Local Black Hole Solutions on the Brane with Induced Gravity

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An analysis of a spherically symmetric braneworld configuration is performed when the intrinsic curvature scalar is included in the bulk action. In the case when the electric part of the Weyl tensor is zero, all the exterior solutions are found; one of them is of the Schwarzschild-$(A)dS_4$ form, which is matched to a modified Oppenheimer-Volkoff interior solution. In the case when the electric part of the Weyl tensor is non zero, the exterior Schwarzschild-$(A)dS_4$ black hole solution is modified receiving corrections from the non-local bulk effects. A non-universal gravitational constant arises, depending on the density of the considered object and the Newton's law is modified for small and large distances; however, the conventional limits are easily obtained.

Keywords: Black Holes, Branes

1 Introduction

Branes are solitonic solutions of ten-dimensional string theories. In the most simplified picture of the braneworld scenario, our physical world is realized as a four-dimensional hypersurface embedded in a five-dimensional space called bulk. All matter and gauge interactions live on the brane, while the gravitational interactions are effective in the whole five-dimensional space. This novel approach of visualizing our world, offers a new understanding of the four fundamental forces. While in the so-called symmetric picture all known interactions were tried to be unified under the same symmetry group, the braneworld scenario treats the weak, electromagnetic and strong interactions differently than gravitational interactions. This allowed to define a gravitational scale of the whole space, which if the extra dimensions are large, can be as low as the TeV scale [1, 2], while the four-dimensional gravitational scale of our world is at the Planck scale. This happens because our four-dimensional world is confined on the brane and can "see" only the four-dimensional localized gravitational field.

Braneworld solutions can give us information about the structure and nature of the extra dimensions. Very recently, we have a plethora of observational data, both cosmological and astrophysical. Consistency of cosmological and local braneworld solutions with these data can give information about the parameters of the theory, like the energy scale, the size of the extra dimensions or the strength of the gravitational force of the extra dimensions. For example, spherically symmetric local braneworld solutions can give information about the crossover scale above which the extra dimensions appear, how the Newton's constant changes with matter density, or what are the corrections of the gravitational potential at high energies.

Braneworld solutions can be obtained following two different approaches. In the first approach, the dynamics and the geometry of the whole space is primarily considered, and then, the dynamics on the brane is extracted using mainly consistency checks like the Israel matching conditions. The second approach is to specify the dynamics and the geometry on the brane first, and then try to

extent the solution to the bulk. A disadvantage of this method is that finding the bulk geometry in which the brane consists its boundary may be a very difficult task. Another difficulty in this approach is, as we will discuss later, that it is not always possible to obtain a closed set of equations on the brane, so that only with data on the brane to be able to predict the behavior of the fields on the brane. However, this method is basically the only way we have for finding non-trivial braneworld solutions (i.e. solutions not arising from factorizable bulk geometries).

The effective brane equations have been obtained [3] when the effective low-energy theory in the bulk is higher-dimensional gravity. However, a more fundamental description of the physics that produces the brane could include [4] higher order terms in a derivative expansion of the effective action, such as a term for the scalar curvature of the brane, and higher powers of curvature tensors on the brane. If the dynamics is governed not only by the ordinary five-dimensional Einstein-Hilbert action, but also by the four-dimensional Ricci scalar term induced on the brane, new phenomena appear. In [5], it was observed that the localized matter fields on the brane (which couple to bulk gravitons) can generate via quantum loops a localized four-dimensional worldvolume kinetic term for gravitons (see also [6, 7, 8, 9]). That is to say, four-dimensional gravity is induced from the bulk gravity to the brane worldvolume by the matter fields confined to the brane. It was also shown that an observer on the brane will see correct Newtonian gravity at distances shorter than a certain crossover scale, despite the fact that gravity propagates in extra space which was assumed there to be flat with infinite extent; at larger distances, the force becomes higher-dimensional.

A realization of the induced gravity scenario in string theory was presented in [10]. Furthermore, new closed string couplings on Dp-branes for the bosonic string were found in [11]. These couplings are quadratic in derivatives and therefore take the form of induced kinetic terms on the brane. For the graviton in particular these are the induced Einstein-Hilbert term as well as terms quadratic in the second fundamental tensor. Considering the intrinsic curvature scalar in the bulk action, the effective brane equations have been obtained in [14]. Results concerning cosmology have been discussed in [15, 16, 17, 18, 19, 20, 21].

