Latest Developments in Noncommutative Field and Gauge Theories

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Some properties of quantum field theories on noncommutative space-time are reviewed. Studying the general structure of the noncommutative (NC) local groups, we present a no-go theorem for NC gauge theories. This no-go theorem imposes strong restrictions on the NC version of the Standard Model and in resolving the standing problem of charge quantization in noncommutative QED. We also consider the phenomenological implications of noncommutativity on the spectrum of the H-atom and derive a bound on the noncommutativity parameter θ. Finally, in the framework of noncommutative quantum field theories (NC QFT), we show the general validity of the CPT and spin-statistics theorems, with the exception of some peculiar situations in the latter case.

1. Introduction

It is generally believed that the notion of space-time as a continuous manifold should break down at very short distances of the order of the Planck length $\lambda_P \approx 1.6 \times 10^{-33} cm$. This would arise, e.g. from the process of measurement of space-time points based on quantum mechanics and gravity arguments [1]. Arguments for noncommutativity arise also from string theory with a constant antisymmetric background field, whose low-energy limit, in some cases, turns up to be a NC QFT [2]. This in turn implies that our classical geometrical concepts may not be well suited for the description of physical phenomena at very small distances. One such direction is to try to formulate physics on some noncommutative space-time [1]-[3]. If the concepts of noncommutative geometry are used, the notion of point as elementary geometrical entity is lost and one first expectation is that an ultraviolet cutt-off appears. In [4] this expectation was shown not to be fulfilled in general. Instead, a peculiar UV/IR mixing appears [5].

The usual way of constructing a noncommutative theory is through the Weyl-Moyal correspondence: in a NC space-time the coordinate operators satisfy the commutation relation:

$$[x^\mu, x^\nu] = i\theta^{\mu\nu} , \tag{1}$$

where $\theta^{\mu\nu}$ is a constant antisymmetric matrix of dimension (length)^2. In QFT the operator character of the space-time coordinates (1) requires that the product of any two field operators be replaced by their $\star$-product, or Weyl-Moyal product. The $\star$-product compatible with the associativity of field products is given by:

$$\phi(x) \star \psi(x) = e^{\frac{i}{2}\theta^{\mu\nu}\frac{\partial}{\partial x^\mu}\frac{\partial}{\partial y^\nu}} \phi(x)\psi(y)\Big|_{x=y} . \tag{2}$$

An important step in constructing a physical noncommutative model is to develop the concept of local gauge symmetry. Intuitively, because of the inherent nonlocality of noncommutative field theories, the notion of *local* symmetry in the noncommutative case should be handled with special care. As a result, the pure noncommutative $U(1)$ theory behaves similarly to the usual non-Abelian gauge theories, but now the structure constants depend on the momenta of the fields [6]. This feature induces a charge quantization problem [7], in the sense that the electric charges in the NC QED based on NC U(1) group are quantized only to $\pm 1, 0$. The solution of this problem was sought in the construction of a noncommutative version of the Standard Model (NC SM) [8], based on a no-go theorem [9], and is discussed in Section II.

In Section III phenomenological implications of the noncommutativity are also addressed on a concrete model of the H-atom, for which we present the noncommutative corrections to the spectrum and, using the data for the Lamb shift, we find a bound on the noncommutativity parameter θ [10].

In Section IV, we show that a breaking of the spin-statistics relation in NC QFT could occur *only* in the case of theories with NC time. We also present in Section V a general proof that the CPT theorem remains valid in NC field theories, for general form of noncommutativity, although the individual symmetries C,T and P are broken [11].

