Extra Dimensions and Density Matrix of Gauge Fields

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We set up a hypothesis in which the matter density of the Universe is much higher than its estimate obtained from the recent astronomical data. On this basis the gravitational phenomena are considered as the macroscopic quantum effects attributed by the low temperature of the weakly interacting particles of the Universe. The theory is constructed in which polarization fields describing the ground state of the Universe and defining the geometrical structure of the space–time (in particular, its dimensionality) play the fundamental role.

1. Introduction

In 1925 Shirokov [1] has shown, that the solution of the Laplace equation dependent only on the spatial interval r and possible only when the space curvature is a constant one has the form: $V(r) = (c_1/R)\cot(r/R) + c_2$, if the curvature is positive (R is a radius of the curvature; c_1, c_2 are arbitrary constants), or [2]

$$V(r) = (c_1/L)\coth(r/L) + c_2, \tag{1.1}$$

when the curvature of space is negative (L is the Lobachevsky constant). For obtaining the Newton (or Coulomb) potential it is necessary to put $c_2 = -c_1/L$, $L \to \infty$ ($c_2 = -c_1/R$, $R \to \infty$). Let's consider hereinafter, that the Newton potential should be exchanged by a potential (1.1), in which $c_2 = -c_1/L$, which was studied by Lobachevskiy [2], in his investigation of the spaces with negative curvature. For such a potential the Seeliger paradox does not take place [3]. For the Lobachevsky potential, as it is easy to note, the divergence is absent (the relativistic generalization of the Lobachevsky theory on the basis of the Einstein theory was proposed by Chernikov [2]).

We represent the Lobachevsky gravity potential in the form:

$$V(r) = \frac{A}{L}(1 - \coth\frac{r}{L}) = -\frac{2A}{L}\,\frac{e^{-2r/L}}{1-e^{-2r/L}} = -\frac{2A}{L}\sum_{n=1}^{\infty} e^{-2rn/L}, \tag{1.2}$$

where $A = G_N m_1 m_2$ ($G_N \approx 6.7\cdot 10^{-39} GeV^{-2}$ is the Newton gravitational constant; the system of units will hereinafter be used $h/(2\pi) = c = 1$, where h is the Planck constant, and c is the speed of light; m_1, m_2 are the masses of interacting bodies). Let's remark, that the asymptotical behavior of the Lobachevsky potential is reduced to the behavior of the potential

$$V(r) = -(A/r)e^{-Br}, \tag{1.3}$$

which, to opinion, was offered for the first time by Neumann [4] and which is more known as the Yukawa potential, introduced for the description of short-range nuclear forces (the similar potential is also used for the description of short-range electromagnetic fields in a plasma). As a result the constant $B = 1/L$ in gravity potential (1.3), as well as L in the potential (1.2), should be determined by properties of a medium (dark matter [5], quintessence etc.), in addition a fundamental role should play the background neutrinos of the Universe. It is possible to assume, that the constant $B = 1/L$ coincides with the value of the Hubble constant $H \approx 1.134 \cdot 10^{-42} GeV$ (or even $B \propto H$), which in this case will serve one of the characteristics of the Universe background matter.

The Bashkin's works [6] on the propagation of spin waves in the polarized gases, which have appeared in eighties, have allowed to make the assumption [7], that similar collective oscillations are possible under certain conditions in the neutrino medium as well. As a result, with help of the Casimir effect, it was possible to associate [8] the gravitational constant G_N with parameters of the electroweak interactions: $G_N \sim \sigma_{e\nu} \sim \alpha\ G_F^2\ T^2$ ($\alpha \approx 1/137$ is the fine structure constant, $G_F \approx 1.166 \cdot 10^{-5} GeV^{-2}$ is the Fermi constant, $T \approx 1.9K \approx 1.64 \cdot 10^{-13} GeV$ is the effective temperature of the neutrinos background of the Universe, $\sigma_{e\nu}$ is the scattering cross–section of a neutrino on an electron, when the energy of a particle is close to T). Let's remark, that in 1937 [9] Gamow and Teller tried to use neutrinos for explanation of the gravitation, but their mechanism is related to for the exchange of neutrino–antineutrino pairs.

