UV/IR Mixing in Noncommutative Theories Via Closed Strings

Adi Armoni*

Theory Division, CERN CH-1211 Geneva 23, Switzerland

Based on work with Esperanza Lopez

I will discuss UV/IR mixing effects in non-supersymmetric non- commutative $U(N)$ gauge theories. I'll show that the55h-370(the5sar5h-370 on-)pandar therms-270(tn)-374(the55h-3702-5h-370nd)]h-3703-eone tfuncton,s nam can be obtained from a manifestly gauge invariant effective action. The action, which involves open Wilson line operators, can be derived from closed strings exchange between two stacks of D-branes. In particular, one of the closed string modes that couple to the field theory operator which is responsible for the infrared poles, is the closed string tachyon.

1 Introduction

Non-commutative gauge theories attracted recently a lot of attention, mainly due to the discovery of their relation to string/M theory [1, 2]. The perturbative dynamics of these theories is very interesting: planar graphs of non-commutative theories are exactly the same as the planar graphs of ordinary theories apart from global phases which depend on external momenta [3]. Non-planar graphs, on the other hand, are regulated by the non-commutativity parameter θ and they are therefore UV-finite. This regularization is however only effective when there is a non-zero momentum inflow into the graph. In particular, as a result of this, the non-planar contribution to the propagator contains, usually, a pole $1/(\theta p)^2$. This pole, which originates from the high momentum region of the integral (UV) seems to affect the large distance dynamics. This unusual phenomenon is called UV/IR mixing [4]. In supersymmetric theories this pole cancels and a softer version of UV/IR mixing exists due to a logarithmic contribution [5].

In this talk we would like to focus on non-supersymmetric non-commutative gauge theories in 4-dimensions. The non-planar pole modifies the dispersion relation of the photon as follows [5]

$$E^2 = \vec{p}^{\,2} - (N_B^{adj} - N_F^{adj})\frac{g^2}{\pi^2}\frac{1}{(\theta p)^2}, \tag{1}$$

where N_B^{adj} and N_F^{adj} are the numbers of bosons and fermions in the adjoint representation, respectively. In the case of pure Yang-Mills theory, or in general when $N_B^{adj} > N_F^{adj}$, the low momentum end of the spectrum acquires imaginary energy. Namely, the one loop analysis suggests that the theory suffers from an instability.

2 Field Theory Calculations - Various Non-Planar Amplitudes

The 4d field theory under consideration is the theory that lives on a stack of N coincident electric D-branes of type 0B string theory. It is obtained by dimensional reduction of pure (non-

*E-mail: adi.armoni@cern.ch.

supersymmetric) 10d non-commutative Yang-Mills theory. The model contains a vector and 6 adjoint scalars and it is described by the following action

$$S = \mathrm{tr} \int d^4x \left(-\frac{1}{2g^2} F_{\mu\nu} \star F^{\mu\nu} + D_\mu \phi^i \star D^\mu \phi^i \right), \tag{2}$$

where

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i(A_\mu \star A_\nu - A_\nu \star A_\mu) \tag{3}$$

and

$$D_\mu \phi^i = \partial_\mu \phi^i - i(A_\mu \star \phi^i - \phi^i \star A_\mu) \tag{4}$$

for $i = 1...6$. The model is invariant under the following non-commutative $U(N)$ gauge transformation

$$\begin{aligned} \delta_\lambda A_\mu &= \partial_\mu \lambda - i(A_\mu \star \lambda - \lambda \star A_\mu), \\ \delta_\lambda \phi^i &= -i(\phi^i \star \lambda - \lambda \star \phi^i) \qquad Td \qquad Td \qquad \star \end{aligned} \tag{5}$$

in at least one of the external legs [7]. The amplitude in the case of 3 external gluons, when all gluons are in the $U(1)$ is

$$A^{\mu\nu\rho}_{(1-1-1)} = \frac{i64g^3\sqrt{N/2}}{(4\pi)^2}\left(\frac{\tilde{p}_1^\mu\tilde{p}_1^\nu\tilde{p}_1^\rho}{\tilde{p}_1^4} + \frac{\tilde{p}_2^\mu\tilde{p}_2^\nu\tilde{p}_2^\rho}{\tilde{p}_2^4} + \frac{\tilde{p}_3^\mu\tilde{p}_3^\nu\tilde{p}_3^\rho}{\tilde{p}_3^4}\right). \tag{9}$$

We have ignored in this expression a factor $\cos\tilde{p}_1p_2/2$, which appears in previous calculations of the leading IR contribution to the 3-point function. The reason is that, in the approximation used to obtain (9), i.e. $\tilde{p}_ip_j \ll 1$, we cannot distinguish between $\cos\tilde{p}_1p_2/2$ and 1. We keep this convention in the following. When one gluon is in the $U(1)$ and the two other gluons are in the $SU(N)$ the amplitude takes the form

$$A^{\mu\nu\rho}_{(1-N-N)} = \frac{i64g^3\sqrt{N/2}}{(4\pi)^2}\,\frac{\tilde{p}_1^\mu\tilde{p}_1^\nu\tilde{p}_1^\rho}{\tilde{p}_1^4}, \tag{10}$$

where $\tilde{p}_1$ is the momentum of the $U(1)$ field. Similarly, the amplitude for two external scalars and one gluon, all in the $U(1)$, is

$$A^{\mu}_{(1-1-1)} = \frac{i32g^3\sqrt{N/2}}{(4\pi)^2}\left(\frac{\tilde{p}_1^\mu}{\tilde{p}_1^2} + \frac{\tilde{p}_2^\mu}{\tilde{p}_2^2} + \frac{\tilde{p}_3^\mu}{\tilde{p}_3^2}\right). \tag{11}$$

