}$	$-\frac{4}{7}$

It turns out that only three out of the four scaling symmetries given in Table 1 are linearly independent. There is a relation

$$\frac{4}{9}\alpha - \frac{8}{3}\beta = \gamma + \frac{1}{2}\delta\,. \tag{5}$$

The massless N=2, D=9 theory also has an $SL(2,\mathbb{R})$ symmetry:

$$\begin{aligned}
&\tau \to \frac{a\tau + b}{c\tau + d}\,, \quad \vec{A} \to \Omega \vec{A}\,, \quad \vec{B} \to \Omega \vec{B}\,, \quad \text{with } \Omega = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in SL(2,\mathbb{R})\,,\\
&\psi_\mu \to \left(\frac{c\,\tau^* + d}{c\,\tau + d}\right)^{1/4} \psi_\mu\,, \qquad \lambda \to \left(\frac{c\,\tau^* + d}{c\,\tau + d}\right)^{3/4} \lambda\,,\\
&\tilde{\lambda} \to \left(\frac{c\,\tau^* + d}{c\,\tau + d}\right)^{-1/4} \tilde{\lambda}\,, \qquad \epsilon \to \left(\frac{c\,\tau^* + d}{c\,\tau + d}\right)^{1/4} \epsilon\,.
\end{aligned} \tag{6}$$

The fields φ and C are invariant.

We now turn to massive deformations of the 9D theory. To obtain these deformations we will apply a SS reduction which can be best illustrated by an example. Consider a single scalar field coupled to gravity:

$$\hat{\mathcal{L}} = -\frac{1}{2}\sqrt{-\hat{g}}\big(\partial\hat{\phi}\big)^2\,, \tag{7}$$

which is invariant under the $\mathbb{R}$–symmetry $\hat{\phi} \to \hat{\phi} + c$. In the SS procedure one gives the field a dependence on the compactification coordinate x which is governed by a global symmetry, in this case the $\mathbb{R}$–symmetry:

$$\hat{\phi}(x, z) = \phi(x) + m_\phi x \,. \tag{8}$$

Using the standard reduction rules[1] the Lagrangian reduces to

$$\mathcal{L} = -\frac{1}{2}\sqrt{-g}\left(\left(D\phi\right)^2 + m_\phi^2\right) \,, \tag{9}$$

