Spontaneously Broken Symmetry and Graviton Mass

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In the framework of relativistic theory of gravity the graviton mass is introduced with help of spontaneously broken gravitational gauge symmetry mechanism for interacting gravitational and nonlinear symmetrical tensor fields.

The gravitational field equations in the massless variant of relativistic theory of gravity (RTG) [1] are the Einstein ones for the effective metric g^{ik}, connected with a tensor gravitational potential ψ^{ik} and Minkowsky metric γ^{ik} with the expression

$$\tilde{g}^{ik} = \sqrt{-g}g^{ik} = \sqrt{-\gamma}(\gamma^{ik} + k\psi^{ik}), \tag{1}$$

where $\gamma = det\gamma_{ik}, g = detg_{ik}$, k^2 - is the Einstein gravitational constant. This theory can be regarded as a gauge theory of the group of Lie variations for dynamical variables. The related transformations are the function form variations for generally covariant transformations. That the action be invariant for this group under the transformations of the dynamic variables alone requires replacing the "nondynamical" Minkowski metric γ^{ik} with effective metric g^{ik}.

This approach may be generalized on a case of massive graviton [2]. In this case the free field Lagrangian has a form

$$L^g = -\frac{1}{k^2}[\sqrt{-g}(G + \frac{1}{2}m^2\gamma_{ik}g^{ik} - m^2) - m^2\sqrt{-\gamma}] \ , \tag{2}$$

where $\sqrt{-g}G$ is Einstein gravitational Lagrangian without second derivatives, m is the graviton mass. In a linear approximation the field equations, following from (2) have the form

$$\gamma^{mn}\partial_m\partial_n\psi^{ik} + m^2\psi^{ik} = 0 \ , \tag{3}$$

that justifies the interpretation of parameter m. To obtain the equation (3), we must to introduce the Minkowski metric to the Lagrangian and hence to break the gauge invariance obviously. Note, that first the Lagrangian (2) was investigated in the frames of GR in [3], where it was emphasized the need to break the geometrical interpretation of GR too.

In present paper we regard the possibility to introduce the graviton mass with help of spontaneously broken gauge symmetry by analogy with the theories of others fundamental interactions.

To receive the Lagrangian (1) after spontaneous breaking of symmetry, we consider a symmetric tensor field f_{ik}, which never can not be equal to zero and has the energy minimum than $f^0_{ik} = \gamma_{ik}$. Significantly that the vacuum state f^0_{ik} is Lorentzinvariant, moreover the only scalar and symmetric tensor fields posses this property.

We choose the Lagrangian of the f_{ik} field in the next form

$$L^f = -\frac{1}{k^2}[R(f)\sqrt{-\gamma} + \frac{1}{2}m^2 f_{ik}\gamma^{ik}\sqrt{-\gamma} - m^2\sqrt{-f}] = L^d + L^1 \ , \tag{4}$$

where R is the scalar curvature for tensor f_{ik} and $f = det(f_{ik})$. The dynamical part L^d is chosen so manner that additional gravitational Lagrangian should not arise after spontaneous braking of symmetry.

If we are interesting in the constant solutions of field equations, the latter have a form

$$\frac{\partial L^1}{\partial f_{ik}} = 0 \ . \tag{5}$$

From eq. (5) we are receiving

$$\frac{\partial L^1}{\partial f_{ik}} = -\frac{m^2}{2k^2}(\gamma^{ik}\sqrt{-\gamma} - f^{ik}\sqrt{-f}) = 0 \ . \tag{6}$$

The only solution of this equation is (in Cartesian coordinates)

$$f_{ik} = f^o_{ik} = \eta_{ik} = diag(1, -1, -1, -1). \tag{7}$$

To show that this solution has a local energy minimum, we must compare its energy with the energies of neighboring solutions. For solution (7) the energy, corresponding to the canonical energy-momentum tensor is equal to $\frac{m^2}{k^2}$. The same result is obtained for metric energy-momentum tensor

$$T_{ik} = \frac{2}{\sqrt{-\gamma}}\frac{\delta L^1}{\delta \gamma^{ik}} \ . \tag{8}$$

For the Lagrangian L^1 we receive

$$k^2 T_{ik} = m^2(\frac{1}{2} f_{mn}\gamma^{mn}\gamma_{ik} - f_{ik}) \ , \tag{9}$$

$$E = T_{00}(f^0_{ik}) = \frac{m^2}{k^2} \ . \tag{10}$$

For an arbitrary solution we have

$$k^2 T_{ik} = R\gamma_{ik} + m^2(\frac{1}{2} f_{mn}\gamma^{mn}\gamma_{ik} - f_{ik}) \ , \tag{11}$$

$$k^2 T_{00} = R + m^2(\frac{1}{2} f_{mn}\gamma^{mn} - f_{00}) \ . \tag{12}$$

Now we will find the field equations, varying the Lagrangian (5). Using the equality

$$\frac{\delta R}{\delta f_{ik}} = -R^{ik} \ . \tag{13}$$

where R^{ik} is Ricci tensor for f_{ik}, we have

$$R^{ik} - \frac{1}{2}m^2\gamma^{ik} + \frac{1}{2}m^2 f^{ik}\sqrt{\frac{f}{\gamma}} = 0 \ . \tag{14}$$

and

$$R = \frac{1}{2}m^2 F - 2m^2\sqrt{\frac{f}{\gamma}} \ , \tag{15}$$

where $F = f_{ik}\gamma^{ik}$ is a trace of the field f_{ik}.