In the case of two scalars and one gluon transforming in $U(1)$ and $SU(N)$ the amplitude is the following

$$A^{\mu}_{(1-N-N)} = \frac{i32g^3\sqrt{N/2}}{(4\pi)^2}\,\frac{\tilde{p}_1^\mu}{\tilde{p}_1^2}, \tag{12}$$

where, again, $\tilde{p}_1$ is the momentum of the $U(1)$ field.

The information about the various non-planar diagrams can be summarized in the following effective action

$$\begin{aligned}
&\frac{\pi^2}{2}S_I = g^2\int d^4p\,\left(2\,\frac{\tilde{p}^\mu\tilde{p}^\nu}{\tilde{p}^4}\,\mathrm{tr}\,A_\mu(-p)\ \mathrm{tr}\,A_\nu(p) + \frac{1}{\tilde{p}^2}\,\mathrm{tr}\,\phi^i(-p)\ \mathrm{tr}\,\phi^i(p)\right)\\
&+\frac{i\,g^3}{(2\pi)^4}\int d^4p_1\,d^4p_2\,d^4p_3\,\delta(p_1+p_2+p_3)\times\\
&\Bigg\{2\,\frac{\tilde{p}_1^\mu\tilde{p}_1^\nu\tilde{p}_1^\rho}{\tilde{p}_1^4}\,\mathrm{tr}\,A_\mu(p_1)\ \mathrm{tr}\,A_\nu(p_2)A_\rho(p_3)\\
&+\frac{\tilde{p}_1^\mu}{\tilde{p}_1^2}\left(\mathrm{tr}\,A_\mu(p_1)\ \mathrm{tr}\,\phi^i(p_2)\phi^i(p_3) + 2\,\mathrm{tr}\,\phi^i(p_1)\ \mathrm{tr}\,\phi^i(p_2)A^\mu(p_3)\right)\Bigg\}.
\end{aligned} \tag{13}$$

Apart from the terms which are summarized in the effective action (13), there are other contributions which are less singular when $\theta \to 0$. In contrast to the poles, these terms (which as we shall see in a moment are log-like terms) do not cancel even in the supersymmetric case, apart from the $\mathcal{N} = 4$ SYM case [5]. These terms have a different Lorentz structure than the poles and they are all proportional to the one-loop beta function coefficient.

The gluon propagator (for the $U(1)$ degrees of freedom) contains the following non-planar contribution

$$M^{\mu\nu}_{(1-1)} = -\frac{26g^2N}{3(4\pi)^2}(p^2g^{\mu\nu} - p^\mu p^\nu)\log m^2\tilde{p}^2, \tag{14}$$

where m^2 is an IR cut-off. We can think about it as a mass term for the scalars (and vectors), given via a Higgs mechanism. Similarly to the gluon, the correction to the scalar propagator is

$$M_{(1-1)} = -\frac{26g^2N}{3(4\pi)^2}p^2\log m^2\tilde{p}^2. \tag{15}$$

The subleading corrections to the 3-point vertices are the following: for 3 gluons, all in the $U(1)$, we have

$$M^{\mu\nu\rho}_{(1-1-1)} = -\frac{i26g^3\sqrt{N/2}}{3(4\pi)^2}\sin(\frac{1}{2}\tilde{p}_1 p_2)(\log m^2\tilde{p}_1^2\; g^{\nu\rho}p_1^\mu + \text{perm.}), \tag{16}$$

where 'perm.' means permutations of the three momenta and Lorentz indices due to the symmetry of the amplitude. Similarly for the case of 1 gluon in the $U(1)$ and 2 gluons are in the $SU(N)$

$$M^{\mu\nu\rho}_{(1-N-N)} = -\frac{i26g^3\sqrt{N/2}}{3(4\pi)^2}\sin(\frac{1}{2}\tilde{p}_1 p_2)(\log m^2\tilde{p}_1^2\; g^{\nu\rho}p_1^\mu + \text{perm.}), \tag{17}$$

where now p_1 is the momentum of the $U(1)$ gluon, and the permutations are with respect to the 2 gluons in the $SU(N)$.

In the case of amplitudes where there are two scalars and one gluon we have

$$M^{\mu}_{(1-1-1)} = -\frac{i26g^3\sqrt{N/2}}{3(4\pi)^2}\sin(\frac{1}{2}\tilde{p}_1 p_2)(\log m^2\tilde{p}_1^2\; p_1^\mu + \text{perm.}), \tag{18}$$

and the same expression for the $SU(N) - SU(N) - U(1)$ amplitude.