2. Weakly interacting particles of Universe

We shall divide the matter of the Universe into a rapid subsystem and a slow one, considering, that all known particles (excluding neutrino) belong to the rapid subsystem and are described by standard fields of the quantum field theory. Considering fundamental particles as coherent elements in open systems, characterizing by a quasi-group structure, we shall use inhomogeneous (quasi-homogeneous) space-time manifold allowed by the geometrical structure of the Riemannian space. For the description of the slow subsystem (weakly interacting particles) we shall apply the condensed matter description through mixtures of gauge fields having non-zero vacuum averages [10] (in particular it is convenient to use fields $\Phi_i^{(k)}(x), \Phi^j_{(l)}(x)$ [11]; indices $i, j, k, l, ...$ and $i, j, k, l, ...$ take the values $1, 2, 3, 4$; a point $x \in M_4$, where M_4 is the space-time manifold; $\Phi_i^{(k)} \Phi^j_{(k)} = \delta_i^j$, $\Phi_i^{(k)} \Phi_j^{(l)} \eta_{(k)(l)} = g_{ij}$, δ_i^j are the Kronecker delta symbols, $\eta_{(k)(l)}$ are the covariant components of the metric tensor of the Minkowski space, g_{ij} are the covariant components of the metric tensor of the Riemannian space-time), using them as gravity potentials. For obtaining the gravitational equations [12] let's write the total Lagrangian $\mathcal{L}_t$

$$\mathcal{L}_t = \mathcal{L}(\Psi) + \eta^{(j)(m)}\ [\kappa_o\ F^{\underline{a}}_{(i)(j)}\ F^{\underline{b}}_{(k)(m)}\ \eta^{(i)(k)}\ \eta_{\underline{ab}} +$$

$$\kappa_1\ (F^{(k)}_{(i)(j)} F^{(n)}_{(l)(m)}\ \eta^{(i)(l)}\ \eta_{(k)(n)} + 2\ F^{(k)}_{(i)(j)}\ F^{(i)}_{(k)(m)} - 4\ F^{(i)}_{(i)(j)}\ F^{(k)}_{(k)(m)})]/4 \qquad (2.1)$$

($\underline{a}, \underline{b}, \underline{c}, \underline{d}, \underline{e} = 5, 6, ..., 4 + \underline{r}$), where $\eta_{\underline{ab}}$ are the covariant components of the metric tensor of the flat space, $\kappa_o = 1/(4\pi)$ and $\kappa_1 = 1/(4\pi G_N)$,

$$F^{(k)}_{(i)(j)} = (\Phi^m_{(i)} \nabla$$

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gravitational constant G_N to be a consequence of a large density of particles (quasi-particles) of the slow subsystem described by fields $\Phi_i^{(k)}(x)$, $\Phi_{(l)}^{j}(x)$, and to rewrite the Lagrangian (2.1) in the form

$$\mathcal{L}_t = \mathcal{L}(\Psi) + \kappa\ \{F^{\gamma}_{\alpha\beta}\ F^{\epsilon}_{\delta\kappa}\ [\eta^{\alpha\delta}\ (\delta^{\kappa}_{\gamma}\ \delta^{\beta}_{\epsilon} - 2\ \delta^{\beta}_{\gamma}\ \delta^{\kappa}_{\epsilon}) + \eta^{\beta\kappa}\ (\delta^{\alpha}_{\epsilon}\ \delta^{\delta}_{\gamma} - 2\ \delta^{\alpha}_{\gamma}\ \delta^{\delta}_{\epsilon}) +$$

$$\eta_{\gamma\epsilon}\ (\eta^{\alpha\delta}\ \eta^{\beta\kappa} - 2\ \eta^{\alpha\beta}\ \eta^{\delta\kappa})] + F^{\underline{a}}_{\alpha\beta}\ F^{\underline{b}}_{\delta\kappa}\ \eta_{\underline{ab}}\ (\eta^{\alpha\delta}\ \eta^{\beta\kappa} - 2\ \eta^{\alpha\beta}\ \eta^{\delta\kappa})\}/4 \tag{2.4}$$

(the cardinality of the values set of the Greek indexes is equal to $\mathcal{N}$), where $\eta_{\alpha\beta}$ are the covariant components of the metric tensor of the flat space, $\eta_{\alpha\beta}\ \eta^{\alpha\gamma} = \delta^{\gamma}_{\alpha}$, κ is constant, and the generalization of the formulae (2.2), (2.3) is obvious.