2. Noncommutative gauge groups. A no-go theorem

2.1. Charge quantization problem in NC QED

In [7] it was shown that in NC QED based on the NC U(1) group, one can encounter only fields with charge +1:

$$\begin{aligned} \psi(x) \to \psi'(x) &= U(x) \star \psi(x)\,, \\ D_\mu \psi &= \partial_\mu \psi - i A_\mu \star \psi\,, \end{aligned} \tag{3}$$

fields with charge -1:

$$\begin{aligned} \psi(x) \to \psi'(x) &= \psi(x) \star U^{-1}(x)\,, \\ D_\mu \psi &= \partial_\mu \psi + i \psi \star A_\mu \end{aligned} \tag{4}$$

and fields with charge 0:

$$\begin{aligned} \chi(x) \to \chi'(x) &= U(x) \star \chi(x) \star U^{-1}(x)\,, \\ D_\mu \chi &= \partial_\mu \chi + i[\chi, A_\mu]_\star\,. \end{aligned} \tag{5}$$

This immediately raises the question about other known charges, i.e. the fractional charges of quarks. The simple extension

$$D_\mu \psi^{(n)} = \partial_\mu \psi^{(n)} - i n A_\mu \star \psi^{(n)}\,, \tag{6}$$

with

$$\psi^{(n)} \to \psi'^{(n)} = U^{\star n} \star \psi^{(n)} \tag{7}$$

for the field ψ with integral multiple n of a (conventional) unit charge fails to transform covariantly. In conclusion in NC QED, charge is quantized *only* to 0,± 1. A possible way out from this situation is to construct a NC version of the Standard Model, to which end we have to choose the gauge groups and their representations and also define the direct product of group factors.

2.2. A no-go theorem

The following result was partially obtained in [12] in the framework of noncommutative gauge groups and extended to a no-go theorem in [9]. In general, as discussed in [13], it is not trivial to define the noncommutative version of usual simple local groups, as the $\star$-productwill destroy the closure condition. Consequently, the only group which admits a minimal noncommutative extension is $U(n)$ (we will denote its extention by $U_\star(n)$). However the $NCSO$ and USp *algebras* have been constructed in a more involved way [13].

To define the pure NC $U_\star(n)$ gauge theory we take as generators of the $u_\star(n)$ algebra: T^a, $a = 1, \cdots, n^2-1$ ($n \times n$ $su(n)$ generators) and $T^0 = \frac{1}{\sqrt{2n}} \mathbf{1}_{n\times n}$. The $u_\star(n)$ Lie-algebra is defined with the star-matrix bracket:

$$[f, g]_\star = f \star g - g \star f \ , \qquad f, g \in u_\star(n) \ . \tag{8}$$

The $U_\star(n)$ gauge theory is described by the $u_\star(n)$ valued gauge fields:

$$G_\mu = \sum_{A=0}^{n^2-1} G^A_\mu(x) T^A \ , \tag{9}$$

with the infinitesimal gauge transformation

$$G_\mu \to \ G'_\mu = G_\mu + i\partial_\mu \lambda + g[\lambda, G_\mu]_\star \ , \ \lambda \in u_\star(n) \ . \tag{10}$$

Under the above tranformation, the field strength

$$G_{\mu\nu} = \partial_{[\mu} G_{\nu]} + ig[G_\mu, G_\nu]_\star \ , \tag{11}$$

transforms covariantly:

$$G_{\mu\nu} \to \ G'_{\mu\nu} = G_{\mu\nu} + ig[\lambda, G_{\mu\nu}]_\star \ . \tag{12}$$

leaving invariant the action of the pure $U_\star(n)$ gauge theory:

$$S = -\frac{1}{4\pi} \int d^D x \ \mathrm{Tr}(G_{\mu\nu} \star G^{\mu\nu}) \ . \tag{13}$$

One peculiar feature to be noticed in the case of the pure $U_\star(n)$ gauge theory is that, fixing the number of gauge field degrees of freedom (which is n^2) the dimension of the matrix representation is automatically fixed, i.e. *the gauge fields must be in the* $n\times n$ *matrix form.* The main physical implication is that the matter fields coupled to the $U_\star(n)$ gauge theory can only be in *fundamental, antifundamental, adjoint and singlet states.*

Another nontrivial point in the noncommutative gauge theories is to define the direct product of NC gauge groups. In the commutative case, if G_1 and G_2 are gauge groups, then $G = G_1 \times G_2$ is defined through:

$$\begin{gathered} g = g_1 \times g_2 \ , g' = g'_1 \times g'_2, \quad g_i, g'_i \in G_i, \ \ g, g' \in G, \\ g \cdot g' = (g_1 \times g_2) \cdot (g'_1 \times g'_2) \equiv (g_1 \cdot g'_1) \times (g_2 \cdot g'_2) \ . \end{gathered} \tag{14}$$

