

# Quantum Roots in Geometry: I

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In the present work, it is shown that the geometrization philosophy has not been exhausted. Some quantum roots are already built in non-symmetric geometries. Path equations in such geometries give rise to spin-gravity interaction. Some experimental evidences (the results of the COW-experiment) indicate the existence of this interaction. It is shown that the new quantum path equations could account for the results of the COW-experiment. Large scale applications, of the new path equations, admitted by such geometries, give rise to tests for the existence of this interaction on the astrophysical and cosmological scales. As a byproduct, it is shown that the quantum roots appeared explicitly, in the path equations, can be diffused in the whole geometry using a parameterization scheme.

## 1 Introduction

Most of the success of physics in the 20th century has been achieved as a result of the applications of two philosophies. The first is the *Quantization Philosophy* and the second is the *Geometrization Philosophy*. The consequences of applying the first is the *Quantum Theory*, while the consequences of applying the second is the *General Theory of Relativity*, (GR). The study and understanding of the four known fundamental interactions are not equally successful using, only, one of these two rival philosophies. Electromagnetism, weak and strong interactions are well understood using the quantization philosophy, while gravity is not understood using this philosophy. In the context of geometrization of physics, GR is considered as a good theory for gravity, while there are no such successful geometric theories for the other three interactions.

It seems that a third philosophy is needed to unify the physics of the four fundamental interactions. This philosophy may lead to new physics. This would be, undoubtedly, a difficult task. It would be of importance to reach the conclusion that the two rival philosophies are completely exhausted, before trying a third one. This may be a less difficult task. It needs a careful examination of applying the existing philosophies. Examination of the geometric approach to physics shows that this approach is **not** exhausted yet. Some types of geometry admit some quantum properties. This is what I am going to show in the present work.

The following statement summarizes the philosophy of geometrization of physics:

*“To understand nature one should start with geometry and end with physics”.*

In applying this philosophy, one should look for an appropriate geometry. Einstein, in applying his geometrization philosophy, used three types of geometry. Some of the main properties of these geometries are summarized in the following table.

Table I: Comparison between 3-types of geometry.

Geometry [Ref.]	Metric	Connection	Building Blocks (#)
Riemannian [1]	Symmetric	Symmetric	Metric tensor (10)
Absolute Parallelism [2]	Symmetric	Non-symmetric	Tetrad vectors (16)
Einstein Non-symmetric [3]	Non-symmetric	Non-symmetric	Metric tensor (16)

We mean by the term “*Building Block*” the geometric object, using which one can construct the whole geometry. In the last column of Table I, we assume that the dimension of space  $n = 4$ .

Riemannian geometry has been used by Einstein to construct his successful theory of gravity, GR. It is well known that the number of building blocks in this geometry is just sufficient to describe gravity. For this reason, we are going to consider the other two geometries, in Table I, since the number of building blocks in each is enough to accommodate other interactions, together with gravity. These interactions may have some quantum properties.

The term “*Non-Symmetric Geometry*” will be used to indicate that the geometry admits non-symmetric connection. In such a geometry, one can define three types of tensor derivatives (derivatives that preserve tensor properties):

$$A^{\mu}_{+|\nu} \stackrel{def.}{=} A^{\mu}_{,\nu} + A^{\alpha} C^{\mu}_{\alpha\nu}, \tag{1.1}$$

$$A^{\mu}_{-|\nu} \stackrel{def.}{=} A^{\mu}_{,\nu} + A^{\alpha} C^{\mu}_{\nu\alpha}, \tag{1.2}$$

$$A^{\mu}_{0|\nu} \stackrel{def.}{=} A^{\mu}_{,\nu} + A^{\alpha} C^{\mu}_{(\alpha\nu)}, \tag{1.3}$$

where  $A^{\mu}$  is an arbitrary vector and  $C^{\mu}_{\nu\alpha}$  is the non-symmetric connection. Braces ( ) are used for symmetrization and brackets [ ] will be used for anti-symmetrization. The comma is used for ordinary (not tensor) partial differentiation.

Now, what is the starting point for examining non-symmetric geometries to look for any quantum features? It is well known that, quantum properties in microscopic world were discovered when Planck tried to interpret black body radiation, a phenomena which is closely connected to the motion of electrons. On the other hand, in the context of geometrization of physics, motion is described using paths (curves) of an appropriate geometry. So, a good starting point, may be a search for path equations in the geometries under consideration.

Bazanski [4],[5] has established a new approach to derive the equations of geodesic and geodesic deviation simultaneously by carrying out variation on the following Lagrangian:

$$L_B = g_{\mu\nu} U^{\mu} \frac{D\Psi^{\nu}}{DS}, \tag{1.4}$$

where  $U^{\mu} \stackrel{def.}{=} \frac{dx^{\mu}}{dS}$ ,  $g_{\mu\nu}$  is the metric tensor,  $\Psi^{\mu}$  is the deviation vector and  $\frac{D}{DS}$  is the covariant differential operator using Christoffel Symbol. We are going to generalize the Bazanski approach, by replacing the covariant derivative, used in his Lagrangian, by tensor derivatives of the types given by (1.1), (1.2) and (1.3), admitted by the geometry under consideration.

The work in this review is organized as follows: Section 2 gives a brief review of the two non-symmetric geometries under consideration, together with the new path equations resulting from

each one. Section 3 gives some remarks about the quantum features appearing in these geometric paths. A method for diffusing the quantum properties, in the whole geometry, is given in Section 4. The general quantum path, of the absolute parallelism geometry, is linearized in Section 5. Section 6 gives confirmation and applications of the quantum paths. the work is discussed and concluded in Section 7.