where $D_\mu \phi = \partial_\mu \phi - m_\phi A_\mu$ with A_μ

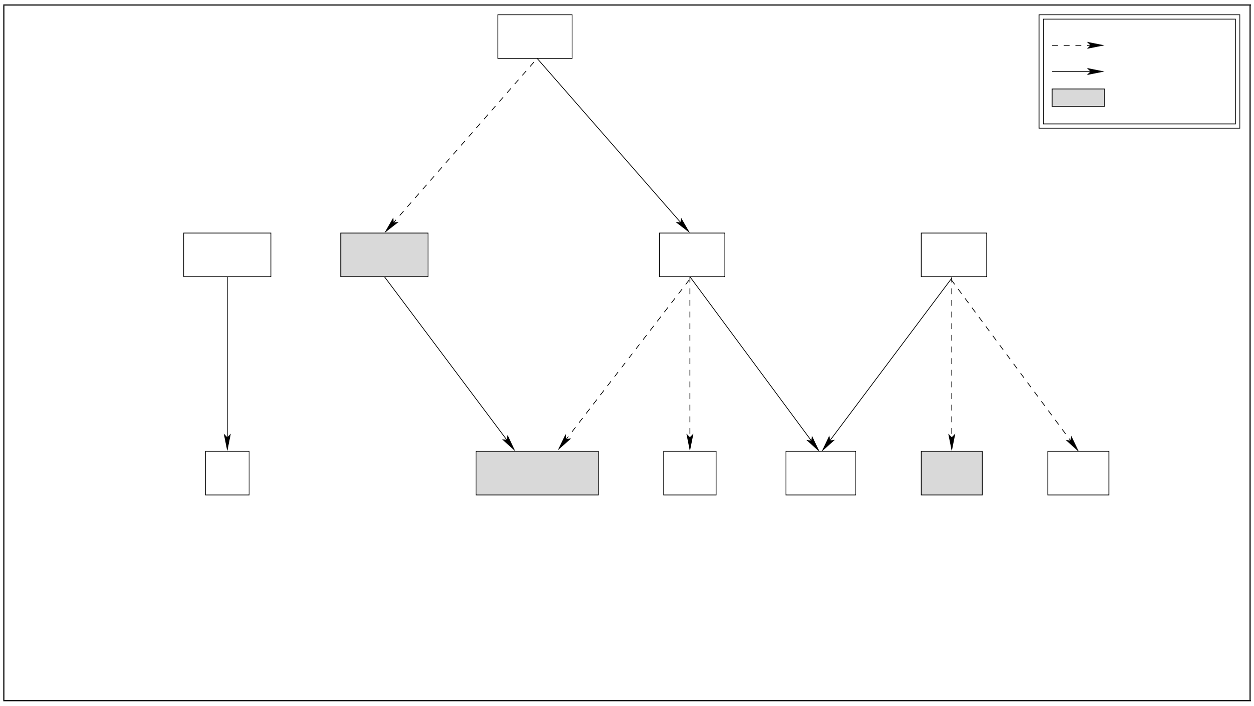

Fig. 1. *Overview of all reductions discussed in this talk. These cases can all be interpreted as gauged supergravities, with gauged symmetry and corresponding gauge field as given in the Figure. Mass parameters in the same box, such as $m_{11}, m_{\rm IIA}$ or m_1, m_2, m_3, form a multiplet under $SL(2,\mathbb{R})$.*

Note that the different massive deformations can be related. Symmetries of the massless theory become field redefinitions in the massive theory that only act on the massive deformations. This means that the mass parameters transform under such transformations: they have a scaling weight under the different scaling symmetries and fall in multiplets of $SL(2,\mathbb{R})$. In Table 2 the multiplet structure of the massive deformations under $SL(2,\mathbb{R})$ is given. The mass parameter $\tilde{m}_4$ is defined as the S-dual partner of m_4 and can not be obtained by a SS reduction of IIA supergravity.

Table 2. *This tables indicates the different multiplets that the D=9 mass parameters form under $SL(2,\mathbb{R})$.*

Mass parameters	$SL(2,\mathbb{R})$
(m_1, m_2, m_3)	triplet
$(m_4, \tilde{m}_4)$	doublet
$(m_{11}, m_{\rm IIA})$	doublet
$m_{\rm IIB}$	singlet

All these deformations correspond to a gauging of a 9D global symmetry. In particular, it is always the symmetry that is employed in the SS reduction Ansatz that becomes gauged upon reduction with the gauge field being the Kaluza-Klein vector. In all but one case this is the complete story and one finds an Abelian gauged supergravity. It turns out that there is one exception, i.e. the case with $m_4 \neq 0$, where we find a *non-Abelian* gauge symmetry. The (non-semi-simple) gauge group is $C\mathbb{R}^1$, the group of scalings and translations of the real line. Further details of the different massive deformations can be found in [8].

3 Combining Massive Deformations

We next try to combine the different massive deformations we found above. Requiring that the fermionic field equations transform under supersymmetry to a complete set of bosonic field equations restricts us to five cases, each containing two nonzero mass parameters:

- **Case 1** with $\{m_{\rm IIA}, m_4\}$: this combination can also be obtained by Scherk-Schwarz reduction of IIA employing a linear combination of the symmetries $\hat{\alpha}$ and $\hat{\beta}$, guaranteeing its consistency. It is also a gauging of both this symmetry and (for $m_4 \neq 0$) the parabolic subgroup of $SL(2,\mathbb{R})$ in 9D, giving the non-Abelian gauge group $C\mathbb{R}^1$.
- **Case 2,3,4** with $\{\vec{m}, m_{\rm IIB}\}$: as in the case with $m_{\rm IIB} = 0$ and only $\vec{m}$ this combination contains three different, inequivalent cases depending on $\vec{m}^2$ (depending crucially on the fact that $m_{\rm IIB}$ is a singlet under $SL(2,\mathbb{R})$):
  - **Case 2** with $\{\vec{m}, m_{\rm IIB}\}$ and $\vec{m}^2 = 0$.
  - **Case 3** with $\{\vec{m}, m_{\rm IIB}\}$ and $\vec{m}^2 > 0$.
  - **Case 4** with $\{\vec{m}, m_{\rm IIB}\}$ and $\vec{m}^2 < 0$.

  All these combinations can also be obtained by Scherk-Schwarz reduction of IIB employing a linear combination of the symmetries $\hat{\delta}$ and (one of the subgroups of) $SL(2,\mathbb{R})$, guaranteeing its consistency. All cases (assuming that $m_{\rm IIB} \neq 0$) correspond to the gauging of an Abelian non-compact symmetry in 9D. Only in the special case $\vec{m}^2 < 0, m_{\rm IIB} = 0$ corresponds to a SO(2)–gauging.
- **Case 5** with $\{m_4 = -\frac{12}{5} m_{\rm IIA}, m_2 = m_3\}$: this case can be understood as the generalized dimensional reduction of Romans' massive IIA theory, employing the $\mathbb{R}^+$ symmetry that is not broken by the $m_{\rm R}$ deformations: $\hat{\beta} - \frac{5}{12}\hat{\alpha}$. It gauges both this linear combination of $\mathbb{R}^+$'s and the parabolic subgroup of $SL(2,\mathbb{R})$ in 9D, giving a non-Abelian gauge group provided $m_4 \neq 0$.

All five cases are gauged theories and have a higher–dimensional origin. Both case 1 and case 5 have a non-Abelian gauge group provided $m_4 \neq 0$.

4 Solutions

We have constructed a variety of gauged supergravities with 32 supersymmetries. They all have in common that there is a scalar potential. Our next goal is to make a systematic search for solutions that are based on this scalar potential. In the next Subsections we will search for two types of solutions: (i) 1/2 BPS domain wall (DW) solutions and (ii) maximally symmetric solutions with constant scalars, i.e. de Sitter (dS), Minkowski (Mink) or anti–de Sitter (AdS) solutions.

4.1 1/2 BPS Domain Wall Solutions

In [7] we already made a systematic search for half-supersymmetric Domain Wall (DW) solutions of the gauged supergravities corresponding to the cases 2, 3 and 4 (with $m_{\rm IIB} = 0$). Due to a one-to-one relationship with 7-branes in D=10 dimensions [10] we could even make a systematic investigation of the quantization of the mass parameters by using the results of [15, 16].

We now want to investigate whether the five massively deformed supergravities we found in the previous Section allow new half-supersymmetric DW solutions. Since we are looking for 1/2 BPS solutions it is convenient to solve the Killing spinor equations, which are obtained by setting the supersymmetry variation of the gravitino and dilatinos to zero. The projector[3] for a DW is given by $\frac{1}{2}(1 \pm \gamma_y)$, where y denotes the transverse direction. We find that, in order to make a projection operator in the Killing spinor equations, we are forced to set all mass parameters to zero except for $\vec{m}$, which corresponds to cases 2, 3 and 4 of Section 3 with $m_{\rm IIB} = 0$. This is a consistent combination of masses and we obtain three classes of domain wall solution which were discussed in detail in [7].

To summarize, we find that there are no new codimension-one 1/2 BPS solutions to the D=9 supergravity theories we obtained in the previous Sections, as compared to the three classes of domain wall solutions given in [7].