In the noncommutative case, let $G_1 = U_\star(n)$ and $G_2 = U_\star(m)$. But now the group products involve the $\star$-product, so that the group elements can not be re-arranged. As a result, the definition of direct product cannot be straightforwardly generalized to the NC case and consequently the *matter fields cannot be in fundamental representations of both* $U_\star(n)$ *and* $U_\star(m)$. The only possibility left is for a matter field to be in the fundamental representation of a gauge group and the antifundamental representation of another:

$$\Psi \to \Psi' = U \star \Psi \star V^{-1}, \ U \in U_\star(n), \ V \in U_\star(m). \tag{15}$$

In the general case of n gauge groups

$$G = \prod_{i=1}^{N} U_\star(n_i) \ , \tag{16}$$

the matter fields can be charged under at most two of the $U_\star(n_i)$ *factors.*

2.3. NC Standard Model. A solution to the charge quantization problem

Based on the above no-go theorem, we have built a noncommutative version of the Standard Model [8]. The model is based on the gauge group $U_\star(3) \times U_\star(2) \times U_\star(1)$ (the general elements of the respective group factors will be denoted by $U \in U_\star(3), V \in U_\star(2), v \in U_\star(1)$) and comprises: **one** gauge field, B_μ, valued in $u_\star(1)$, **four** gauge fields of $u_\star(2)$:

$$W_\mu(x) = \sum_{I=0}^{3} W_\mu^I(x)\sigma^I \ , \tag{17}$$

where $\sigma^i,\ i = 1,2,3$ are the Pauli matrices and $\sigma^0 = \mathbf{1}_{2\times 2}$ and **nine** gauge fields of $u_\star(3)$:

$$G_\mu(x) = \sum_{A=0}^{8} G_\mu^A(x)T^A \ , \tag{18}$$

where $T^a,\ a = 1,2,\cdots,8$ are the Gell-Mann matrices and $T^0 = \mathbf{1}_{3\times 3}$. This choice of the gauge group is due to the fact that there is no straightforward noncommutative extension of the $SU(n)$ groups. However, compared to the commutative Standard Model, two additional gauge fields have appeared, corresponding to the extra $U(1)$ factors. The reduction of the extra $U(1)$ factors is achieved through a Higgs-type of mechanism, in two stages. First the mechanism is run with the symmetry-reducing scalar field

$$\Phi_1(x) \to U_1(x)\Phi_1(x)V_1^{-1}(x) \ , \tag{19}$$

with $U_1 \in U(1) \subset U_\star(3)$ and $V_1 \in U(1) \subset U_\star(2)$. In the second stage, the symmetry is reduced eventually to that of hyper-charge, throught the scalar particle

$$\Phi_2(x) \to s(x)\Phi_2(x)v^{-1}(x) \ , \tag{20}$$

with $s \in U(1)_{residual}$ and $v \in U_\star(1)$. After the symmetry reduction, two of the gauge fields become massive (G^0 and W^0) and the gauge field corresponding to the residual $U(1)$ symmetry will be the (masless) hyper-photon Y.

When coupling matter fields to the $U_\star(3) \times U_\star(2) \times U_\star(1)$ theory, we have to keep in mind that, according to [9], the fields can be only in the fundamental and/or anti-fundamental representation of the group factors. It is interesting to note that the no-go theorem allows six different types of charged particles in the case of three simple group factors and the matter content of the original Standard Model (including the Higgs particle) exhausts those possible types of charges. By properly taking the representations of the matter fields and performing the $U(1)$ symmetry reduction introduced earlier, it is straightforward to show that *the couplings of all matter fields to the hyper-photon Y_μ are realized through the usual hypercharges* [8]. Moreover, after performing the spontaneous symmetry breaking of the original Standard Model, *all particles will couple to the photon A_μ through the usual electric charges, i.e. 1, -1, 0, -1/3, 2/3*, so this model provides a *solution to the NC charge quantization problem.*