## 2 Geometries with Built-in Quantum Roots

### 21. The Absolute Parallelism Geometry

Absolute parallelism (AP)space is an  $n$ -dimensional manifold each point of which is labelled by  $n$ -independent variables  $x^\nu$  ( $\nu = 1, 2, 3, \dots, n$ ) and at each point we define  $n$ -linearly independent contravariant vectors  $\lambda_i^\mu$  ( $i = 1, 2, 3, \dots, n$ , denotes the vector number and  $\mu = 1, 2, 3, \dots, n$ , denotes the coordinate component) subject to the condition,

$$\lambda_i^+{}^\mu{}_\nu = 0, \quad (2.1)$$

where the stroke and the (+) sign denote absolute differentiation, using a non-symmetric connection to be defined later. Equation (2.1) is the condition for the absolute parallelism. The covariant components of  $\lambda_i^\mu$  are defined such that

$$\lambda_i^\mu \lambda_\nu = \delta_\nu^\mu, \quad (2.2)$$

and

$$\lambda_i^\nu \lambda_\nu = \delta_{ij}. \quad (2.3)$$

Using these vectors, the following second order symmetric tensors are defined:

$$g^{\mu\nu} \stackrel{\text{def}}{=} \lambda_i^\mu \lambda_i^\nu, \quad (2.4)$$

$$g_{\mu\nu} \stackrel{\text{def}}{=} \lambda_\mu \lambda_\nu, \quad (2.5)$$

consequently,

$$g^{\mu\alpha} g_{\nu\alpha} = \delta^\mu_\nu. \quad (2.6)$$

These second order tensors can serve as the metric tensor and its conjugate of Riemannian space, associated with the AP-space, when needed. This type of geometry admits, at least, four affine connections. The first is a non-symmetric connection given as a direct solution of the AP-condition(2.1), i.e.

$$\Gamma^\alpha{}_{\mu\nu} = \lambda_i^\alpha \lambda_{i\mu,\nu}. \quad (2.7)$$

The second is its dual  $\hat{\Gamma}^\alpha{}_{\mu\nu}$  ( $= \Gamma^\alpha{}_{\nu\mu}$ ) since (2.7) is non-symmetric. The third one is the symmetric part of (2.7),  $\Gamma^\alpha{}_{(\mu\nu)}$ . The fourth is Christoffel symbol defined using (2.4),(2.5) ( as a result of imposing a metricity condition). The torsion tensor is the skew symmetric part of the affine connection (2.7), i.e.

$$\Lambda^\alpha{}_{\mu\nu} \stackrel{\text{def}}{=} \Gamma^\alpha{}_{\mu\nu} - \Gamma^\alpha{}_{\nu\mu}. \quad (2.8)$$

Another third order tensor (contortion) is defined by,

$$\gamma^\alpha{}_{\mu\nu} \stackrel{\text{def}}{=} \lambda_i^\alpha \lambda_{i\mu;\nu}. \quad (2.9)$$

The semicolon is used to characterize covariant differentiation using Christoffel symbol. The two tensors are related by the formula,

$$\gamma_{\cdot\mu\nu}^{\alpha} = \frac{1}{2}(\Lambda_{\cdot\mu\nu}^{\alpha} - \Lambda_{\nu\cdot\mu}^{\alpha} - \Lambda_{\mu\cdot\nu}^{\alpha}). \quad (2.10)$$

A basic vector could be obtained by contraction of one of the above third order tensors, i.e.

$$C_{\mu} \stackrel{\text{def}}{=} \Lambda_{\cdot\mu\alpha}^{\alpha} = \gamma_{\cdot\mu\alpha}^{\alpha}. \quad (2.11)$$

The curvature tensor of the AP-space is, conventionally, defined by,

$$B_{\cdot\mu\nu\sigma}^{\alpha} \stackrel{\text{def}}{=} \Gamma_{\cdot\mu\sigma,\nu}^{\alpha} - \Gamma_{\cdot\mu\nu,\sigma}^{\alpha} + \Gamma_{\epsilon\nu}^{\alpha}\Gamma_{\cdot\mu\sigma}^{\epsilon} - \Gamma_{\cdot\epsilon\sigma}^{\alpha}\Gamma_{\cdot\mu\nu}^{\epsilon} \equiv 0. \quad (2.12)$$

This tensor vanishes identically because of (2.1).

The autoparallel paths, of this geometry, are given by the equation,

$$\frac{d^2 x^{\mu}}{d\lambda^2} + \Gamma_{\alpha\beta}^{\mu} \frac{dx^{\alpha}}{d\lambda} \frac{dx^{\beta}}{d\lambda} = 0. \quad (2.13)$$

The AP-geometry, in its conventional form, has two main problems concerning applications: The first is the identical vanishing of its curvature tensor and the second is that its path equations (2.13) do not represent any known physical trajectory. These problems will be treated in Section 4.

Many authors believe that, because of (2.12), the AP-space is flat. It is shown [6] that AP-spaces are, in general, curved. The problem of curvature in AP-spaces is a problem of definition. In any affinely connected space there is, at least, two methods for defining the curvature tensor. The first method is by replacing Christoffel symbol, in the definition of Riemann-Christoffel tensor, by the affine connection defined in the space concerned. The second method is to define curvature as a measure of non-commutation of tensor differentiation using the affine connection of the space. It is known that, the two methods give identical results in case of Riemannian space. But the situation is different for spaces with non-symmetric connections. The two methods are not identical.

The application of the second method in non-symmetric geometries implies a problem. That is, we usually use an arbitrary vector in order to study the non-commutation of tensor differentiation, and the resulting expression will not be free from this vector. Fortunately, this problem is solved in AP-spaces [7]. We can replace the arbitrary vector by the vectors defining the structure of AP-spaces. In this case we can define the following curvature tensors:

$$\lambda_i^{\mu} |_{\nu\sigma} - \lambda_i^{\mu} |_{\sigma\nu} = \lambda_i^{\alpha} B_{\cdot\alpha\nu\sigma}^{\mu}, \quad (2.14)$$

$$\lambda_i^{\mu} |_{\nu\sigma} - \lambda_i^{\mu} |_{\sigma\nu} = \lambda_i^{\alpha} L_{\cdot\alpha\nu\sigma}^{\mu}, \quad (2.15)$$

$$\lambda_i^{\mu 0} |_{\nu\sigma} - \lambda_i^{\mu 0} |_{\sigma\nu} = \lambda_i^{\alpha} N_{\cdot\alpha\nu\sigma}^{\mu}, \quad (2.16)$$

$$\lambda_i^{\mu} ;_{\nu\sigma} - \lambda_i^{\mu} ;_{\sigma\nu} = \lambda_i^{\alpha} R_{\cdot\alpha\nu\sigma}^{\mu}, \quad (2.17)$$

here we use the stroke , a (+) sign and (-) sign to characterize absolute differentiation using the connection (2.7) and its dual, respectively. We use the stroke without signs to characterize absolute differentiation using the symmetric part of (2.7), while the semicolon is used to characterize covariant differentiation using the Christoffel symbols. The curvature tensors defined by (2.14), (2.15), (2.16) and (2.17) are in general non-vanishing except the first one, which vanishes (because of the AP-condition (2.1)).