The equations (14) may be presented now in the following form

$$H^{ik} = R^{ik} - \frac{1}{2}m^2\gamma^{ik} + \frac{1}{8}m^2 F f^{ik} - \frac{1}{4}R f^{ik} = 0 \ . \tag{16}$$

They satisfy the identity

$$H^{ik} f_{ik} = 0 \tag{17}$$

and don't allow to determine the value F (or R). Consequently we may to choose the one from values F or R arbitrary, then the second is defined from the condition (15).

Substituting (15) to (11), we have

$$k^2 T_{ik} = m^2(F\gamma_{ik} - f_{ik} - 2\sqrt{\frac{f}{\gamma}}\gamma_{ik}) \ . \tag{18}$$

Now we will proceed to variable ϕ_{ik}, doing a shift

$$f_{ik} = \gamma_{ik} + \phi_{ik} \ . \tag{19}$$

For the values of ϕ_{ik} near to γ_{ik} we have (in Cartesian coordinates)

$$F = 4 + \phi \ , \sqrt{-f} = 1 + \frac{1}{2}\phi \ , \tag{20}$$

$$k^2 T_{00} = m^2(1 - \phi_{00}) \ . \tag{21}$$

The value $\phi = \phi_{ik}\gamma^{ik}$ may be chosen arbitrarily. We will put $\phi = c$, where c is a negative constant and $|c| < 1$. Then this constant may be chosen so that $\phi_{00} < 0$, $T_{00} > \frac{m^2}{k^2}$, and all solutions will have possess the energy larger than the vacuum solution.

We will regard the equation (14) in linear approximation, presenting this equation in the form

$$R_{ik} - \frac{1}{2}m^2\gamma^{mn} f_{im} f_{kn} + \frac{1}{2}m^2 f_{ik}\sqrt{\frac{f}{\gamma}} = 0 \ . \tag{22}$$

Taking into account the equations (19), (20) we get

$$R^L_{ik} - \frac{1}{2}m^2\phi_{ik} + \frac{\phi}{2}m^2\gamma_{ik} = 0 \ . \tag{23}$$

Here R^L_{ik} is the linearized Ricci tensor. For variables $h_{ik} = \phi_{ik} - \frac{1}{2}\gamma_{ik}\phi$ this equation describes a tensor particle with the mass equal to m.

Now we consider the Lagrangian of f_{ik}-field, interacting with a gravitational one

$$L^{fg} = -\frac{1}{k^2}[\sqrt{-g}(G(g) + R(f) + \frac{1}{2}m^2 f_{ik} g^{ik} - m^2) - m^2\sqrt{-f}] \tag{24}$$

and will proceed from the field f_{ik} to field ϕ_{ik}

$$L^{\phi g} = -\frac{1}{k^2}[\sqrt{-g}(G(g) + \frac{1}{2}m^2\gamma_{ik}g^{ik} - m^2) +$$

$$+ R(\phi)\sqrt{-g} - m^2\sqrt{-f(\phi)} + \frac{1}{2}m^2\phi_{ik}g^{ik}\sqrt{-g}] \ . \tag{25}$$

So, we obtained the RTG Lagrangian (2) with a massive graviton interacting with the ϕ-field, moreover the total Lagrangian remains gauge invariant.

The energy-momentum tensor and field equations for the field ϕ_{ik} has a form

$$k^2 T^{\phi}_{ik} = R(\phi) g_{ik} - m^2 \phi_{ik} + \frac{1}{2} m^2 \phi g_{ik} \ , \tag{26}$$

$$R^{ik}(\phi) - \frac{1}{2} m^2 (g^{ik} - \sqrt{\frac{f}{g}} f^{ik}) = 0 \ . \tag{27}$$

In the linear approximation for the field ϕ_{ik} the expression (26) takes the form

$$k^2 T^{\phi}_{ik} = m^2 [(1-\sigma)\phi g_{ik} - \phi_{ik} - 2\sigma g_{ik}] \ , \tag{28}$$

where $\sigma = \sqrt{\frac{\gamma}{g}}$ and the field ϕ_{ik} satisfies the linearized equation (27).

Recontly due to new cosmological data a number of generalizations of GR, connected with introducing new fundamental fields, for example dynamical scalar field “quintessence” [4], [5], appeared. The introducing the tensor field f_{ik} is related with some kind of generalizations but it dependends on internal reasons of the theory, namely the demand to introduce a graviton mass without obviously breaking gravitational gauge symmetry. In RTG the graviton mass imitates the λ - term of GR, though does not coinside with it. The last cosmological data show that λ – term is different from zero, hence the graviton mass must be different from zero too. The consideration of energy-momentum tensor of the field ϕ_{ik} in the gravitational field equations must change the theory results concerning strong gravitational fields and the cosmological scenario. In particular in this approach the cosmological expansion acceleration problem [6], [7] may be considered.

References

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