We will discuss static spherically symmetric solutions of braneworlds with induced gravity [12], and we will present an exterior Schwarzschild-$(A)dS_4$ solution which is matched to a modified interior Oppenheimer-Volkoff solution. In this solution the gravitational constant get corrected for very small matter densities. The conventional solar system bounds of General Relativity set the crossover scale below the TeV scale. The above results were obtained by setting the electric part of the Weyl tensor, $\mathsf{E}_{\mu\nu}$ to zero.

Then, we will generalize our study by including the non-local bulk effects [13], as they are expressed by a non-vanishing electric part of the Weyl tensor on the brane. By choosing $g_{tt} = -g_{rr}^{-1}$, the system of equations on the brane becomes closed and all the possible static black hole solutions are found for these metrics. These solutions have generic new terms which give extra attractive force compared to the Newtonian -$(A)dS_4$ force, and represent the strong-gravity corrections to the Schwarzschild-$(A)dS_4$ spacetime.

2   Four-Dimensional Static Spherically Symmetric Solutions

We consider a 3-dimensional brane Σ embedded in a 5-dimensional spacetime. Capital Latin letters $A, B, ... = 0, 1, ..., 4$ will denote full spacetime, lower Greek $\mu, \nu, ... = 0, 1, ..., 3$ run over brane worldvolume. For convenience, we can quite generally, choose a coordinate y such that the hypersurface $y = 0$ coincides with the brane. The total action for the system is taken to be:

$$S = \frac{1}{2\kappa_5^2} \int \sqrt{-^{(5)}g}\, (^{(5)}R - 2\Lambda_5) d^5x + \frac{1}{2\kappa_4^2} \int \sqrt{-^{(4)}g}\, (^{(4)}R - 2\Lambda_4) d^4x$$
$$+ \int \sqrt{-^{(5)}g}\, L_5^{mat}\, d^5x + \int \sqrt{-^{(4)}g}\, L_4^{mat}\, d^4x. \tag{1}$$

For clarity, we have separated the cosmological constants Λ_5, Λ_4 from the rest matter contents L_5^{mat}, L_4^{mat} of the bulk and the brane respectively. Λ_4/κ_4^2 can be interpreted as the brane tension of the standard Dirac-Nambu-Goto action and can include quantum contributions to the four-dimensional cosmological constant. We basically concern on the case with no fields in the bulk, i.e. $^{(5)}T_{AB} = 0$. From the dimensionful constants κ_i^2, the Planck masses M_i are defined as $\kappa_i^2 = 8\pi G_{(i)} = M_i^{i-2}$ Then, a distance scale r_c is defined as $r_c = M_4^2/M_5^3$.

Varying (1) with respect to the bulk metric g_{AB}, and reducing the resulting equations to four dimensions [14], we get four-dimensional Einstein gravity, coupled to a well-defined modified matter content. More explicitly, one gets

$$^{(4)}G_\nu^\mu = \kappa_4^{2\,(4)}T_\nu^\mu - \left(\Lambda_4 + \frac{3}{2}\alpha^2\right)\delta_\nu^\mu + \alpha\left(L_\nu^\mu + \frac{L}{2}\delta_\nu^\mu\right), \tag{2}$$

where $\alpha \equiv 2/r_c$, while the quantities L_ν^μ are related to the matter content of the theory through the equation

$$L_\lambda^\mu L_\nu^\lambda - \frac{L^2}{4}\delta_\nu^\mu = \mathcal{T}_\nu^\mu - \frac{1}{4}\left(3\alpha^2 + 2\mathcal{T}_\lambda^\lambda\right)\delta_\nu^\mu, \tag{3}$$

and $L \equiv L_\mu^\mu$. The quantities $\mathcal{T}_\nu^\mu$ are given by the expression

$$\mathcal{T}_\nu^\mu = \left(\Lambda_4 - \frac{1}{2}\Lambda_5\right)\delta_\nu^\mu - \kappa_4^{2\,(4)}T_\nu^\mu +$$
$$+ \frac{2}{3}\kappa_5^2\left(^{(5)}\overline{T}_\nu^\mu + \left(^{(5)}\overline{T}_y^y - \frac{^{(5)}\overline{T}}{4}\right)\delta_\nu^\mu\right) - \overline{\mathsf{E}}_\nu^\mu. \tag{4}$$