The curvature tensors defined above can be written explicitly in terms of torsion, or contortion via (2.10), i.e.

$$B^{\alpha}_{\cdot\mu\nu\sigma} = R^{\alpha}_{\cdot\mu\nu\sigma} + Q^{\alpha}_{\cdot\mu\nu\sigma} \equiv 0, \quad (2.18)$$

$$L^{\alpha}_{\cdot\mu\nu\sigma} \stackrel{\text{def}}{=} \Lambda^{\alpha}_{\cdot\mu\nu|\sigma} - \Lambda^{\alpha}_{\cdot\mu\sigma|\nu} + \Lambda^{\beta}_{\cdot\mu\nu}\Lambda^{\alpha}_{\cdot\sigma\beta} - \Lambda^{\beta}_{\cdot\mu\sigma}\Lambda^{\alpha}_{\cdot\nu\beta}, \quad (2.19)$$

$$N^{\alpha}_{\cdot\mu\nu\sigma} \stackrel{\text{def}}{=} \Lambda^{\alpha}_{\cdot\mu\nu|\sigma} - \Lambda^{\alpha}_{\cdot\mu\sigma|\nu} + \Lambda^{\beta}_{\cdot\mu\nu}\Lambda^{\alpha}_{\cdot\beta\sigma} - \Lambda^{\beta}_{\cdot\mu\sigma}\Lambda^{\alpha}_{\cdot\nu\beta}, \quad (2.20)$$

$$Q^{\alpha}_{\cdot\mu\nu\sigma} \stackrel{\text{def}}{=} \gamma^{\alpha}_{\cdot\mu\nu|\sigma} - \gamma^{\alpha}_{\cdot\mu\sigma|\nu} + \gamma^{\beta}_{\cdot\mu\sigma}\gamma^{\alpha}_{\cdot\beta\nu} - \gamma^{\beta}_{\cdot\mu\nu}\gamma^{\alpha}_{\cdot\beta\sigma}, \quad (2.21)$$

It is clear that the vanishing of the torsion will lead to the vanishing of (2.19), (2.20). Also this will lead to vanishing of (2.21) via (2.10) and consequently the vanishing of  $R^{\alpha}_{\cdot\mu\nu\sigma}$  via (2.18). This represents another problem facing field theories written in AP-spaces. Such theories will not have GR limit as the torsion vanishes, if this condition is needed.

## 22. Quantum Properties of the AP-Geometry

Recently [8], using the affine connections defined in the AP-space to generalize the Bazanski Lagrangian (1.4), three path equations were discovered in the AP-geometry . These equations can be written in the form:

$$\frac{dU^{\mu}}{dS^{-}} + \{\mu_{\alpha\beta}\} U^{\alpha} U^{\beta} = 0, \quad (2.22)$$

$$\frac{dW^{\mu}}{dS^0} + \{\mu_{\alpha\beta}\} W^{\alpha} W^{\beta} = -\frac{1}{2} \Lambda_{(\alpha\beta)\cdot}^{\mu} W^{\alpha} W^{\beta}, \quad (2.23)$$

$$\frac{dV^{\mu}}{dS^{+}} + \{\mu_{\alpha\beta}\} V^{\alpha} V^{\beta} = -\Lambda_{(\alpha\beta)\cdot}^{\mu} V^{\alpha} V^{\beta}, \quad (2.24)$$

where  $S^{-}, S^0$  and  $S^{+}$  are the parameters varying along the corresponding curves whose tangents are  $J^{\alpha}, W^{\alpha}$  and  $V^{\alpha}$ , respectively. We can write the new set of the path equations, obtained in this geometry, in the following form:

$$\frac{dB^{\mu}}{d\hat{S}} + a \left\{ \begin{matrix} \mu \\ \alpha\beta \end{matrix} \right\} B^{\alpha} B^{\beta} = -b \Lambda_{(\alpha\beta)\cdot}^{\mu} B^{\alpha} B^{\beta}, \quad (2.25)$$

where  $a, b$  are the numerical coefficients of the Christoffel symbol term and of the torsion term, respectively. Thus we can construct the following table.

Table II: Numerical Coefficients of The Path Equation in AP-Geometry.

Affine Connection Used	Coefficient $a$	Coefficient $b$
$\hat{\Gamma}^{\alpha}_{\cdot\mu\nu}$	1	0
$\Gamma^{\alpha}_{\cdot(\mu\nu)}$	1	$\frac{1}{2}$
$\Gamma^{\alpha}_{\cdot\mu\nu}$	1	1

The first column in this table contains the affine connections used to generalize the Bazanski Lagrangian. The set of equations (2.22), (2.23) and (2.24) possesses some interesting features:

1. It gives the effect of the torsion on the curves (paths) of the geometry.
2. This set is irreducible i.e. no one of these equations can be reduced to the other unless the torsion vanishes. This vanishing will lead to flat space (in view of the definitions (2.18-21)), which is not suitable for applications.
3. The coefficient of the torsion term jumps by a step of one-half from one equation to the next (as clear from Table II).

The last feature is tempting to conclude that:

“paths in AP – geometry are naturally quantized”.

### 23. Einstein Non-symmetric Geometry

Einstein generalized the Riemannian geometry by dropping the symmetry conditions imposed on the metric tensor and on the affine connection [3]. In this geometry the non-symmetric metric tensor is given by:

$$g_{\mu\nu} \stackrel{def.}{=} h_{\mu\nu} + f_{\mu\nu}, \quad (2.26)$$

where

$$h_{\mu\nu} \stackrel{def.}{=} \frac{1}{2}(g_{\mu\nu} + g_{\nu\mu}),$$

$$f_{\mu\nu} \stackrel{def.}{=} \frac{1}{2}(g_{\mu\nu} - g_{\nu\mu}).$$

Since the connection of the geometry,  $U^\alpha_{\mu\nu}$ , is assumed to be non-symmetric, we can define the following 3-types of covariant derivatives:

$$A^\mu_{+||\nu} \stackrel{def.}{=} A^\mu_{,\nu} + A^\alpha U^\mu_{\alpha\nu}, \quad (2.27)$$

$$A^\mu_{-||\nu} \stackrel{def.}{=} A^\mu_{,\nu} + A^\alpha U^\mu_{\nu\alpha}, \quad (2.28)$$

$$A^\mu_{0||\nu} \stackrel{def.}{=} A^\mu_{,\nu} + A^\alpha U^\mu_{(\alpha\nu)}, \quad (2.29)$$

where  $A^\mu$  is any arbitrary vector. Now the connection  $U^\alpha_{\mu\nu}$  is defined such that [3],

$$g^\mu_{\nu}{}_{+||\sigma} = 0, \quad (2.30)$$

$$i.e. \quad g_{\mu\nu,\sigma} = g_{\mu\alpha} U^\alpha_{\sigma\nu} + g_{\alpha\nu} U^\alpha_{\mu\sigma}. \quad (2.31)$$

