Infinite Integrals in Quantum Electrodynamics Can Be Made Finite

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Electric field of zero fluctuations influences electron, and the electron oscillates. The energy of these oscillations contributes to the self-energy of the electron. If one performed the calculations in the frames of perturbation theory, the energy of oscillation would be infinite. This is one of the fundamental difficulties in the theory. In this paper we propose an new exact calculation, which is based on the fluctuate-dissipative theorem and the suppression of high energy quanta because of the vacuum polarization. This calculation leads to the finite contributions to the electron self-energy. The energy of coulomb field is also finite. Thus the difficulty of the theory is overcome.

Appearance of infinite integrals is the principal difficulty of quantum electrodynamics. If the upper limit of the integral ω_{max} goes to infinity when integrating in ω then integral also becomes infinite or divergent [1 -8]. In 1948 it was able to classify arising infinities and learn how to throw them out in concrete calculations. This formal way is not correct, R. Feinman called it "sweeping difficulties under the carpet". Recently author of this note managed to remove these infinities by more correct method of calculations at the first point. It was discussed on the conference, articles are accepted to scientific journals. I would like to give a popular talk on it to the readers of NTR with minimal number of formulas.

Consider the motion of the classical pointlike charged particle influenced by zero fluctuations of electromagnetic field [5, 7, 8]. Under the action of field fluctuations particle moves and simultaneously loses the energy in radiation. Let us find the kinetic energy of particle oscillations. For that one should use FDT -fluctuate-dissipative theorem instead of the method of consecutive approximations [3 §50, 8 §3, 4]. FDT has been proved not so long ago (1951) and therefore it was not used in quantum electrodynamics before the work of ref. [7]. FDT automatically takes into account the reaction of the field to the electron motion and so it is more precise than the method of consecutive approximations. We consider nonrelativistic particle with linear equations of motion and thereby FDT is completely applicable [9 §87; 10 §45]. In the method of consecutive approximations [8, eq. 10]:

$$E_{\text{фл}} = \frac{m < v^2 >}{2} = \frac{e^2 h}{\pi m c^2} \int_0^\infty \omega d\omega \sim (\omega_{max})^2 \quad . \tag{1}$$

Integral for $E_{\text{фл}}$ has the quadratic divergence in the upper limit. Integral for the shift squared diverges logarithmically. Formal calculation in FDT improves the convergence of the resulting integral. It diverges only logarithmically.

$$E_{\text{фл}} \sim \ln \omega_{max} \quad . \tag{2}$$

The next step is to take into account that, due to vacuum polarization, the interaction of the electron with the electromagnetic field becomes smaller at frequencies $h\omega > mc^2$. Approximate calculation made by V. Veiskopff more than 50 years go leads to the factor

$$F \sim (mc^2/ h\omega)^2 \tag{3}$$

in the integral. In this case integral diverges logarithmically, but the coefficient before the logarithm decreases strongly. Exact calculation has not been done yet. There are some reasons to expect that F will decrease faster with increasing frequency than (3), for example $\exp -(mc^2/ h\omega)^2$, that gives the first term of the expansion in series which coincides with (3). It makes the integral finite. Electron vibrate under the influence of fluctuations, and the average velocity of the vibration v is equal to

$$(v/c)^2 \sim 1/137 \quad . \tag{3}$$

Let us show now **that the electric field of the vibrating electron differs from the field of static pointlike charge at small distances**. An exactly static charge was taken in articles [4, 11] that resulted in contradiction «Moscow zero**». But there will no contradiction for the vibrating charge**. When an electron moves in the heterogeneous ($\omega = 0$, $k \neq 0$) field, Doppler's effect is arising in the reference frame related to the electron $\omega' = kv \neq 0$. And if $h\omega' \sim mc^2$, then the interaction of the electron with the field decreases. Ther efore the field exciting by the electron charge is also decreases at small distances (large k). And so the structure of the electrostatic (Coulomb) field changes at small distances, and the electrostatic energy will also be finite.

Shortly: **electron in a vacuum is not a static point, but something dynamical. If there is a vacuum polarization, then it has sluggishness**.

FDT computations allow calculating exactly the interaction of an electron with radiation and rejecting the method of consecut ive approximations that was the dream of L.D. Landau [4, 11]. **So it is possible to overcome these principal difficulties of quantum electrodynamics.**

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