Another proposal for a noncommutative version of the Standard Model is based on the Seiberg-Witten (SW) map [14], which assigns to commutative gauge configurations the noncommutative equivalent configurations, linked by field-dependent noncommutative gauge transformations. This version of the NC SM is constructed from NC fields realized by SW map as a *tower of commutative fields*, transforming under $G = U(1) \times SU(2) \times SU(3)$. There are no additional $U(1)$ gauge fields, so there is no need for the $U(1)$ factor reduction. The gauge symmetry is considered on *Lie algebra level* and not Lie group level. Consequently, arbitrary (fractional) $U(1)$ charges are admissible. However, this last point can be considered as a disadvantage: in the NC SM based on the no-go theorem, the $U(1)$ factor reduction fixes the correct (hyper) charges for all SM particles.

3. Lamb shift in NC QED. Bounds on θ

In this section we focus on the hydrogen atom and, using the non-relativistic limit of NC QED results, we propose the Hamiltonian describing the NC H-atom. Given the Hamiltonian and assuming that the noncommutativity parameter (θ_{ij}) is small, we study the spectrum of H-atom. We show that because of noncommutativity, even at field theory tree level, we have some corrections to the Lamb shift ($2P_{1/2} \to 2S_{1/2}$ transition) [10].

Hereafter, we shall consider the electron of the H-atom moving in the external field of the proton. However, similar results (up to a numerical factor) would be obtained by treating the proton as a composite particle, e.g., in the naive quark model [15]. The latter analysis infirms the treatement of [16], where the proton is taken as an elementary particle, thereby obtaining no noncommutative corrections for the H-atom spectrum at tree level.

To start with, we propose the following Hamiltonian for the noncommutative H-atom:

$$H = \frac{\hat{p}.\hat{p}}{2m} - \frac{Ze^2}{\sqrt{\hat{x}\hat{x}}} \ , \tag{21}$$

with

$$[\hat{x}_i, \hat{x}_j] = i\theta_{ij} \ , \quad [\hat{x}_i, \hat{p}_j] = i\hbar\delta_{ij} \ , \quad [\hat{p}_i, \hat{p}_j] = 0 \ . \tag{22}$$

The NC Coulomb potential

$$V(r) = -\frac{Ze^2}{r} - \frac{e}{4\hbar}(\theta \times p) \cdot (-\frac{Zer}{r^3}) + O(\theta^2) \ , \tag{23}$$

with $\theta_{ij} = \frac{1}{2}\epsilon_{ijk}\theta_k$ can be obtained either as the nonrelativistic limit from the NC photon exchange diagram or from the change of variables:

$$x_i = \hat{x}_i + \frac{1}{2\hbar}\theta_{ij}\hat{p}_j, \quad p_i = \hat{p}_i \ , \tag{24}$$

where the new variables satisfy the usual canonical commutation relations:

$$[x_i, x_j] = 0 \ , \quad [p_i, p_j] = 0 \ , \quad [x_i, p_j] = i\hbar\delta_{ij} \ . \tag{25}$$

Using the usual perturbation theory, the leading corrections to the energy levels due to noncommutativity, i.e. first order perturbation and in field theory tree level, are:

$$\begin{aligned} \Delta E_{NC}^{H-atom} &= -\frac{m_e c^2}{4}(Z\alpha)^4 \frac{\theta}{\lambda_e^2} j_z (1 \mp \frac{1}{2l+1}) \\ &\times \ f_{n,l} \ \delta_{ll'}\delta_{j_z j'_z} \end{aligned} \tag{26}$$

for $j = l \pm \frac{1}{2}$ and $f_{n,l} = \frac{1}{n^3 l(l+\frac{1}{2})(l+1)}$. The case of our interest, the $2P_{1/2} \to 2S_{1/2}$ transition (Lamb shift), for the noncommutative H-atom, besides the usual loop effects, depends on the j_z quantum number (only for the $2P_{1/2}$ level, as the levels with $l = 0$ are not affected) and is there, even in the field theory tree level. Hence we call it *polarized Lamb shift*. New transition channels are opened (notation $nl_j^{j_z}$), i.e. $2P_{1/2}^{-1/2} \to 2P_{1/2}^{1/2}$ and a split of the usual Lamb shift occurs: $2P_{1/2}^{1/2} \to 2S_{1/2}$ and $2P_{1/2}^{-1/2} \to 2S_{1/2}$.