The log-like amplitudes can be summarized by the following effective action

$$\begin{aligned}
&-\frac{24\pi^2}{13}S_{II} = g^2\int d^4p\ \left((p^2g^{\mu\nu} - p^\mu p^\nu)\log m^2\tilde{p}^2(\text{tr } A^\mu(-p))(\text{tr } A^\nu(p))\right.\\
&\left.+p^2\log m^2\tilde{p}^2(\text{tr } \phi^i(-p))(\text{tr } \phi^i(p))\right)\\
&+\frac{ig^3}{(2\pi)^4}\int d^4p_1\ d^4p_2\ d^4p_3\ \delta(p_1+p_2+p_3)\times\sin(\frac{1}{2}\tilde{p}_1p_2)\times\log m^2\tilde{p}_1^2\ p_1^\mu\ \times\\
&\left\{(\text{tr } A^\nu(p_1))(\text{tr } A^\mu(p_2)A^\nu(p_3)) + (\text{tr } \phi^i(p_1))(\text{tr } A^\mu(p_2)\phi^i(p_3))\right\}.
\end{aligned} \tag{19}$$

The actions (13)(19) are not gauge invariant. In order to have a (non-commutative) gauge invariant action, higher order terms in A_μ should be added. In the following sections we will derive a manifestly gauge-invariant action which includes (13) and (19) as part of it.

3 The Effective Action – Field Theory Derivation

3.1 The Poles

We will start by considering the pole-like IR-divergent contributions to the 2- and 3-point functions with only gluons as external legs. We observe that in both cases each vector field $A_\mu(p_i)$ is contracted with $\tilde{p}^\mu = \theta^{\mu\nu}p_\nu$, where p_ν is the total momentum flowing on each trace operator. This suggests that these terms are related to the simplest gauge-invariant operators carrying non-zero momentum, the straight open Wilson line defined by [11, 12]

$$W(p)\ =\ \text{tr}\int d^4x\ P_*\left(e^{i\,g\int_0^1 d\sigma\,\tilde{p}^\mu A_\mu(x+\tilde{p}\,\sigma)}\right) * e^{ipx}\,. \tag{20}$$

Indeed, the gluon 2- and 3-point functions (13) can be obtained as the first terms in the expansion of the following gauge-invariant expression

$$S^I_{eff}\ =\ \frac{2+N_s}{2\pi^2}\int d^4p\, W'(-p)f(\tilde{p})\,W'(p)\,, \tag{21}$$

with $f(\tilde{p})$ a function that tends in the IR to $1/\tilde{p}^4$; N_s is the number of scalars in the adjoint representation ($N_s = 6$ in the type 0 case). We denote by $W'(p)$ the Wilson loop operator (20) once the $O(g^0)$ term has been subtracted

$$W'(p) = i\, g\, \tilde{p}^\mu \mathrm{tr}\, A_\mu(p) - g^2 \int \frac{d^4l}{(2\pi)^4} \frac{\sin\frac{\tilde{l}p}{2}}{\tilde{l}p} \tilde{p}^\mu \tilde{p}^\nu \mathrm{tr}\, A_\mu(p-l) A_\nu(l) + \ldots . \tag{22}$$

By inserting (22) in (21), we immediately recover the gluon 2-point function. The expressions (9),(10) for the gluon 3-point function are valid in the limit $\tilde{p}_i p_j << 1$. In that limit $\sin\frac{\tilde{l}p}{2} / \frac{\tilde{l}p}{2} \to 1$ and thus also the 3-point function is correctly obtained from (21). This was to be expected since the IR divergent contribution to the 3-point function satisfies the Ward identity [8].

For the pure $U(1)$ non-commutative theory, (21) can be obtained from a direct calculation of the 1-loop N-point functions. This will allow us to determine the function f in (21). Due to the structure of the argument in the exponential of the Wilson loop, (21) contributes to the N-point function with terms proportional to $\tilde{p}^{\mu_1} ... \tilde{p}^{\mu_N}$. The N-point functions will have in general a complicated Lorentz index structure. However it is easy to isolate the terms of the mentioned form. They can only come from diagrams with 3-point vertices. Diagrams with 4-point vertices will give rise to a tensor structure containing $g^{\mu_i \mu_j}$, and therefore are not of the desired form. We will like to point out that diagrams with 4-point vertices can produce as strong an IR divergence as those with only 3-point vertices. Indeed, the tadpole induces a quadratic pole-like contribution to the 2-point function of the form $g^{\mu\nu}/\tilde{p}^2$. However the role of this term is to cancel a similar contribution coming from diagrams with 3-point vertices, and which would otherwise violate the Ward identity [5]. The same applies to the 3-point function. We will thus ignore diagrams with 4-point vertices when analizing the leading UV/IR mixing effects.