Bars over $^{(5)}T_\nu^\mu$ and the electric part E_ν^μ of the Weyl tensor mean that the quantities are evaluated at $y = 0$. $\overline{\mathsf{E}}_\nu^\mu$ carries the influence of non-local gravitational degrees of freedom in the bulk onto the brane [3] and makes the brane equations (2) not to be, in general, closed. This means that there are bulk degrees of freedom which cannot be predicted from data available on the brane. One needs to solve the field equations in the bulk in order to determine E_ν^μ on the brane [28]. Due to the contracted Bianchi identities, the following differential equations among L_ν^μ arise from (2) $L_{\nu;\mu}^\mu + L_{;\nu}/2 = 0$. We are looking for solutions of (2) for spherically symmetric braneworld metrics

$$ds_{(4)}^2 = -B(r)dt^2 + A(r)dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2). \tag{5}$$

The matter content of the 3-universe is considered to be a localized spherically symmetric perfect fluid. We first consider the case $\overline{\mathsf{E}}_\nu^\mu = 0$ as the boundary condition of the propagation equations in the bulk space. All the solutions outside a static localized matter distribution were found. One of these is the Schwarzschild-$(A)dS_4$ metric which is matched to a modified Oppenheimer-Volkoff interior. The exterior solution was found to be

$$B_>(r) = \frac{1}{A_>(r)} = 1 - \frac{\gamma}{r} - \beta r^2 \ , \ r \geq R, \tag{6}$$

where γ is an integration constant and

$$\beta = \frac{1}{3}\Lambda_4 + \frac{1}{2}\alpha^2 - \frac{\alpha}{2\sqrt{3}}\sqrt{4\Lambda_4 - 2\Lambda_5 + 3\alpha^2}. \tag{7}$$

Considering a uniform distribution $\rho(r) = \rho_o = \frac{3M}{4\pi R^3}$ for the object, the interior solution was found, and its matching to the above exterior specified the integration constant γ. The result is

$$\frac{1}{A_<(r)} = 1 - \left(\beta + \frac{\gamma}{R^3}\right) r^2 \ , \ \ r \le R \, , \tag{8}$$

$$B_<(r) = \frac{1 - \frac{\gamma}{R} - \beta R^2}{\left(1 + \frac{4\pi R^3}{3M} p(r)\right)^2} \ , \ \ r \le R \, , \tag{9}$$

$$p(r) = -\rho_o \frac{\sqrt{1 - (\beta + \frac{\gamma}{R^3})r^2} - \sqrt{1 - (\beta + \frac{\gamma}{R^3})R^2}}{\sqrt{1 - (\beta + \frac{\gamma}{R^3})r^2} - \omega\sqrt{1 - (\beta + \frac{\gamma}{R^3})R^2}} \, , \tag{10}$$

where

$$\gamma = \frac{\kappa_4^2 M}{4\pi} + \frac{\alpha R^3}{2\sqrt{3}} \sqrt{4\Lambda_4 - 2\Lambda_5 + 3\alpha^2} - \frac{\alpha R^3}{2\sqrt{3}} \sqrt{4\Lambda_4 - 2\Lambda_5 + 3\alpha^2 + \frac{3\kappa_4^2 M}{\pi R^3}} \, , \tag{11}$$

$$\omega^{-1} = 1 - \frac{2}{\kappa_4^2 \rho_o} \left(\beta + \frac{\gamma}{R^3}\right) \left(1 - \frac{\sqrt{3}\alpha}{\sqrt{4\Lambda_4 - 2\Lambda_5 + 3\alpha^2 + 4\kappa_4^2\rho_o}}\right)^{-1} \, . \tag{12}$$