The non-symmetric connection can be written in the the following form:

$$U^\alpha_{\mu\nu} \stackrel{def.}{=} U^\alpha_{(\mu\nu)} + U^\alpha_{[\mu\nu]} = \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} + K^\alpha_{\mu\nu}, \quad (2.32)$$

where

$$U^\alpha_{(\mu\nu)} \stackrel{def.}{=} \frac{1}{2}(U^\alpha_{\mu\nu} + U^\alpha_{\nu\mu}), \quad (2.33)$$

$$U^\alpha_{[\mu\nu]} \stackrel{def.}{=} \frac{1}{2}(U^\alpha_{\mu\nu} - U^\alpha_{\nu\mu}) = K^\alpha_{[\mu\nu]} = \frac{1}{2}S^\alpha_{\mu\nu}, \quad (2.34)$$

where  $S^\alpha_{\mu\nu}$  is a third order tensor representing the torsion of the Einstein non-symmetric (ENS) geometry.

The contravariant metric tensor  $g^{\mu\nu}$  is defied such that :

$$g^{\mu\alpha}g_{\nu\alpha} = g^{\alpha\mu}g_{\alpha\nu} = \delta_{\nu}^{\mu}. \quad (2.35)$$

The tensor derivatives (2.27), (2.28) and (2.29) are connected to the parameter derivatives by the relations :

$$\frac{\nabla A^{\mu}}{\nabla\tau^{-}} = A^{\mu}_{-||\alpha} \tilde{J}^{\alpha}, \quad (2.36)$$

$$\frac{\nabla A^{\mu}}{\nabla\tau^0} = A^{\mu}_{0||\alpha} \tilde{W}^{\alpha}, \quad (2.37)$$

$$\frac{\nabla A^{\mu}}{\nabla\tau^{+}} = A^{\mu}_{+||\alpha} \tilde{V}^{\alpha}, \quad (2.38)$$

where  $\tilde{J}^{\mu}, \tilde{W}^{\mu}$  and  $\tilde{V}^{\mu}$  are tangents to the paths whose evolution parameters are  $\tau^{-}, \tau^0$  and  $\tau^{+}$ , respectively.

## 24. Quantum Properties of ENS-Geometry

Applying the Bazanski approach to the Lagrangian functions:

$$\Xi^{-} = g_{\mu\alpha} \tilde{J}^{\mu} \frac{\nabla\Psi^{\alpha}}{\nabla\tau^{-}}, \quad (2.39)$$

$$\Xi^0 = g_{\mu\alpha} \tilde{W}^{\mu} \frac{\nabla\Theta^{\alpha}}{\nabla\tau^0}, \quad (2.40)$$

$$\Xi^{+} = g_{\mu\alpha} \tilde{V}^{\mu} \frac{\nabla\Phi^{\alpha}}{\nabla\tau^{+}}, \quad (2.41)$$

where  $\Psi^{\alpha}, \Theta^{\alpha}$  and  $\Phi^{\alpha}$  are the deviation vectors, we get [9] the following set path equations respectively,

$$\frac{d\tilde{J}^{\alpha}}{d\tau^{-}} + \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} \tilde{J}^{\mu} \tilde{J}^{\nu} = -K^{\alpha}_{\mu\nu} \tilde{J}^{\mu} \tilde{J}^{\nu}, \quad (2.42)$$

$$\frac{d\tilde{W}^{\alpha}}{d\tau^0} + \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} \tilde{W}^{\mu} \tilde{W}^{\nu} = -\frac{1}{2} g^{\alpha\sigma} g_{\mu\rho} S^{\rho}_{\nu\sigma} \tilde{W}^{\mu} \tilde{W}^{\nu} - K^{\alpha}_{\mu\nu} \tilde{W}^{\mu} \tilde{W}^{\nu}, \quad (2.43)$$

$$\frac{d\tilde{V}^{\alpha}}{d\tau^{+}} + \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} \tilde{V}^{\mu} \tilde{V}^{\nu} = -g^{\alpha\sigma} g_{\mu\rho} S^{\rho}_{\nu\sigma} \tilde{V}^{\mu} \tilde{V}^{\nu} - K^{\alpha}_{\mu\nu} \tilde{V}^{\mu} \tilde{V}^{\nu}. \quad (2.44)$$

This set of equations can be written in the following general form:

$$\frac{dC^{\alpha}}{d\tau} + a \left\{ \begin{matrix} \alpha \\ \mu\nu \end{matrix} \right\} C^{\mu} C^{\nu} = -b g^{\alpha\sigma} g_{\mu\rho} S^{\rho}_{\nu\sigma} C^{\mu} C^{\nu} - c K^{\alpha}_{\mu\nu} C^{\mu} C^{\nu}, \quad (2.45)$$

where  $a, b$  and  $c$  are the numerical coefficient of the Christoffel symbol, torsion and K-terms, respectively. Thus, we can construct the following table:

Table III: Coefficients of The Path Equations in ENS-Geometry.

Affine Connection used	Coefficient $a$	Coefficient $b$	Coefficient $c$
$\hat{U}^\alpha_{\cdot\mu\nu}$	1	0	1
$U^\alpha_{\cdot(\mu\nu)}$	1	$\frac{1}{2}$	1
$U^\alpha_{\cdot\mu\nu}$	1	1	1

The first column in this table contains the affine connections used to generalize the Bazanski Lagrangian.

From this table, it is clear that, the jumping coefficient of the torsion term (column 3) has the same values obtained in the case of the AP-geometry (Table II, column 3). So, one can draw a similar conclusion given in Subsection 2.2:

“Paths in ENS – geometry are naturally quantized”

### 3 Features of Quantum Roots

(i) We consider the jump of the coefficient of the torsion term in the path equations of Subsections 2.2 and 2.4, by a step of one-half, as quantum roots emerging in non-symmetric geometries. Such path equations, are usually used to represent trajectories of test particles, in the context of the scheme of geometrization of physics. So, if such trajectories do exist in nature, then one can conclude that space-time is quantized and the geometry describing nature should be non-symmetric.