4.2 Maximally Symmetric Solutions with Constant Scalars

The second category of vacuum solutions we consider are the solutions with all three scalars constant. This is a consistent truncation in two cases which both have two mass parameters. In this truncation one is left with the metric only satisfying the Einstein equation with a cosmological term

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = -\Lambda g_{\mu\nu} \,, \tag{14}$$

with Λ quadratic in the two mass parameters. Depending on the sign of this term one thus has anti-de Sitter, Minkowski or de Sitter geometry.

We find that solutions with constant scalars are possible in the following massive supergravities:

- **D=10** with $\{m_{11}\}$ has $\Lambda = +36 {m_{11}}^2 e^{-3\hat{\phi}/2}$, which gives rise to de Sitter$_{10}$ [17], breaking all supersymmetry. The D=11 origin of this solution is Mink$_{11}$ written in a basis where the x–dependence is of the required form [17]:

$$\text{Mink}_{11}: \qquad ds^2 = e^{2m_{11}x}\big(-dt^2 + e^{2m_{11}t} dx_9^2 + dx^2\big) \,. \tag{15}$$

- **D=9, Case 1** with $\{m_{\rm IIA} = -\frac{2}{3} m_4\}$ has $\Lambda = +\frac{63}{4} {m_4}^2 e^{\phi - 3\varphi/\sqrt{7}}$, which gives rise to De Sitter$_9$, breaking all supersymmetry. This case follows from the reduction of Mink$_{10}$ by using a combination of IIA scale symmetries that leave the dilaton invariant so that. This particular scale symmetry allows a SS reduction of a configuration with a zero dilaton sothat, after reduction, one is left with a non-trivial metric field only.

- **D=9, Case 4** with $\{m_{\rm IIB}, m_3\}$ has $\Lambda = +28 {m_{\rm IIB}}^2 e^{4\varphi/\sqrt{7}}$, which gives rise to de Sitter$_9$ for non-vanishing $m_{\rm IIB}$. This case follows from the reduction of Mink$_{10}$ by using a combination of IIB scale symmetries that leave the dilaton invariant. Note that for vanishing $m_{\rm IIB}$ this reduces to Mink$_9$, despite the presence of m_3 [12]. For either $m_{\rm IIB}$ or m_3 non-zero this solution breaks all supersymmetry.

5 Conclusions

We have constructed five different D=9 massive deformations with 32 supersymmetries, each containing two mass parameters. All these five theories have a higher–dimensional origin via SS reduction from D=10 dimensions. Furthermore, the massive deformations gauge a global symmetry of the

[3]From a general analysis of the possible projectors in 9 dimensions, we find that there is a second projector given by $\frac{1}{2}(1 \pm i\gamma_t)$. This projector would give a *Euclidean* DW, i.e. a DW having time as a transverse direction. Note that such a Euclidean DW can never be 1/2 BPS since if there existed a Killing spinor it would square to a Killing vector in the *transverse* direction, i.e. time, which is not an isometry of the euclidean DW.

massless theory. The gauge group we have obtained are the Abelian groups $SO(2), SO(1,1)^+, \mathbb{R}, \mathbb{R}^+$ and the unique two–dimensional non-Abelian Lie group $C\mathbb{R}^1$ of scalings and translations on the real line.

We have analyzed the possibility of combining massive deformations to obtain more general massive supergravities that are not gauged or do not have a higher–dimensional origin. Our analysis shows that the only possible combinations are the five two–parameter deformations, which are all gauged and can be uplifted. We have not made a systematic search for massive D=9 supergravities that are not the combination of gaugings and we cannot exclude that there are more possibilities. This requires a separate calculation.