The parameters γ and β of the Schwarzschild-$(A)dS_4$ exterior solution (6) can be constrained by solar system experiments. The bounds obtained fix the crossover scale below the TeV range. The γ parameter in the $1/r$ term modifies the Newton's gravitational constant which, as it is seen from (11), for small matter densities deviates significantly from its conventional value, as it is shown in Fig. 1.
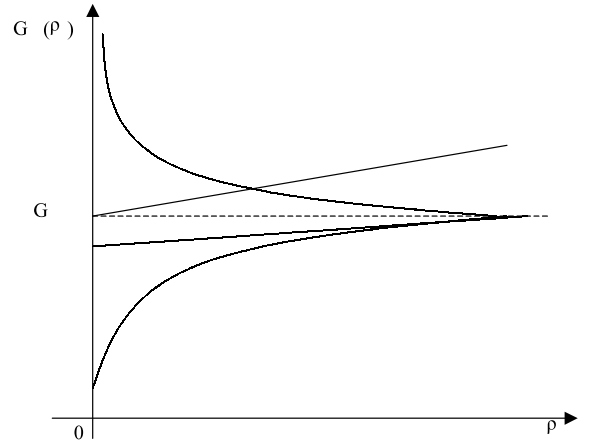
Fig. 1. ρ_0 *dependence of Newton's constant in various models:(1) The Newton's constant variation in our model; If* $4\Lambda_4 - 2\Lambda_5 \gg 3\alpha^2$ *then (1a), otherwise (1b). (2) The behaviour of Newton's constant in another class of our solutions. (3) The Newton's constant variation in RS model.*

where $h_{\mu\nu} = g_{\mu\nu} + u_\mu u_\nu$ is the projection operator normal to u^μ, while $\mathcal{P}_{\mu\nu} u^\nu = \mathcal{Q}_\mu u^\mu = 0$. $\mathcal{U}$ is the non-local energy density, $\mathcal{P}_{\mu\nu}$ the non-local anisotropic stress, and $\mathcal{Q}_\mu$ the non-local energy flux on the brane. Static spherical symmetry implies [28] that

$$\mathcal{Q}_\mu = 0 \quad , \quad \mathcal{P}_{\mu\nu} = \mathcal{P}(r)\left(r_\mu r_\nu - \frac{1}{3} h_{\mu\nu}\right), \tag{14}$$

where r_μ is the unit radial vector. Thus, the non-vanishing components of the electric part of the Weyl tensor are

$$\overline{\mathsf{E}}^0_0 = -\mathcal{U} \ , \ \overline{\mathsf{E}}^r_r = \frac{1}{3}(\mathcal{U} + 2\mathcal{P}) \ , \ \overline{\mathsf{E}}^\theta_\theta = \overline{\mathsf{E}}^\phi_\phi = \frac{1}{3}(\mathcal{U} - \mathcal{P}). \tag{15}$$

One class of solutions of (2) with non-zero $\mathsf{E}_{\mu\nu}$ is

$$
\begin{aligned}
B = \frac{1}{A} \ = \ & 1 - \frac{\gamma}{r} - \beta r^2 \\
& + \ sg(\zeta)\frac{\delta}{r}\left[\frac{128}{105}\,{}_1F_1\left(\frac{15}{8}, \frac{23}{8}; sg(\zeta)z\right)z \right. \\
& + \left. \frac{9}{8}\left(\frac{1}{z} - sg(\zeta)\frac{8}{7}\right)e^{\,sg(\zeta)\,z}\right]z^{\frac{7}{8}},
\end{aligned}
\tag{16}
$$

where γ is another integration constant (typically interpreted as $2G_N M$ with M being the mass of the point particle, G_N the Newton's constant),

$$\beta = \frac{1}{3}\Lambda_4 + \frac{1}{2}\alpha^2 \quad , \quad \zeta = \frac{\alpha^2}{9}(4\Lambda_4 - 2\Lambda_5 + 3\alpha^2), \tag{17}$$

and $\delta = \frac{4}{9}\left(\frac{9}{8|\mathcal{P}|}\right)^{\frac{1}{8}}|\mathcal{P}|\alpha c^3 > 0$. While

$$r = \left(\frac{\delta}{\sqrt{|\zeta|}}\right)^{\frac{1}{3}}z^{\frac{1}{8}}\,e^{\,sg(\zeta)\,z/3}. \tag{18}$$

The electric components of the Weyl tensor are given by the equation, $\mathcal{P} = -2\mathcal{U} = 9\zeta/(2\alpha^2)$.