We will use the background field method in the following analysis; for the associated Feynman rules see [8]. The diagrams we are interested in are those depicted in Fig.1. We have

$$\begin{aligned} &(a) + (b) = \\ &(-2ig)^N \int \frac{d^4l}{(2\pi)^4} \prod_{i=1}^{N} \frac{(2l + 2p_1 + .. + 2p_{i-1} + p_i)_{\mu_i}}{(l + p_1 + .. + p_{i-1})^2} \sin\frac{\tilde{p}_i(l+p_1+..+p_{i-1})}{2} . \end{aligned} \tag{23}$$

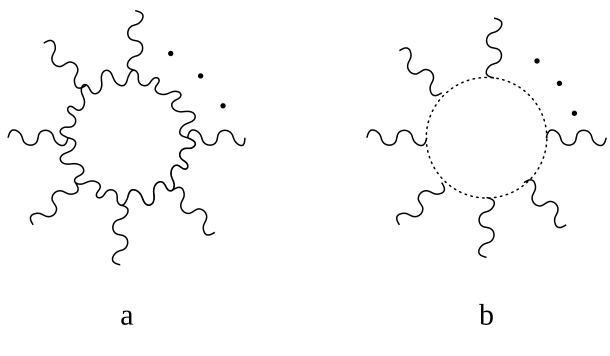

Fig. 1. *Amplitudes containing terms* $\sim \tilde{p}^{\mu_1} ... \tilde{p}^{\mu_N}$. *Wavy lines refer to gauge bosons and doted lines to ghost. The end points of the external lines are background vector fields* B_μ.

As explained, we will disregard those parts of (23) whose tensor structure is such that they cannot contribute to (21). This allows us to discard all the terms in the numerator proportional to external momenta, and keep only $2l_{\mu_i}$ for each i. Expression (23) then reduces to

$$(-2ig)^N \sum_{\nu_i} (-)^n \int \frac{d^4l}{(2\pi)^4} \frac{l_{\mu_1} .. l_{\mu_N}\, e^{-i\tilde{p}l - \frac{i}{2}\sum_{j<k} \tilde{p}_j p_k \nu_k}}{l^2 (l+p_1)^2 ... (l + p_1 + .. + p_{N-1})^2} , \tag{24}$$

where the summation on ν_i, $\nu_i = \pm 1$ for $i = 1,..,N$, comes from expanding the sine. We have defined $p = \sum_i p_i \frac{1-\nu_i}{2}$ and $n = \sum_i \frac{1-\nu_i}{2}$. We can interpret the N vertices as twisted or untwisted depending if $\nu = -1$ or 1 respectively. Thus n is the number of twisted vertices and p the total momentum flowing in the twisted vertices. The l_{μ_i} in the numerator can now be substituted by derivatives with respect to $\tilde{p}^\mu$ acting on the exponential. In order to simplify the analysis will we consider $\tilde{p}$ as an independent variable, and only in the end we will set $\tilde{p}^\mu = \theta^{\mu\nu} p_\nu$ with $p = \sum_i p_i \frac{1-\nu_i}{2}$. This allows us to bring the derivatives out of the integral, and rewrite (24) as

$$(-2ig)^N \, \partial_{\mu_1} ... \partial_{\mu_N} \sum_{\nu_i} (-)^n \int \frac{d^4 l}{(2\pi)^4} \frac{e^{-i\tilde{p}l - \frac{i}{2}\sum_{j<k} \tilde{p}_j p_k \nu_k}}{l^2 (l+p_1)^2 ... (l+p_1+..+p_{N-1})^2} \,. \tag{25}$$

The evaluation of the previous integral gives[1]

$$J_{*n}(-p) \left(\frac{\tilde{p}}{m}\right)^{N-2} K_{N-2}\left(m\tilde{p}\right) J_{*N-n}(p) \,, \tag{26}$$

where K_{N-2} are modified Bessel functions. We have denoted by $J_{*n}(-p)$ the kernel of the $*_n$-product defined in [13], i.e. $J_{*n}(-p) \equiv J(-p_{r(1)},..,-p_{r(n)})$ where $p_{r(j)}$ are the n momenta entering the twisted vertices and $p = \sum p_{r(j)}$. A comment is now in order. Expression (26) is not the complete answer, but the leading term in the infrared. Subleading terms are suppressed by powers of $\tilde{p}^2 p_i p_j$ and therefore they do not give rise to infrared divergences for any N[2]. In the following, we will keep in the evaluation of the N-point functions only the infrared-leading term. Then, (25) reduces to

$$\frac{1}{2\pi^2} (-ig)^N \sum_{\nu_i} (-)^n J_{*n}(-p) \left[\partial_{\mu_1} ... \partial_{\mu_N} \left(\frac{\tilde{p}}{m}\right)^{N-2} K_{N-2}\left(m\tilde{p}\right) \right] J_{*N-n}(p) \,. \tag{27}$$

Using the properties of the modified Bessel functions, it is easy to see that the term in square brackets gives rise to a contribution of the form

$$(-)^N \, \tilde{p}^{\mu_1} ... \tilde{p}^{\mu_N} \, \frac{m^2}{\tilde{p}^2} \, K_2(m\tilde{p}) \,. \tag{28}$$

Adding up all such contributions to the effective action we get

$$\begin{aligned} S^I_{eff} \;\; &= \;\; \frac{1}{2\pi^2} \sum_{N=2}^{\infty} (ig)^N \int d^4 p \sum_{n=1}^{N-1} \frac{(-)^n}{n!(N-n)!} \\ & \frac{m^2}{\tilde{p}^2} K_2(m\tilde{p}) \, \tilde{p}^{\mu_1} ... \, \tilde{p}^{\mu_N} \, [A_{\mu_1}..A_{\mu_n}]_{*n}(-p) \, [A_{\mu_{n+1}}..A_{\mu_N}]_{*N-n}(p) \,. \end{aligned} \tag{29}$$