One can use the data on the Lamb shift to impose some bounds on the value of the noncommutativity parameter θ. Of course, to do it, we only need to consider the classical (tree level) results, (26). Comparing these results, the contribution of (26) should be of the order of $10^{-6} - 10^{-7}$ smaller than the usual one loop result and hence,

$$\frac{\theta}{\lambda_e^2} \gtrsim 10^{-7}\alpha \quad \text{or} \quad \frac{1}{\sqrt{\theta}} \gtrsim 10\ TeV\ . \tag{27}$$

The same bound is obtained also from the violation of Lorentz invariance, based on the clock-comparison experiments, which monitor the difference between two atomic hyperfine or Zeeman transition frequencies, searching for variations as the Earth rotates [17].

4. Noncommutative quantum field theory and spin-statistics theorem

Pauli's spin-statistics relation [18] is responsible for the entire structure of the matter and for its stability. Experimentally, the relation has been verified to high accuracy. Theoretically up to now there has been no compelling argument or logical motivation for its breaking. However, the violation of Lorentz invariance as well as the intrinsic nonlocality of noncommutative field theories may suggest that a (presumably very small, of the order of $|\theta^{\mu\nu}|m^2$) breaking of this fundamental theorem, as well as of the CPT theorem, might be possible.

Pauli demonstrated [18] the spin-statistics relation based on the following requirements: *i)* The vacuum is the state of lowest energy; *ii)* Physical quantities (observables) commute with each other in two space-time points with a space-like distance; *iii)* The metric in the physical Hilbert space is positive definite.

In the noncommutative case the physical quantities (observables) which are in general products of several field operators, are no more local quantities and could therefore fail to fulfil the above requirement *ii)*. For instance, taking the normally ordered product $:\phi^2(x):$ for a real scalar field with mass m, its noncommutative version $:\phi(x)\star\phi(x):$ could give a nonvanishing equal-time commutation relation (ETCR). In particular, the matrix element between vacuum and a two-particle state, on a d-dimensional space, when Bose statistics is used, is [11]:

$$\begin{aligned}\langle 0|[:\phi(x)\star\phi(x):\,,\ :\phi(y)\star\phi(y):]\Big|_{x_0=y_0}|p,p'\rangle &= -\frac{2i}{(2\pi)^{2d}}\frac{1}{\sqrt{\omega_p\omega_{p'}}}\\ \times(e^{-ip'x-ipy}+e^{-ipx-ip'y})\int\frac{d\vec{k}}{\omega_k}\sin[\vec{k}(\vec{x}-\vec{y})]&\cos(\frac{1}{2}\theta^{\mu\nu}k_\mu p_\nu)\cos(\frac{1}{2}\theta^{\mu\nu}k_\mu p'_\nu),\end{aligned} \tag{28}$$

where $\omega_k = k_0 = \sqrt{\vec{k}^2+m^2}$ and $\vec{k} = (k_1, ..., k_d)$. The r.h.s. of (28) is nonzero only when $\theta^{0i} \neq 0$. This holds for observables expressed as any power of both bosonic fields and their derivatives, with $\star$-product analogous to (28), and spinor fields and their derivatives, with anti-commutation relation used in the latter case. The study of NC QFT also showed a violation of both causality [19] and unitarity [20] conditions, for theories with noncommutative time ($\theta^{0i} \neq 0$). Indeed, while the low-energy limit of string theory in a constant antisymmetric background field B^{mn}, which exhibits noncommutativity, reduces to field theory with the $\star$-product when $\theta^{0i} = 0$, for the case $\theta^{0i} \neq 0$ there is no corresponding low-energy field theory limit.

The field theories with light-like noncommutativity, $\theta^{\mu\nu}\theta_{\mu\nu} = 0$, i.e. $\theta^{0i} = -\theta^{1i}$, become very interesting from this point of view as they preserve unitarity [21]. In this case, however, the microcausality in the sense of ETCR (28) is still violated [11].