Comparing the solution (16) with the solution (6) having $\overline{\mathsf{E}}^\mu_\nu = 0$, we note that there is a new term, besides the conventional Newtonian and $(A)dS_4$ terms, which carries the information of the gravitational field in the bulk. For $\zeta > 0$, the asymptotic behavior $r \to \infty$ of this new term in the solution (16) is seen to be AdS_4–like, i.e. $\sqrt{\zeta}\,r^2$. Thus, asymptotically, the effective cosmological constant is $\beta - \sqrt{\zeta}$. For $\zeta < 0$, the asymptotic behavior $r \to 0$ of the new term in the solution (16) is Newtonian, i.e. $-2\Gamma(7/8)\delta/r$. Thus, the effective Newton's constant in this regime appears larger.

We know that for non-relativistic particles the effective potential is $2\Phi = B - 1$. As it can be seen, the new force corresponding to the above non-local term is always attractive. For $\zeta > 0$, its magnitude is monotonically increasing with distance, while for $\zeta < 0$, this happens in decreasing distances (after a characteristic scale). In order for the new term not to disturb the well-measured Newtonian law at distances from the cm to the solar-distance scale, one has in both cases to adjust the quantity δ/γ as small as desired. For $\zeta > 0$ and for larger distances, the sum of the Newtonian and the new force decreases (in magnitude) slower than the Newtonian force, while for even larger distances, this sum grows to infinity (Figure 2a). For $\zeta < 0$, deviations between the total and the Newtonian force appear only at small distances (Figure 2b). The $(A)dS_4$ term βr^2 is generally considered to be of cosmological origin and is not considered here to be of importance at the local level.

We notice also that this solution may have some interesting physical implications. For $\zeta > 0$, because the total gravitational force grows slower than the conventional Newtonian law, this force

may serve as a possible qualitative explanation for the yet unresolved problem of galactic rotation curves. However, numerical fittings with real data remain to be done. On the other hand, the solution with $\zeta < 0$ could be considered if submillimeter deviations from the Newtonian law are discovered.
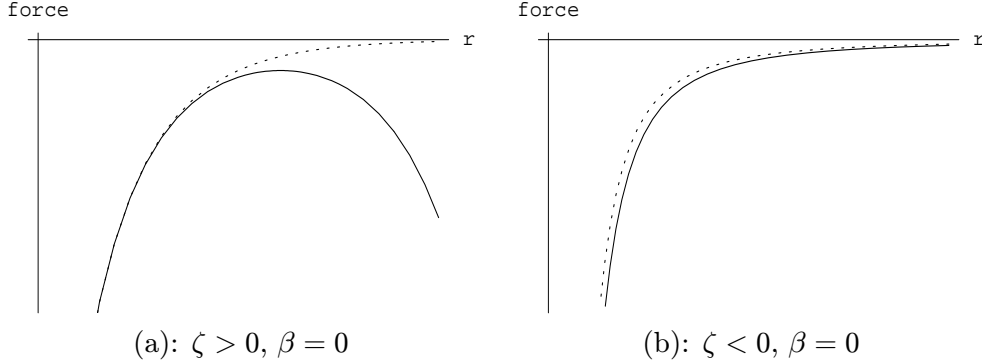

(a): $\zeta > 0$, $\beta = 0$ (b): $\zeta < 0$, $\beta = 0$

Fig. 2. *Dotted lines represent the Newtonian force. Continuous lines represent the total force, i.e. the sum of the Newtonian and the new force.*