This expression reproduces (21) by setting $2f(\tilde{p}) = \frac{m^2}{\tilde{p}^2} K_2(m\tilde{p})$. Although the $*_n$ also appear in the effective action of the non-commutative Φ^3 theory, they only combine to form the scalar analog of Wilson loop operators in the limit of large non-commutative parameter. On the contrary, the invariance of the effective action with respect to gauge transformations of the background field suggests that, in gauge theories, Wilson loop operators will play an important role for any value of θ. As a first example, a Wilson loop completion of the non-planar contributions to the F^4

[1] This result is not affected by considering p and $\tilde{p}$ as independent variables.

[2] It is interesting that the subleading terms do not seem to have such a simple expression in terms of $*_n$ products as (26).

terms in $\mathcal{N}=4$ gauge theory has been proposed in [14]. We have just seen that the puzzling pole-like divergent terms originating from UV/IR mixing are part of the simplest gauge-invariant double-trace operator that can appear in the effective action. We will leave for the next section the extension of the previous considerations to gauge theories with adjoint matter.

3.2 The Logarithms

We would like to comment on the IR logarithmic-divergent terms arising from UV/IR mixing. As already mentioned, these subleading contributions occur also in the supersymmetric case. We suggest here a gauge invariant completion of the IR logarithmic divergent terms. This suggestion is not as rigorous as the derivation in the previous subsection, but our result is fixed by the requirement of gauge invariance.

It was shown in [8] that the logarithmic singularities of the 2-, 3- and 4-point function of pure NC $U(1)$, in the limit $|\tilde{p}_i| \sim |\tilde{p}_i+\tilde{p}_j| \sim \theta\Lambda_{IR} \to 0$, combine into the following contribution to the effective action:

$$S_{eff}^{II} = \frac{1}{4}\beta_0 \log{(\theta\Lambda_{\mathrm{IR}})^2} \int \mathrm{d}^4\mathrm{x}\, \mathrm{F}^{\mu\nu}\mathrm{F}_{\mu\nu}\,, \tag{30}$$

with Λ_{IR} an infrared cut-off and β_0 the coefficient of the 1-loop beta function. It is tempting to propose the following gauge-invariant completion of (30), which generalizes to the $U(N)$ case

$$S_{eff}^{II} = \frac{1}{4}\beta_0 \int d^4p\, \mathcal{O}^{\mu\nu}(-p) K_0(m\tilde{p}) \mathcal{O}_{\mu\nu}(p)\,, \tag{31}$$

where the operator $\mathcal{O}_{\mu\nu}$ is defined by

$$\mathcal{O}_{\mu\nu}(p) = \mathrm{tr} \int \mathrm{d}^4\mathrm{x}\ \mathrm{L}_* \left(\mathrm{F}_{\mu\nu}(\mathrm{x})\, \mathrm{e}^{\mathrm{i\,g} \int_0^1 \mathrm{d}\sigma\, \tilde{\mathrm{p}}^\mu \mathrm{A}_\mu(\mathrm{x}+\tilde{\mathrm{p}}\,\sigma)}\right) * \mathrm{e}^{\mathrm{ipx}}\,. \tag{32}$$

Following the notation of [14], L_* denotes integration of $F_{\mu\nu}$ along the open Wilson line together with path ordering with respect of the $*$-product of all terms inside the parenthesis. The action (31) reproduces the pure gluonic log-like N-point functions (19) in the small m limit.

4 The Effective Action via Closed Strings Exchange

The recent interest in the study of non-commutative field theories has been mainly motivated by their connection to string theory. The world-volume coordinates of D-branes in the presence of a constant B-field background turn out to satisfy the relation $[x^\mu, x^\nu] = i\theta^{\mu\nu}$, with $\theta^{\mu\nu} \sim 1/B_{\mu\nu}$. As a consequence, the low energy theory on the brane is a non-commutative gauge theory. In this section we would like to analyze (21) and (31) (or (13) and (19)) from a string-inspired point of view. In a series of recent papers it has been shown that closed string modes couple to non-commutative D-branes through open Wilson line operators [14, 15, 16, 17]. This result was obtained by evaluating the disk amplitude between a closed string and open string modes.

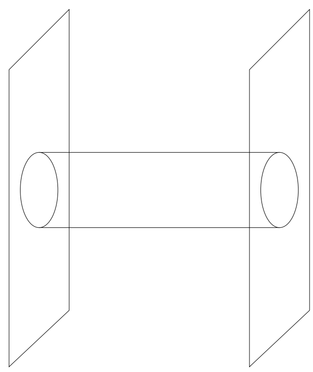

Fig. 2. *The annulus amplitude.*

Let us consider the annulus diagram with boundaries on non-commutative D-branes as in Fig.2. It can be seen as a loop of open strings or a tree level exchange of closed strings. In the limit of a large cylinder the closed string channel picture is more adequate since the annulus diagram factorizes to closed string insertions on a disk connected by a closed string propagator [18].