Let's consider, that violation of symmetry in weak interactions is induced by a high density of right-handed polarized neutrinos of different flavors and, accordingly, left-handed polarized antineutrinos, which at low energies do not participate in reactions owing to the large pressure in matching degenerate Fermi — gases. In addition it is necessary to recall the Dirac hypothesis of 1930, in which the dilemma (existence of negative-energy solutions to his equations) is resolved by filling by electrons all states with negative energies in accordance with Pauli principle. In outcome a state of vacuum is identifiable as state, in which all levels with negative energies are filled by particles (weakly interacting particles), and all levels with positive energies are free, that corresponds to the completely degenerate Fermi — gas at the zero temperature. The slightest increase of a temperature, which can cause a fluctuation (here again it is necessary to recall a Boltzman hypothesis, asserting, that observed by us the area of the Universe is outcome of a huge fluctuation) will cause the appearance of excited states, i.e. known elementary particles with positive energy having colour and (or only) electrical charges. If in addition the space-division of weakly interacting particles is descending, we shall receive the charge asymmetrical Universe with a possible predominance of a matter over an antimatter. Naturally, what exactly the predominance of u- and d-quarks (from which the observed baryon matter is composed in known to us regions of the Universe) probably indicates first of all the predominance of the conforming flavors of right-handed polarized neutrinos with the enough high density of a de(energies)-1.242 Td7(h)neisiontsin addition the de542Ω33.175 0 Tdfl5ivde5

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the broken symmetry [14]. As the Higgs scalar particles till now are not found, it is possible to offer a hypothesis, in which masses of fundamental particles, belonging to the rapid subsystem, are induced by their interaction with particles of the slow subsystem (dark matter [5], quintessence, etc.). In this connection we shall mark the huge value of the masses of the vector bosons W^+, W^-, Z^o, which (contrary to the massless photon) can interact with background neutrinos directly.

We shall prove the given statement for the vector boson from the Lagrangian (2.1), considering M_4 Minkowski space, and fields $\Phi_i^{(k)}(x), \Phi_{(j)}^l(x)$ as constant, owing to large density of weakly interacting particles and their homogeneous distribution in a space. Let $\underline{r} = 1$, that assumes $C_{\underline{ab}}^{\underline{c}} = 0$. For obtaining equations of fields $A_i^{\underline{b}}(x)$ in Feynman perturbation theory the gauge should be fixed. For this we shall add the following

$$\mathcal{L}_q = \kappa_o \, q_{\underline{bb}} \, g^{ij} \, g^{kl} \, (\partial_i A_j^{\underline{b}} - q_o \, C_i \, A_j^{\underline{b}}) \, (\partial_k A_l^{\underline{b}} - q_o \, C_k \, A_l^{\underline{b}})/2, \tag{3.1}$$

where $q_o = \eta_{\underline{bb}}/q_{\underline{bb}}$, $C_i = C_{i\underline{b}}^{\underline{b}}$. Besides let

$$T_{a}{}_{(k)}^{(i)} \, \eta^{(j)(k)} + T_{a}{}_{(k)}^{(j)} \, \eta^{(i)(k)} = \epsilon_{\underline{a}}^{\underline{b}} \, t_{\underline{b}} \, \eta^{(i)(j)}. \tag{3.2}$$

As a result the equations of the vector field $A_i^{\underline{b}}(x)$ will be written as:

$$g^{jk}[\partial_j \partial_k A_i^{\underline{a}} - (1 - 1/q_o)\partial_i \partial_j A_k^{\underline{a}} + (1 - q_o) C_i C_j A_k^{\underline{a}}] + m^2 A_i^{\underline{a}} = I_i^{\underline{a}}/\kappa_o, \tag{3.3}$$

where $I_i^{\underline{a}} = \frac{g_{ij}}{\eta_{\underline{aa}}} \frac{\partial \mathcal{L}(\Psi)}{\partial A_j^{\underline{a}}}$ and

$$m^2 = (n-1)(n-2)\kappa_1 t_{\underline{a}}^2/(2\kappa_o \eta_{\underline{aa}}) - g^{jk} C_j C_k. \tag{3.4}$$

Notice that due to the vacuum polarization ($C_i \neq 0$) the propagator of the vector boson has rather cumbersome form:

$$D_{ij}(p) = [(1 - q_o)\frac{(p_i p_j - C_i C_j)(p^k p_k - q_o m^2) + (1 - q_o) p^k C_k (p_i C_j + C_i p_j)}{(p^l p_l - q_o m^2)^2 + (1 - q_o)^2 (p^l C_l)^2}$$
$$- g_{ij}]/(p^m p_m - m^2), \tag{3.5}$$

which is simplified and gets the familiar form $(-g_{ij}/(p_k p^k - m^2)$, p^k is the 4-momentum, and m is the mass of the vector boson) only in the Feynman gauge ($q_o = 1$).