These is a second class of solutions of (2) which are given by

$$B = \frac{1}{A} = 1 - \frac{\gamma}{r} - \beta r^2 \oplus \frac{\delta}{r} \int |v - \sqrt{3}|^{-\frac{3(3-\sqrt{3})}{8}} \left(v + \sqrt{3}\right)^{-\frac{3(3+\sqrt{3})}{8}} \frac{v}{|v - 3|^{7/4}} \, dv \,, \qquad (19)$$

where γ is another integration constant (typically interpreted as $2G_N M$ with M being the mass of the point particle, G_N the Newton's constant) and

$$\beta = \frac{1}{3}\Lambda_4 + \frac{1}{2}\alpha^2 \,. \qquad (20)$$

The symbol $\oplus$ means $-$ for $\epsilon = +1$, $v > 3$ or for $\epsilon = -1$, $1 < v < \sqrt{3}$; for $\epsilon = +1$, $2 < v < 3$ or for $\epsilon = -1$, $\sqrt{3} < v < 2$ it means $+$. The above integral cannot be computed in terms of known functions. However, this can be done in the asymptotic regimes $r \to \infty$ and $r \to 0$. For $r \to \infty$ ($\epsilon = +1$), the new term in the solution (19) becomes AdS_4 –like, i.e. $(\sqrt{2\zeta}/3\sqrt{3}) \, r^2$ and thus, asymptotically, the effective cosmological constant is $\beta - (\sqrt{2\zeta}/3\sqrt{3})$. For $r \to 0$ ($\epsilon = -1$), approximating the integral in (19) around $v = \sqrt{3}$, we find that the new term scales as $r^{2(2-\sqrt{3})}$, giving therefore extra attractive force $1/r^{2\sqrt{3}-3}$. Numerical evaluation of the integral in (19) leads for $\epsilon = +1$ qualitatively to the same picture as that of Figure 2a, where by adjusting the quantity δ/γ as small as desired, deviations from Newtonian law appear only at large distances. Similarly, for $\epsilon = -1$, the picture for the solutions resembles qualitatively to that of Figure 2b, where deviations from Newtonian law appear only at very small distances.

We have considered so far the motion of non-relativistic particles. However, the motion of a freely falling photon in a static isotropic gravitational field (5) is described [33] by the equation

$$\left(\frac{d\phi}{dr}\right)^2 = \frac{A}{r^4}\left(\frac{1}{J^2 B} - \frac{1}{r^2}\right)^{-1} \,, \qquad (21)$$

where J is an integration constant. In the cases where the solutions (16), (19) deviate from Newton's law at large distances, it is seen from (21) that $d\phi/dr \to 0$ as $r \to \infty$, and thus, the photon moves in a "straight" line of the background geometry in that region (even when a second horizon exists, we consider it of cosmological size compared to the local distances of interest). More specifically, at

large distances, it arises from (21) that $\phi(r) - \phi(\infty) \simeq (\frac{1}{J^2} + \beta - \sqrt{\zeta})^{-1/2} \frac{1}{r}$ for the solution (16), while for the solution (19), $\sqrt{\zeta}$ is replaced by $\sqrt{2\zeta}/3\sqrt{3}$ in the last expression. This means that the "impact parameter" b is $b = (\frac{1}{J^2} + \beta - \sqrt{\zeta})^{-1/2}$ for (16) (and respectively for (19) with the change of $\sqrt{\zeta}$). For our solutions, the total deflection angle in (21) cannot be computed explicitly. However, we can understand the influence of the new term on the motion of a photon and compare to the Newtonian deflection. For doing so, we have to refer to two photons with the same "initial conditions", i.e. the same impact parameter b, one moving in a Schwarzschild-$(A)dS_4$ background (denoted by the subscript 1) and the other in the background defined by the solutions (16), (19) (denoted by the subscript 2). The following equations are easily obtained for the solutions (16), (19) respectively:

$$\frac{1}{\sqrt{\zeta}\, r^4} \frac{(dr_1)^2 - (dr_2)^2}{(d\phi)^2} = \frac{\sqrt{z}}{e^z}\left[\frac{128}{105}\, {}_1F_1\left(\frac{15}{8}, \frac{23}{8}; z\right) z + \frac{9}{8}\left(\frac{1}{z} - \frac{8}{7}\right) e^z \right] - 1\,, \qquad (22)$$

$$\frac{3\sqrt{3}}{\sqrt{2\zeta}\, r^4} \frac{(dr_1)^2 - (dr_2)^2}{(d\phi)^2} =$$
$$\oplus \frac{3\sqrt{3}}{\sqrt{2}} \frac{|v-3|^{\frac{3}{4}} (v+\sqrt{3})^{\frac{3(\sqrt{3}-1)}{8}}}{(v-\sqrt{3})^{\frac{3(\sqrt{3}+1)}{8}}} \int (v-\sqrt{3})^{-\frac{3(3-\sqrt{3})}{8}} (v+\sqrt{3})^{-\frac{3(3+\sqrt{3})}{8}} \frac{v\,dv}{|v-3|^{7/4}} - 1\,. \qquad (23)$$