If the field theory with light-like noncommutativity is indeed the low-energy limit of string theory, as stated in [21], it is then intriguing that the theory is unitary but acausal (as it is known that a low-energy effective theory should not necessarily be unitary, as is the case, e.g., for the Fermi four-spinor interaction).

5. CPT Theorem in NC Field Theories

The CPT theorem [22] (see also [23] for a review) is of a universal nature in that it is valid in all the known field theories. The *general* validity of the CPT theorem for any noncommutative quantum field theory of the type (1) was shown in [11] (for partial results, see [24]). Based on the anti-unitary character of the CPT transformation (denoted hereafter by $\diamond$):

$$\begin{aligned}(c_1 A + c_2 b)^\diamond &= c_1 A^\diamond + c_2 B^\diamond \ (linearity) \ , \\ (AB)^\diamond &= B^\diamond A^\diamond, \end{aligned} \tag{29}$$

where c_1 and c_2 are c-number coefficients, and the set of CPT transformations for local elementary fields [25]:

$$\begin{gathered}\psi^\diamond_\alpha(x) = (i\gamma_5)_{\alpha\beta}\psi_\beta(-x) \ , \ \bar{\psi}^\diamond_\alpha(x) = \bar{\psi}_\beta(-x)(i\gamma_5)_{\beta\alpha} \ , \\ \phi^\diamond_{\lambda_1 ... \lambda_n}(x) = (-1)^n \phi_{\lambda_1 ... lambda_n}(-x)\end{gathered} \tag{30}$$

it is easy to show that

$$\mathcal{H}^\diamond_{int}(x) = \mathcal{H}_{int}(-x) \ , \tag{31}$$

where

$$\mathcal{H}(x) = \sum_{i_1 ... i_n} f_{i_1 ... i_n} \phi^1_{i_1}(x) \star ... \star \phi^n_{i_n}(x) \, . \tag{32}$$

Here i_j with $j = 1, ..., n$ stand for spinorial or tensorial indices and the coefficients $f_{i_1 ... i_n}$ are so chosen as to make $\mathcal{H}(x)$ a scalar under proper Lorentz transformations, *in the local limit.*

The CPT theorem is valid for any form of noncommutativity, including the case $\theta^{0i} \neq 0$.

6. Conclusions

In the framework of noncommutative gauge theories, we present a no-go theorem according to which the closure condition of the gauge algebra implies that: 1) the local NC $u(n)$ *algebra* only admits the irreducible $n \times n$ matrix-representation. Hence the gauge fields are in $n \times n$ matrix form, while the matter fields *can only be* in fundamental, adjoint or singlet states; 2) for any gauge group consisting of several simple-group factors, the matter fields can transform nontrivially under *at most two* NC group factors. In other words, the matter fields cannot carry more than two NC gauge group charges. This no-go theorem imposes strong restrictions on the NC version of the Standard Model and in resolving the standing problem of charge quantization in noncommutative QED.

Elaborating on the phenomenological implications of noncommutativity we have calculated the noncommutative corrections to the spectrum of the H-atom and obtained a bound on θ from the data on the Lamb shift.

We have found that the CPT theorem is *generally* valid in NC FT, irrespective of the form of the noncommutativity parameter $\theta^{\mu\nu}$ involved, although Lorentz invariance is violated. The spin-statistics theorem holds in the case of field theories with space-space noncommutativity, which can be obtained as a low-energy limit from the string theory.

A violation of the spin-statistics relation in the case of NC time can not be justified, given the pathological character of such theories. The case of light-like noncommutativity (compatible with unitarity) deserves, however, more attention.

In conclusion, it is of importance to study further the light-like case, as to determine whether it can indeed be obtained as a low-energy limit of string theory. Questions concerning a possible breaking of the spin-statistics relation are of outmost importance, since such a violation, no matter how small, would have a crucial impact on the structure and the stability of matter in the Universe. The issue, on the other hand, is of fundamental interest by itself, since up to now no theoretical argument or motivation for such a breaking has been presented.

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