(ii) The quantum properties shown in Tables II and III, are properties built in the examined geometries. In other words, these properties are intrinsic properties characterizing the type of geometry used. The properties mentioned above are not consequences of applying any known quantization schemes.

(iii) In the scheme performed to discover these properties, certain Lagrangian functions are used. Such functions contain, in their structure, covariant derivatives, in which certain affine connections are used. The quantum properties discovered are closely related to such connections. It is well known that, in any non-symmetric geometry, one can define more affine connections by adding any third order tensor to any affine connection already defined in the geometry. If we do so, in the geometries examined in Section 2, one would not get any values (for the coefficients given in Tables II, III) different from those listed in the two tables. As a check one can try the connection,

$$\Omega^\alpha_{\cdot\mu\nu} \stackrel{\text{def}}{=} \{\alpha_{\mu\nu}\} + \Lambda^\alpha_{\cdot\mu\nu}, \quad (3.1)$$

defined in the AP-geometry.

(iv) As stated above, the quantum properties discovered are closely connected to the affine connection, or more strictly, to its skew part, the torsion tensor. The coefficients of Christoffel symbol term (the second column of Tables II and III) are the same for all paths. Also, the coefficient of the symmetric part of the tensor  $K^\alpha_{\mu\nu}$  has no such jumping properties (last column of Table III).

### 4 Parameterization and Diffusion of Quantum Roots

It is now obvious that the quantum roots discovered in non-symmetric geometries depend mainly on the existence of non-symmetric connections admitted by such geometries. Furthermore, these roots, explicitly, appear in the path equations and not in other geometric entity.



In order to extend these roots to the whole geometry, we are going to reconstruct the geometry using a general affine connection. This connection is defined as a linear combination of the connections, already, admitted in the geometry. The combination is carried out using certain parameters. The general expression obtained may not represent an affine connection, in a conventional sense. In other words, it might not be transformed as an affine connection, under the group of general coordinate transformation, unless certain conditions are imposed on the values of the parameters used. The version of the geometry obtained in this way is a *parameterized* version.

In the case of the AP-geometry, using the affine connections mentioned in Subsection 2.1 and carrying out the parameterization scheme mentioned above, the following results are obtained [10]: Combining linearly the above mentioned connections we get, after some reductions, the following parameterized expression,

$$\nabla^{\mu}_{.\alpha\beta} = (a + b)\{\alpha^{\mu}_{\beta}\} + b\gamma^{\mu}_{.\alpha\beta} \quad (4.1a)$$

where  $a$  and  $b$  are parameters. As a consequence of imposing a metricity condition, using (4.1a), we get

$$a + b = 1. \quad (4.1b)$$

So, expression (4.1a) will reduce to,

$$\nabla^{\mu}_{.\alpha\beta} = \{\alpha^{\mu}_{\beta}\} + b\gamma^{\mu}_{.\alpha\beta}, \quad (4.2)$$

which is a general parameterized affine connection. Using (4.2) to generalize the Bazanski Lagrangian (1.4), we get

$$\frac{dZ^{\mu}}{d\tau} + \{\alpha^{\mu}_{\beta}\}Z^{\alpha}Z^{\beta} = -b \Lambda_{(\alpha\beta).}^{\mu} Z^{\alpha}Z^{\beta}, \quad (4.3)$$

where  $\tau$  is a parameter varying along the path and  $Z^{\mu}$  is the tangent to the path.

All curvature tensors defined in this parameterized version of geometry, are non-vanishing. For example if we redefine the curvature (2.12) using the connection (4.2) we get [10]

$$B^{*\alpha}_{.\mu\nu\sigma} = R^{\alpha}_{.\mu\nu\sigma} + b \hat{Q}^{\alpha}_{.\mu\nu\sigma}. \quad (4.4)$$

where  $R^{\alpha}_{.\mu\nu\sigma}$  is Riemann-Christoffel curvature tensor and  $Q^{\alpha}_{.\mu\nu\sigma}$  is defined by,

$$\hat{Q}^{\alpha}_{.\mu\nu\sigma} \stackrel{\text{def}}{=} \gamma^{\alpha}_{.\mu\nu}{}_{\sigma} - \gamma^{\alpha}_{.\mu\sigma}{}_{\nu} + b (\gamma^{\beta}_{.\mu\sigma} \gamma^{\alpha}_{.\beta\nu} - \gamma^{\beta}_{.\mu\nu} \gamma^{\alpha}_{.\beta\sigma}). \quad (4.5)$$

This tensor is, in general non-vanishing although the corresponding one (2.18) vanishes identically in the old version of the geometry. The torsion and the basic vector of AP-geometry are also parameterized and defined by [12],

$$\Lambda^{*\alpha}_{.\mu\nu} \stackrel{\text{def}}{=} \nabla^{\alpha}_{.\mu\nu} - \nabla^{\alpha}_{.\nu\mu} = b \Lambda^{\alpha}_{.\mu\nu}, \quad (4.6)$$

$$\hat{C}_{\mu} \stackrel{\text{def}}{=} \hat{\Lambda}^{\alpha}_{.\mu\alpha} = b \Lambda^{\alpha}_{.\mu\alpha}. \quad (4.7)$$

The tangent of the new path (4.3) can be written in the form,

$$Z^{\mu} = U^{\mu} + b \zeta^{\mu}, \quad (4.8)$$

where  $U^{\mu}$  is the tangent vector of the geodesic of metric and the vector  $\zeta^{\mu}$  represents a deviation from geodesic. The affine parameter ( $\tau$ ) varying along (4.3) can be related to that varying along the geodesic ( $s$ ) by the relation [12],

$$s = \tau (1 + b U^{\mu} \zeta_{\mu}). \quad (4.9)$$

For physical reasons [11], the parameter  $b$  is suggested to take the form

$$b = \frac{n}{2} \alpha \gamma, \quad (4.10)$$

where  $n$  is a natural number,  $\alpha$  is the fine structure constant and  $\gamma$  is a dimensionless parameter of order unity. The presence of  $\frac{n}{2}$  in the parameter  $b$  will preserve the jumping step appeared in Table II. We are going to call (4.3) the “*Quantum Path Equations*”.