So, the transition to the hot state of Universe was connected with the destruction of the Bose condensate and the increase of the Fermi gas pressure accordingly. In addition during some time the temperature of background particles of Universe could remain equal or close to zero (the stage of inflation). As a result the rest–mass of W^+, W^-, Z^o bosons have decreased so, that weak interaction has stopped to be weak and all (or nearly so all) particles from a ground (vacuum) state started to participate in an installation of a thermodynamic equilibrium. The given phenomenon also has become the cause of an apparent increase of a density of particles in the early Universe. Suggesting, that mean density n_o of particles in the Universe did not vary at the same time and the hot model in general is correct, we come to the following estimation $n_o \geq m_\pi^3 \sim 10^{-3} GeV^3$ (m_π is a mass of a π meson). This result allows us to give explanation to a known ratio [15] $H_o/G_N \approx m_\pi^3$, if to consider, that the Hubble constant H_o gives an estimation $1/H_o$ to the length $l \sim 1/(n_o \sigma_\nu)$ of a free path of particle in "vacuum" at present stages of the Universe evolution (σ_ν is a scattering cross–section of neutrinos on a charged particle) and to take into account the estimation given earlier for the gravitational constant G_N ($G_N \sim \sigma_\nu$). Thus the gravitational constant G_N is inversely proportional to the time of a free run of a charged particle in the neutrinos medium characterizing a kinetic phase of the relaxation process in the Universe.

4. The hot stage of the Universe evolution and the total Lagrangian

The given requirement causes us the return to the foundations of the very construction. Let's consider the Boltzmann hypothesis of the Universe birth from to a gigantic fluctuation not in an empty space but in a medium which consists of weakly interacting particles characterized by zero temperature and forming the Bose condensate. Certainly, if the particles are fermions they should be in the bound state. For the description of such state of the Universe matter (this state we shall consider pure one) it is necessary to introduce an amplitude of probability $\mathcal{B}$ with components $\mathcal{B}_a^b(\omega)$ $(a, b, c, d, e, f, g, h = 1, 2..., r)$ dependent on points ω of a manifold M_r (including the limiting case $r \to \infty$). In this case we can not define the metric but for its definition we need a density matrix $\rho(\mathcal{B})$ (the rank of which equals to 1 for a pure state), determining its standard mode $\mathcal{B}\mathcal{B}^+ = \rho\,\mathrm{tr}(\mathcal{B}\mathcal{B}^+)$ ($\mathrm{tr}\,\rho = 1$, $\rho^+ = \rho$, the top index "$+''$ is the symbol of the Hermitian conjugation).

Since the influencing of the macroscopic observer will display in an approximation of the transition operator by the differential operators ∂_i then it is necessary to introduce differentiable fields given in a differentiable manifold M_n which we shall call space–time and the points x of which will have coordinates x^i $(i, j, k, l... = 1, 2..., n)$. Probably the rank of the density matrix ρ equals n, but it is impossible to eliminate that the generally given equality is satisfied only approximately when some components of a density matrix can be neglected. In any case we shall consider that among fields $\mathcal{B}$ the mixtures Π_a^i were formed with non-zero vacuum means h_a^i which determine differentiable vector fields $\xi_a^i(x)$ as:

$$\Pi_a^i = \mathcal{B}_a^b\ \xi_b^i \tag{4.1}$$

for considered area $\Omega_n \subset M_n$ (field $\xi_a^i(x)$ determine a differential of a projection $d\pi$ from $\Omega_r \subset M_r$ in Ω_n). It allows to define a space–time M_n as a Riemannian manifold, the basic tensor $g_{ij}(x)$ of which we shall introduce through a reduced density matrix $\rho'(x)$.

So let components ρ_i^j of a reduced density matrix $\rho'(x)$ are determined by the way [12]:

$$\rho_i^j = \xi$$

(η_{ab} are metric tensor components of the flat space and η^{ab} are tensor components of a converse to basic one) then the given Lagrangian is most suitable one at the description of the hot stage of the Universe evolution because it is most symmetrical one concerning intensities of the gauge fields $\mathcal{F}^c_{ab}$. What is more we shall require the realization of the correlations: ${T_a}^b\ \eta^{cd} + {T_a}^d_c\ \eta^{cb} = 0$, that the transition operators ${T_a}^b$ generate the symmetry, which follows from the our assumptions. In absence of fields $\Pi^i_a(x)$ and $\Psi(x)$ at earlier stage of the Universe evolution the Lagrangian (5) becomes even more symmetrical ($\mathcal{L}_t \propto \mathcal{B}^4$), so that the formation of fermions (the appearance of fields Ψ in a total Lagrangian $\mathcal{L}_t$) from primary bosons is a necessary condition (though not a sufficient one) of the transition of Universe to the present stage of its development with a spontaneous symmetry breaking. Only the formation of the Bose condensate from pairs of some class of fermions (the neutrinos of different flavors) has resulted in a noticeable growth of rest–masses of those vector bosons (W^+, W^-, Z^o), which interact with this class of fermions. In parallel there could be an increase of masses of other fundamental particles, though not all (photon and gluon, which do not interact with neutrino directly remain massles).

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