In the opposite limit of a small cylinder, the exchange of the lowest open string modes dominates. This provides the field theory limit, and the annulus amplitude reproduces the 1-loop field theory effective action. Thus in general we could expect in the field theory effective action more complicated contributions than (21) and (31), which structurally are reminiscent of a closed string exchange. Notice that a similar structure was proposed as the gauge-invariant completion of the non-planar F^4 terms in the effective action of $\mathcal{N}=4$ non-commutative Yang-Mills [13, 14]. In that case, the function f had the interpretation of a closed string propagator in type II string theory. This however comes as no surprise since the F^4 terms in the maximally supersymmetric case are protected by non-renormalization theorems [18]. In contrast, it is remarkable that (21) emerges in a non-supersymmetric theory. We will show below that the function f appearing in the IR-divergent terms can also be directly related to a closed string propagator.

In the rest of this section we will consider type 0 string theory. This theory can be obtained as a world-sheet orbifold of type II, which projects out space-time fermions. It contains a closed string tachyon arising from the twisted sectors. There are however no open string tachyons on D-branes in type 0 theory. This makes it especially adequate for our considerations. We will work with the gauge theory on N electric D3-branes. It is given by the dimensional reduction of pure Yang-Mills in 10 dimensions, i.e. gauge fields plus 6 scalars in the adjoint representation.

We start by analysing which closed string modes couple to the open Wilson line operator (20). The first candidate is the type 0 tachyon. In the absence of B-field and at leading order in α', it couples to the brane tension as [19, 20]

$$\frac{N}{4\,(2\pi\alpha')^2}\,. \tag{33}$$

Following the same analysis done for bosonic and type II string theory [14, 16], it is easy to see that the trivial field theory operator (33) gets promoted to an open Wilson loop in the presence of a B-field. The coupling of the type 0 tachyon to the D-brane field theory at leading α' order is described by

$$S^I=\frac{\kappa_{10}}{g_{YM}^2}\int\frac{d^{10}P}{(2\pi)^{10}}\,\sqrt{\det \mathrm{G}}\;T(P)\;\mathcal{O}(-P)\,, \tag{34}$$

where G is the open string metric and

$$\mathcal{O}(P)=\frac{1}{4\,(2\pi\alpha')^2}\,\mathrm{tr}\int d^4x\,W(x,C)*e^{ipx}\,. \tag{35}$$

We denote by P_M the 10-dimensional momentum, p_μ the momentum along the 4-dimensional world-volume of the D3-brane and $p_{\perp i}$ the momentum in the transverse directions. In the previous expressions $W(x,C)$ is a generalization of (20) which involves the transverse scalars

$$W(x,C)=P_*\left(e^{i\,g\int_0^1 d\sigma\,\tilde{p}^\mu A_\mu(x+\tilde{p}\sigma)+y_i\phi^i(x+\tilde{p}\sigma)}\right)\,, \tag{36}$$

where we have defined $y_i=2\pi\alpha' p_{\perp i}$ and $\phi^i=X^i/2\pi\alpha'$ for $i=1,...,6$, which provides the correct normalization for the field theory scalar fields.

The on-shell condition for the type 0 tachyon is $P_M g^{MN}P_N=-2/\alpha'$, with g the closed string metric. Closed and open string metrics are related by $g^{-1}=G^{-1}-\theta G\theta/(2\pi\alpha')^2$ [2]. In the Seiberg-Witten limit, i.e. $\alpha'\to 0$ keeping G and θ fixed, the on-shell condition becomes [14, 16]

$$\tilde{p}^2+y^2=8\pi^2\alpha'\,. \tag{37}$$

The closed string mass is a subleading effect with respect to the momentum in the non-commutative directions in the Seiberg-Witten limit. In spite of that, it will be crucial in the following to keep its contribution to the mass-shell condition. We want to analyse how the tachyon exchange contributes to the annulus amplitude. For two D3-branes separated by a distance r we obtain

$$S^I_{\mathit{eff}} = \frac{\kappa_{10}^2}{g_{YM}^4} \int \frac{d^{10}P}{(2\pi)^{10}} \frac{\det G}{\sqrt{\det g}} \, \mathcal{O}(P) \, \mathcal{O}(-P) \, \frac{e^{ip_\perp r}}{\frac{M^2}{(2\pi\alpha')^2} + p_\perp^2} \,. \tag{38}$$

The quantity $M^2/(2\pi\alpha')^2$ is the effective mass of the closed string tachyon propagating in the six transverse dimensions; from (37) $M^2 = \tilde{p}^2 - 8\pi^2\alpha'$.