The torsion term, on the R.H.S. of (4.3), is suggested [11] to represent a type of interaction between the torsion of the background gravitational field and the quantum spin of the moving test particle, **Spin-Gravity Interaction**. We are going to take  $n = 0, 1, 2, 3, \dots$  for particles with spin  $0, \frac{1}{2}, 1, \frac{3}{2}, \dots$ , respectively. For slowly rotating macroscopic objects, we are going to take  $n = 0$ .

## 5 Quantum Paths and Their Linearization

The path equation (4.3) can be used as an equation of motion for any field theory, constructed in the AP-geometry, provided that the theory has good Newtonian limits. In such theories, (e.g. [7], [13], [14]), the tetrad vectors  $\lambda_{\mu}^i$  are considered as field variables. So, if we write,

$$\lambda_{\mu}^i = \delta_{i\mu} + \epsilon h_{i\mu}, \quad (5.1)$$

where  $\epsilon$  is a small parameter,  $\delta_{i\mu}$  is Kronecker delta and  $h_{i\mu}$  represents deviations from flat space, then the weak field condition can be fulfilled by neglecting quantities of the second and higher orders in  $\epsilon$  in the expanded field quantities. For a static field assumption, we are going to assume the vanishing of time derivatives of the field variables. The vector components  $Z^{\mu}$  ( $\stackrel{\text{def}}{=} \frac{dx^{\mu}}{d\tau}$ ) will have the values,

$$Z^1 \approx Z^2 \approx Z^3 \approx \epsilon, \quad Z^0 \approx 1 - \epsilon, \quad (5.2)$$

where  $\epsilon$  is a parameter of the order  $(\frac{v}{c})$ . If we want to add the condition of slowly moving particle to the previous conditions we should neglect quantities of second and higher orders of the parameter  $\epsilon$ . Thus, in expanding the quantities of the path equation (4.3) we are going to neglect quantities of orders  $\epsilon^2$ ,  $\epsilon^2$ ,  $\epsilon\epsilon$  and higher, and also time derivatives of the field variable are to be neglected. To the first order of the parameters, the only field quantities that will contribute to the path equation (4.3) are given by [11],

$$\Lambda_{00}^i = -\epsilon h_{00,i}, \quad (i = 1, 2, 3) \quad (5.3)$$

$$\{0^i_0\} = \frac{\epsilon}{2} Y_{00,i}, \quad (i = 1, 2, 3), \quad (5.4)$$

where  $Y_{\mu\nu}$  is defined by,

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon Y_{\mu\nu},$$

$g_{\mu\nu}$  is given by (2.5) and  $\eta_{\mu\nu}$  is the Minkowski metric tensor. Substituting from (5.3), (5.4) into (4.3) we get, after some manipulations:

$$\frac{d^2 x^i}{d\tau^2} = -\frac{1}{2} \epsilon \left(1 - \frac{n}{2} \alpha \gamma\right) Y_{00,i} Z^0 Z^0. \quad (5.5)$$

In the present case, the metric of the Riemannian space, associated to AP-space, can be written in the form [11],

$$\left(\frac{d\tau}{dt}\right)^2 = c^2 (1 + \epsilon Y_{00}). \quad (5.6)$$

Substituting from (5.6) into (5.5) we get after some manipulations:

$$\frac{d^2 x^i}{dt^2} = -\frac{c^2}{2} \epsilon \left(1 - \frac{n}{2} \alpha \gamma\right) Y_{00,i} \quad (i = 1, 2, 3)$$

which can be written in the form,

$$\frac{d^2 x^i}{dt^2} = -\frac{\partial \Phi_s}{\partial x^i} \quad (i = 1, 2, 3) \quad , \quad (5.7)$$

where

$$\Phi_s \stackrel{\text{def}}{=} \frac{c^2}{2} \epsilon \left(1 - \frac{n}{2} \alpha \gamma\right) Y_{00}. \quad (5.8)$$

Equation (5.7) has the same form as Newton's equation of motion of a particle in a gravitational field having the potential  $\Phi_s$  given by (5.8), which differs from the classical Newtonian potential. In the case of motion of macroscopic particles ( $n = 0$ ), we get from (5.8):

$$\Phi_s = \frac{c^2}{2} \epsilon Y_{00} = \Phi_N \quad (5.9)$$

where  $\Phi_N$  is the Newtonian gravitational potential obtained from a similar treatment of the geodesic equation. Thus (5.8) can be written in the form,

$$\Phi_s = \left(1 - \frac{n}{2} \alpha \gamma\right) \Phi_N. \quad (5.10)$$

This last expression shows that the gravitational potential felt by the spinning particle is less than that felt by a spinless particle or a macroscopic test particle. In other words, the Newtonian potential is reduced, for spinning particles, by a factor  $(1 - \frac{n}{2} \alpha \gamma)$ .

## 6 Confirmation and Applications of the Quantum Paths

In the context of geometrization of physics, path equations of an appropriate geometry are used to represent trajectories of test particles. It appears clearly, from the previous section, that in the case of a static weak field and a slowly moving test particle, we get Newtonian motion, provided that the particle is spinless. In the following subsections, we are going to use the quantum path equation (4.3), and its linearization consequences, to study the motion of spinning test particles in gravitational fields.

### 61. The COW-Experiment

Colella, Overhauser and Werner suggested and carried out experiments concerning the quantum interference of thermal neutrons [15], [16], [17]. This type of experiments is known, in the literature, as the COW-experiment. The aim of the experiment is to test the effect of the Earth's gravitational field on the phase difference between two beams of thermal neutrons, one is more closer to the Earth's surface than the other. The second version of the COW experiment was carried out by Werner et al.[18]. This version is characterized by a high accuracy in the measurements of the phase shift (1 part in 1000). The measurements show that the experimental results are lower than the theoretical calculations (using the Newtonian gravity) by about 8 parts in 1000. This is a real discrepancy, which may indicate the presence of a type of non-Newtonian effects.

Now one can use equation (4.3) to give an interpretation for the discrepancy in the COW-experiment. In fact we are going to use the consequence of equation (4.3) given by equation (5.10) since the following conditions, under which (5.10) is derived, hold:

-thermal neutrons can be considered as *slowly* moving test particles, and  
-the Earth's gravitational field can be considered as *weak* and *static*.