It is obvious that for the branch $\epsilon = +1, v > 3$ which extends to infinity, the right-hand side of equation (23) is negative, giving $(dr_2)^2 > (dr_1)^2$. Therefore, extra deflection of light compared to the Newtonian deflection arises. This situation of increased deflection (compared to that caused from the luminous matter) has been well observed in galaxies or clusters of galaxies and the above solution might serve as a possible way for providing an explanation. One can easily check that equation (22) provides less deflection compared to Newtonian deflection at the distances of interest. On the other hand, it is easily checked that equation (22) provides less deflection compared to Newtonian deflection at the distances of interest.

3   Conclusions

We presented a new class of brane black hole solutions with induced gravity. It is known that the non-local bulk effects, as they are expressed via the projection of the Weyl tensor on the brane, do not make the brane dynamics closed. We need to know the geometry of the bulk space in order to be able to deal with the dynamics on the brane. In the case where $g_{tt} = -g_{rr}^{-1}$, the system of equations consisting of the modified Einstein equations and the Bianchi identities is closed and we found all the possible black hole solutions. If we had to look for more general spherically symmetric solutions, some extra information would be needed for the non-local energy density $\mathcal{U}$ or the non-local anisotropic stress $\mathcal{P}$.

There has been argued [29, 30] on kinematical grounds, irrespectively from the gravitational dynamics, that the only spherically symmetric geometries which may be candidates for explaining from one side the extra deflection of light observed in galaxies and clusters of galaxies and from the other side the galactic rotation curves are of the form $g_{tt} = -g_{rr}^{-1}$. However, severe criticism has appeared on this [31]. In the present paper, we use this interesting and reasonable condition to make the brane dynamics autonomous.

The black hole solutions with non-zero $E_{\mu\nu}$ are representing strong-gravity corrections to the spherically symmetric Schwarzschild-$(A)dS_4$ braneworld solutions. Their characteristic is that they predict a new attractive force. There are classes of solutions with increasing r, where this attractive

force combined with the Newtonian one, results to a net force which decreases slower than the Newton's force. This might have interesting physical implications for the explanation of galactic rotation curves. Within this class, a solution giving extra deflection of light compared to General Relativity predictions at galactic scales was found. It is interesting to observe that this solution has non-trivial (i.e. not constant) non-local energy $\mathcal{U}$ and anisotropic stress $\mathcal{P}$. In another class of solutions with decreasing r, the new force starts to deviate from the Newton's force at small distances, indicating that at submillimeter scale we could have testable deviations from the Newtonian law.

In our solution with $E_{\mu\nu}=0$, we also had deviations from the Newton's law at large distances. This deviation was caused by the presence of the $(A)dS_4$ term βr^2, which for $\beta < 0$ can also give extra attraction. In our new solutions, the extra attractive force appears because of the presence of a new term which only asymptotically (when defined in this regime) behaves like AdS_4. This new term arises because of the presence of the electric part of the Weyl tensor and for an observer on the brane is a pure non-local effect. We had also found in [12] modifications to Newton's law as a result of a change of the Newton's constant due to the finite interior of the rigid object. This effect must have an analogous contribution here if one solves the interior problem.

We have followed a braneworld viewpoint for obtaining braneworld solutions, ignoring the exact bulk space. We have not provided a description of the gravitational field in the bulk space, but confined our interest to effects that can be measured by brane-observers. By making assumptions for obtaining a closed brane dynamics, there is no guarantee that the brane is embeddable in a regular bulk. This is the case for a Friedmann brane [32], whose symmetries imply that the bulk is Schwarzschild-AdS_5 [34, 35]. A Schwarzschild brane can be embedded in a "black string" bulk metric, but this has singularities [36, 37, 38]. The investigation of bulk backgrounds which reduce to Schwarzschild-$(A)dS_4$ or more general black holes is in progress.

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