In order to make contact with the previous section we will first consider the dependence of $\mathcal{O}$ on the gauge fields only. Then $\mathcal{O} = \frac{1}{(4\pi\alpha')^2} W(p)$, with $W(p)$ given by (20). Using (see for example [14])

$$\frac{\kappa_{10}^2}{g_{YM}^4} = \pi(2\pi\alpha')^4 \frac{\sqrt{\det g}}{\sqrt{\det G}} \,, \tag{39}$$

and defining $m = r/2\pi\alpha'$, the previous expression can be rewritten as

$$S^I_{\mathit{eff}} = \frac{\pi}{(4\pi\alpha')^4} \int \frac{d^4p}{(2\pi)^4} \sqrt{\det G} \; \mathrm{tr}\, W(p) \; \mathrm{tr}\, W(-p) \; G(p) \,, \tag{40}$$

where

$$G(p) = \int \frac{d^6y}{(2\pi)^6} \frac{e^{iym}}{M^2 + y^2} = \frac{1}{(2\pi)^3} \frac{M^2}{m^2} K_2(mM) \,. \tag{41}$$

$G(p)$ represents the closed string propagator in the transverse dimensions, rescaled appropriately to the field theory limit. Indeed, it is finite in the limit $\alpha' \to 0$. However (40) diverges in this limit due to the $O(\alpha'^{-2})$ dependence of the brane tension to which the tachyon couples. We can define a finite contribution to (40) by expanding $G(p)$ to $O(\alpha'^4)$, using the explicit dependence of M^2 on α'. We then obtain a contribution to the field theory effective action of the form (21), with

$$f(p) \equiv G(p)|_{\alpha'^4} = c \, \frac{m^2}{\tilde{p}^2} K_2(m\tilde{p}) \,, \tag{42}$$

with $c = \frac{4\pi^5}{3}$. This agrees with the result derived from the field theory calculation, up to a global coeficient. We will comment on this below.

Notice that in order to obtain the IR divergent terms from the string exchange, it was essential that the field theory operator that couples to the tachyon carries negative powers of α'. The reason for this is that at $\mathcal{O}(\alpha'^0)$, $G \sim 1/m^4$ as $\tilde{p} \to 0$. Such a term is related to the ordinary infrared problems of a field theory with massless degrees of freedom. However, remarkably, $G(p)$ contains information about the new divergences due to UV/IR mixing effects in non-commutative field theories when expanded to higher orders in α'. We have analyzed above the coupling of the type 0 tachyon to the D-brane field theory at leading order in α'. At $\mathcal{O}(\alpha'^0)$ it couples to the field theory operators $\mathrm{tr}F^2$ and $(D\phi^i)^2$ [19, 20]. For the reasons just exposed, the coupling of the tachyon to these operators would contribute non-singular terms in the effective action and therefore we will not consider them.

Equation (40) differs from (21), as we subtracted the '1' from the open Wilson line in (21). The '1' in coordinate space is in fact $\delta(p)$, as we work in Fourier space, and therefore this difference affects only the $p_\mu = 0$ component of W. We would like to stress that the expansion of $G(p)$ in α' powers requires that $\tilde{p}$ is non zero. At $\tilde{p} = 0$ and in the limit $\alpha' \to 0$, the string propagator is $G \sim 1/m^4$. The associated contribution to the effective action is proportional to

$$\frac{1}{(\alpha' m)^4} \, \delta^{(4)}(0) \; \to \; \Lambda^4 \int d^4x \,, \tag{43}$$

where $1/(\alpha' m) \sim \Lambda$ can be interpreted as a field theory scale. Therefore the difference between (40) and (21) reflects the vacuum energy of the gauge theory, which is taken into account in the string theory calculation. Once this infinity is substracted, the string exchange just reproduces the field theory result (21).

We will now show that (38) can also reproduce the pole-like divergent terms associated with the adjoint scalars. Expanding $\mathcal{O}$ to linear order in the fields, we obtain the following contribution involving the adjoint scalars

$$S'^{I}_{\mathit{eff}} \sim \int \frac{d^4p}{(2\pi)^4} \sqrt{\det G}\, \mathrm{tr}\, \phi_i(p)\ \mathrm{tr}\, \phi_j(-p)\ f_{ij}(p)\,, \tag{44}$$

where

$$\begin{aligned} f_{ij} &= \int \frac{d^6y}{(2\pi)^6} \frac{e^{iym}\, y_i y_j}{M^2+y^2}\bigg|_{\alpha'^4} = \\ &= -\partial_{m_i}\partial_{m_j} G(p) = c\left(\delta_{ij}\frac{m}{\tilde{p}} K_1(m\tilde{p}) + m_i m_j K_0(m\tilde{p})\right)\,. \end{aligned} \tag{45}$$

The first term in (45) leads to the action

$$S'^{I}_{\mathit{eff}} \sim \int \frac{d^4p}{(2\pi)^4} \sqrt{\det G}\, \mathrm{tr}\, \phi_i(p)\ \mathrm{tr}\, \phi_i(-p)\ \frac{m}{\tilde{p}} K_1(m\tilde{p})\,, \tag{46}$$

which corresponds, in the $m\tilde{p} \to 0$ limit, to the pole-like contribution in the effective action of the scalars (13). The second term in (45) yields a $m^2 \log$ contribution which vanishes when $m \to 0$. Notice that while f in (42) tends to $2c/\tilde{p}^4$ in the infrared limit, f_{ij} tends to $c/\tilde{p}^2$. This reproduces the relative factor of two between the pole-like contributions to the propagator of the gauge field and adjoint scalars, eq.(7),(8). The same applies to the linear poles of the 3-point functions. Therefore the gauge invariant expression (40), defined such that we only keep the finite terms in the $\alpha' \to 0$ limit, accounts for all pole-like divergent terms of the field theory up to a global coefficient.