The phase difference ( $\Delta\Omega$ ) between the two beams of neutrons in the COW-experiment is given by (cf. [19]),

$$(\Delta\Omega)_N = -\frac{1}{\hbar} \int_{ACD}^{ABD} \Phi_N dt, \quad (6.1)$$

where ABD and ACD are the trajectories of the upper and lower beams of neutrons, in the interferometer, respectively. The index  $N$  is used to indicate that (6.1) is obtained using the Newtonian potential  $\Phi_N$ , and  $\hbar$  is the Planck's constant. Since neutrons are spinning particles they will be affected by the torsion of space-time, as suggested. Thus we replace  $\Phi_N$  in (6.1) by  $\Phi_S$  given by (5.10). In this case (6.1) will take the form [20]:

$$(\Delta\Omega)_S = -(1 - \frac{n}{2}\gamma\alpha) \frac{1}{\hbar} \int_{ACD}^{ABD} \Phi_N dt, \quad (6.2)$$

i.e.,

$$(\Delta\Omega)_S = (\Delta\Omega)_N - \frac{n}{2}\gamma\alpha(\Delta\Omega)_N. \quad (6.3)$$

The index  $S$  is used to indicate that (6.2) is obtained using the potential  $\Phi_S$ . Taking the value of  $\alpha = \frac{1}{137}$ ,  $n = 1$  for spin  $\frac{1}{2}$ - particles (neutrons), we easily get the following results [20]:

- (1) the theoretical value of the COW-experiment will decrease by about 4 parts in 1000, if we take  $\gamma = 1$ ,
- (2) the theoretical value will coincide with the experimental one if we take  $\gamma = 2$ .

## 62. SN1987A

Carriers of astrophysical information are massless spinning particles. These carriers are photons, neutrinos, and expectedly, gravitons. These three types of carriers are assumed to be emitted from supernovae events. In February, 23<sup>rd</sup>, 1987 a supernova, in the Large Magellanic Cloud, was observed (cf. [21]). Observations of the arrival time of neutrinos, at the Kamiokande detectors, was recorded in February 23<sup>rd</sup>, 1987, 7<sup>h</sup>35<sup>m</sup>UT, while the arrival time of photons was on the same day at 10<sup>h</sup>40<sup>m</sup>UT. The bar of the gravitational waves antennae in Rome and Maryland recorded relatively large pulses, 1.2 seconds earlier than neutrinos (cf. [22], [23]). Although the three types of particles have different spins, general relativity assumes that they follow the same trajectory (null-geodesic of the metric), since they are all massless.

In the context of general relativity, it is well known that the time interval required for a massless particle to traverse a given distance is longer in the presence of gravitational field having the potential  $\Phi(r)$ . The time delay is given by (cf. [24])

$$\Delta t_{GR} = const. \int_e^a \Phi(r) dt, \quad (6.4)$$

where  $e$  and  $a$  are the emission and arrival times of the carrier, respectively. In SN1987A's time delay (cf. [24], [25], [26]),  $\Phi(r)$  is taken to be the Newtonian potential  $\Phi_N$  (spin independent). In this case we can construct a spin-independent model, for the emission times of the carriers. If we assume that  $\Phi(r)$  is the spin dependent gravitational potential  $\Phi_s$  (5.10), we then get the spin-dependent model. The results of these two models [27] are summarized in table IV.

Table IV: Emission Times Given By The Models.

Particles Emitted (Cause)	Spin Independent Model (Null-Geodesic)	Spin Dependent Model (Quantum Path)
Neutrino (core collapse)	0.0	0.0
Photons (maximum brightness)	+3 <sup>h</sup> 5 <sup>m</sup>	+15 <sup>h</sup> 18 <sup>m</sup>
Gravitons (?)	-1 <sup>s</sup> .2	+36 <sup>h</sup> 28 <sup>m</sup>

From Table IV, we can conclude that, the two models assume two different scenarios for the emission of carriers of astrophysical information. *The spin-independent model* shows an indication that neutrinos were emitted due to core collapse, associated with gravitons as a result of sudden change in the space-time symmetry, probably, due to a kicked born neutron star. About three hours later photons were emitted as a result of maximum brightness of the envelope. *The spin-dependent model* shows that: neutrinos were emitted due to a core collapse, preserving sphericity of the core. After 15 hours photons were emitted due to maximum brightness of the envelope, in agreement with SN theories, then 21 hours later the envelope explodes asymmetrically producing a sudden change of space-time symmetry which causes the emission of gravitational waves. It could be seen that *the spin-dependent model* is more preferable than *the spin-independent model*.

### 63. The Cosmological Parameters

Cosmological information are usually carried by, and extracted from, massless spinning particles, “*carriers of cosmological information*”. The photon (spin 1-particle) is a good candidate representing one type of these carriers. Recently, the neutrino (spin  $\frac{1}{2}$ -particle) entered the playground as another type. We expect, in the near future, that a third type of carriers, the graviton (spin 2-particle), to be used for extracting cosmological information. Two factors affect the properties of these carriers. The first is the source of the carrier. The second factor is the trajectory of the carrier, in the cosmic space, from its source to the receiver. The first factor implies the information carried, which reflect the properties of the source. The second factor represents the impact of the cosmic space-time on the properties of the carrier. So, information carried by these particles contain, a part connected to their sources, and another part related to the space-time through which these particles travelled.

Cosmological parameters are quantities extracted from the information carried by the above mentioned particles. Consequently, the values of such parameters are certainly affected by the second factor. In the present work, we are going to explore the impact of this factor on these parameters.

It is well known that, the red-shift of spectral lines, coming from distant objects, plays an important role in measuring the cosmological parameters. Theoretical calculations of the red-shift, in the context of GR, treats it as a metric phenomena, since the metric of space-time is the first integral of the geodesic equation. But, if the trajectories of test particles, the carriers, are spin-dependent, then the red-shift of spectral lines is no longer a metric phenomena. In this case one should look for an alternative scheme for calculating this quantity.