Finally, not all gauged supergravities we constructed are necessarily the leading terms in a low-energy approximation to (compactified) superstring theory. In particular, the relation to string theory of those massive deformations that are based on a symmetry that is broken by α' corrections is less clear. In contrast, the massive deformations that are based on symmetries that are preserved by the higher–order string corrections to supergravity can be considered as the low–energy limits of compactified string theory. We have two such symmetries:

- The $SL(2,\mathbb{R})$ (or rather its $SL(2,\mathbb{Z})$ subgroup) symmetry of IIB. Thus the $\vec{m} = (m_1, m_2, m_3) \neq 0$ deformations correspond to the low–energy limits of three different sectors of compactified IIB string theory (depending on $\vec{m}^2 = \frac{1}{4}({m_1}^2 + {m_2}^2 - {m_3}^2)$). In [7] vacuum solutions were constructed for all three sectors. Of these only the D7–brane has a well–understood role in IIB string theory.

- The linear combination $m_{\rm IIA} = \frac{1}{12} m_4 \neq 0$ of SO(1,1)–symmetries of IIA. Thus one can define a massive deformation m_s within Case I with $\{m_{\rm IIA} = \frac{1}{12} m_s, m_4 = m_s\}$ which corresponds to the low–energy limit of a sector of compactified IIA string theory. No vacuum solution has been constructed for this sector. It would be very interesting to try to find a vacuum solution and understand which role it plays in IIA string theory.

In fact, one can have a better understanding of the m_s massive deformation of IIA from the following point of view. The particular m_s–deformation is based on a scale symmetry of IIA that can be understood from its 11D origin as the general coordinate transformation $x^{11} \to \lambda\, x^{11}$. This explains why all α' corrections transform covariantly under this specific scale symmetry: the higher–order corrections in 11D are invariant under general coordinate transformations and upon reduction they must transform covariantly under the reduced g.c.t.'s, among which is the scale symmetry that leads to the m_s–deformation.

The transformation $x^{11} \to \lambda\, x^{11}$ can also be used for a Scherk–Schwarz reduction from 11D to 9D with a different procedure to give internal coordinate dependence to the fields. Let us call this an SS2 reduction as opposed to the SS1 reduction, which is the method we have used throughout the paper and which is based on *global, internal symmetries* of the higher–dimensional theory. The SS2 procedure [18] instead uses *a symmetry of the compactification manifold* for the reduction Ansatz[4]. The massive deformations resulting from a SS2 reduction can be expressed in terms of the structure constants of the corresponding non–Abelian gauge group. Using the tranformation $x^{11} \to \lambda\, x^{11}$ in the SS2 reduction from 11D to 9D we obtain massive deformations which are equal to the m_s deformations upon relating the components of ${f_{ab}}^c$ to m_s. Indeed, this explains why the m_s deformations correspond to a gauging of the 2D non–Abelian Lie group $\mathcal{C}\mathbb{R}^1$ rather than only a scale symmetry.

The understanding of the m_s deformation in terms of a SS2 reduction employing $x^{11} \to \lambda\, x^{11}$ also explains why $\tilde{m}_4$ (see Table 2) cannot be obtained from a SS1 reduction. Since S-duality

[4]It was already noted by Scherk and Schwarz that SS1 reduction with a symmetry that originates from a higher–dimensional g.c.t. is equivalent to the corresponding SS2 reduction.

interchanges x^{10} and x^{11}, it is the g.c.t. $x^{10} \to \lambda x^{10}$ that would give rise to a $m_{11} = \frac{1}{12}\tilde{m}_4$ deformation. However, this transformation is not an internal symmetry of 10D IIA supergravity and thus cannot be exploited in a SS1 reduction. Since m_{11} does have a 10D origin, this implies that $\tilde{m}_4$ cannot be obtained from 10D IIA.

The D=9 gauged supergravities involving $m_{11}, m_{\rm IIB}$ or $m_{\rm IIA} \neq \frac{1}{12}m_4$ have the same status as the D=10 gauged supergravity discussed above, i.e. these theories are based upon symmetries that are broken by α'–corrections. Note that all the de Sitter space solutions we found in Section 4 involve either m_{11}, $m_{\rm IIB}$ or $m_{\rm IIA} \neq \frac{1}{12}m_4$. It would be interesting to see whether these de Sitter spaces could occur as the $\ell_s \to 0$ limit of an exact solution of string theory.

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