The discrepancy in the global coefficient can be related to the fact that not only the tachyon, but also massive scalar closed strings couple to the brane tension. In the Seiberg-Witten limit these contributions are of the same form as that of the tachyon, since momentum in the non-commutative directions dominates over the oscillator mass. Thus they will renormalize the overall coefficient in front of the effective action. To summarize, we have seen that the gauge invariant effective action containing the infrared poles can be directly related to a closed string exchange between D-branes. It is tempting to think of this as the exchange of an "effective closed string mode". Remarkably, among the original closed string modes that contribute to this effect is the tachyon mode.

We will briefly address the log like contributions which appear also in the supersymmetric field theory (19). Consider a two-form (denoted by M_{MN}) closed string which couples to the operator $\mathcal{O}^{MN}$ (separated into 4d and 6d indices):

$$S^{II} = \frac{\kappa_{10}}{g_{YM}^2} \int \frac{d^{10}P}{(2\pi)^{10}} \sqrt{\det \mathrm{G}}\ \left(M_{\mu\nu}(P)\mathcal{O}^{\mu\nu}(-P) + M_{\mu i}(P)\mathcal{O}^{\mu i}(-P)\right)\,, \tag{47}$$

with

$$\begin{aligned} \mathcal{O}^{\mu\nu}(P) &= \frac{1}{2\pi\alpha'} \mathrm{tr} \int d^4x\, L_*(F^{\mu\nu} W(x,C)) * e^{ipx}\,, \\ \mathcal{O}^{\mu i}(P) &= \frac{1}{2\pi\alpha'} \mathrm{tr} \int d^4x\, L_*(D^{\mu}\phi^i W(x,C)) * e^{ipx}\,. \end{aligned} \tag{48}$$

Repeating the same steps as for the tachyon field we can write the effective action due to an exchange of a massive 2-form as

$$S^{II}_{eff} \sim \int \frac{d^4pd^6y}{(2\pi)^{10}} \sqrt{\det G}\; \mathcal{O}^{MN}(p,y)\,\mathcal{O}_{MN}(-p,-y)\frac{e^{iym}}{M^2+y^2}\,, \tag{49}$$

with $M^2 = \tilde{p}^2+8\pi^2 l\alpha'$ and l some positive integer number which corresponds to the string excitation number. For simplicity let us set the adjoint scalar fields to zero in $W(x,C)$, which does not affect gauge invariance. We get then

$$S^{II}_{eff} \sim \int \frac{d^4p}{(2\pi)^4}\left(\mathcal{O}_{\mu\nu}(p)\mathcal{O}^{\mu\nu}(-p)+\mathcal{O}_{\mu i}(p)\mathcal{O}^{\mu i}(-p)\right)G(p)\,, \tag{50}$$

but now we should simply keep the terms in $G(p)$ which are $O(\alpha'^2)$. This yields

$$G(p)|_{\alpha'^2} \sim K_0(m\tilde{p})\,. \tag{51}$$

which reproduces the action (31) and in addition the log like pieces of the scalars (19).

Thus, we have shown that the logarithmic like (K_0, in fact) contribution to effective action can be understood from massive 2-form closed strings exchange. Note that the massless NS-NS 2 form does not contribute here. Only massive modes. Another comment is that we could not reproduce the overall factor in front of the effective action, β_0. The understanding of the overall factor, from the string theory point of view, is equivalent to the understanding of the weight of each individual massive string in the coupling to the operator F^{MN} on the brane. We will not address this problem here.

5 Discussion

In this talk we have discussed UV/IR effects in a non-supersymmetric gauge theory. Our main results are two effective actions which involve logs and poles.

The log like contributions exist also in the supersymmetric theory, apart from the $\mathcal{N}=4$ SYM theory. The picture that emerges from our work is that one can understand these effects as due to an exchange of massive two-form closed strings which couple to the operator tr $F_{\mu\nu}$.

The more interesting contributions are the poles. These poles cancel in the supersymmetric gauge theory. Our picture here is that these terms can be understood as due to an exchange of a tachyon and massive scalar closed strings that couple to the brane tension. In the superstring theory there is no tachyon. Moreover, the contributions from the NS-NS sector cancel the contributions from the R-R sector and this is our explanation of why we do not see such effects in the (super-)gauge theory side.

The (partial) contribution of the closed string tachyon to the tachyonic instabilities of the non-commutative theory suggests that the two phenomena are related. Indeed, it is true that the poles are also due to massive closed strings, since in the Seiberg-Witten limit all the massive tower contributes similarly to the exchange between the D-branes [6]. Therefore we do not argue that the tachyon in the field theory has a one to one correspondence with the closed string tachyon.

In the light of our picture, we would like to address the problem of the stability of the non-commutative non-supersymmetric Yang-Mills theory. Since this theory is tachyonic, similarly to type 0 string theory, we suggest that the consistency issue is related to the fate of the closed string tachyon. It is tempting to suggest that if there is tachyon condensation in type 0 string theory, there will be examples of non-commutative non-supersymmetric gauge theories which are consistent and that (1) is a consequence of the expansion around the perturbative vacuum.

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