Kermack, McCrea and Whittaker [28] developed two theorems on null-geodesics which were applied to get the standard red-shift of relativistic cosmology, using the following formula,

$$\frac{\lambda_o}{\lambda_1} = \frac{{}^1\eta^\mu \rho_\mu}{{}^0\eta^\mu \varpi_\mu}, \quad (6.5)$$

where  ${}^1\eta^\mu$  is the transport vector along the null-geodesic  $\Gamma$  connecting two observers A and B, evaluated at A,  ${}^0\eta^\mu$  is the transport vector evaluated at B,  $\rho^\mu$  is the unit tangent along the trajectory of A,  $\varpi^\mu$  is the unit tangent along the trajectory of B,  $\lambda_1$  is the wave length of the spectral line as measured at A,  $\lambda_o$  is the wave length of the spectral line as measured at B, and  $\Gamma$  represents the trajectory of a massless particle from A (source) to B (receiver). If the universe is expanding then  $\lambda_o > \lambda_1$ . It can be shown that the two theorems, mentioned above, are applicable to any null-path. So, they can be used for massless spinning particles following the trajectory (4.3).

In order to evaluate the red-shift using (6.5) one has to know first the values of the vectors used in this formula. Such vectors are obtained as solution of the spin-dependent path equation (4.3). Robertson [29] constructed two geometric AP-structures for cosmological applications. Using one of these structures, and performing the necessary calculations we get [30],

$$\frac{\lambda_o}{\lambda_1} = \left(\frac{R_o}{R_1}\right)^{(1-\frac{\alpha}{2}\alpha\gamma)}. \quad (6.6)$$

Now, we define the spin-dependent scale factor as,

$$R^* = R^{(1-\frac{\alpha}{2}\alpha\gamma)}. \quad (6.7)$$

Using  $R^*$ , in place of  $R$  in the standard definitions of the cosmological parameters, we can list the resulting spin-dependent parameters in Table V. The second column of this Table, gives the values of the parameters as if they are extracted from massless spinless particles. The values of the parameters extracted from photons should match the values listed in column 4. It is worth of mention that the matter parameter is not affected by the spin-gravity interaction. This is due to its independence on Hubble's parameter.

Table V: Spin-Dependent Cosmological Parameters.

Parameter	Spin-0	Spin- $\frac{1}{2}$ (neutrino)	Spin-1 (photon)	Spin-2 (graviton)
Hubble	$H_o$	$(1 - \frac{\alpha}{2})H_o$	$(1 - \alpha)H_o$	$(1 - 2\alpha)H_o$
Age	$\tau_o$	$\frac{\tau_o}{(1-\frac{\alpha}{2})}$	$\frac{\tau_o}{(1-\alpha)}$	$\frac{\tau_o}{(1-2\alpha)}$
Acceleration	$A_o$	$(1 - \frac{\alpha}{2})(A_o - \frac{\alpha}{2}H_o)$	$(1 - \alpha)(A_o - \alpha H_o)$	$(1 - 2\alpha)(A_o - 2\alpha H_o)$
Deceleration	$q_o$	$\frac{(q_o - \frac{\alpha}{2}H_o)}{(1-\frac{\alpha}{2})}$	$\frac{(q_o - \frac{\alpha}{2}H_o)}{(1-\alpha)}$	$\frac{(q_o - \frac{2\alpha}{2}H_o)}{(1-2\alpha)}$

There are some evidences for the existence of the spin-gravity interaction on the laboratory scale (the results of the COW-experiment), and on the galactic scale (the data of SN1987A). Now, to verify the existence of this interaction on the cosmological scale, observations of one parameter at least, using two different types of carriers, are needed. For example, if we observe neutrinos and photons to get Hubble's parameter, a discrepancy of order 0.001 would be expected, if this interaction exists on the cosmological scale.

## 7 General Discussion and Concluding Remarks

In the present work, it is shown that, starting within the geometrization philosophy, some quantum properties appeared very naturally in the structure of two types of non-symmetric geometries (see the third column of Tables II and III). These properties emerged without imposing any known quantization schemes on the geometry. The properties characterize the torsion term of two new sets of path equations discovered in each geometry, (2.25) and (2.45). The natural appearance of such properties can be considered as quantum roots built in non-symmetric geometry.

It is shown that these roots could be extended and diffuse in the whole geometry, using a certain parameterization scheme, suggested in Section 4. This scheme, applied to the AP-geometry, could be applied with some efforts to the ENS-geometry. The application of the parameterization scheme, not only diffuses the quantum properties in the whole geometric structure, but also solves the two main problems of the AP-geometry, mentioned in Subsection 2.1. We can summarize the main advantages of this scheme in the following points:

1. As stated above, it extends the quantum roots, appeared in the path equations, of a non-symmetric geometry, to other geometric entities.

2. It solves completely the curvature problems (the identical vanishing of the curvature (2.12)), mentioned in Subsection 2.1, by defining a general parameterized non-vanishing curvature tensor (4.4).

3. From the application point of view and depending on the curvature (4.4), field theories written in the parameterized AP-geometry do not need the condition for a vanishing torsion (which leads to a flat AP-space via (2.18-21)), in order to get a correct GR-limit. In other words, to switch-off the effect of the torsion, in such theories, we only take the parameter  $b = 0$ .

4. It solves the second problem of conventional AP-geometry, i.e. the non-physical applicability of the path equations (2.13). The new quantum paths (4.3) could be used for physical applications, as shown in Section 6.

5. The parameterized absolute parallelism (PAP) geometry is more general than both the Riemannian and conventional AP-geometries. It could account for both geometries as two limiting cases. These limits can be obtained using (4.1b). The first limit  $a = 0 \implies b = 1$ , which corresponds to the conventional AP-geometry. The second is  $a = 1 \implies b = 0$ , which corresponds to the Riemannian geometry. Figure 1 [12], is a schematic diagram giving the complete spectrum of geometries admitted by the PAP-geometry.

This Figure is plotted using equations (4.4) and (4.6).

The new quantum paths (4.3) can be reduced to the geodesic equation of Riemannian geometry (or null-geodesic upon reparameterization), upon setting  $b = 0$ , which switch-off the effect of the torsion term. In this case the equation can account for classical mechanics and relativistic mechanics. But if  $b \neq 0$ , then the torsion of the background gravitational field will interact with some of the properties of the moving particle. Recalling that the parameter  $b$  jumps by steps of one-half, (4.10), then one can conclude that the property of the test particle, by which it interacts with the torsion, is its quantum spin. For this reason, the torsion term in (4.3) is suggested to represent "Spin-Gravity Interaction". The linearization carried out in Section 5, shows clearly that this interaction will reduce Newton's gravitational potential, as obvious from (5.10). This equation shows that, the gravitational potential felt by a spinning particle is less than that felt by a spinless particle, or by a macroscopic object. In other words, one can say that, spinning particles feel the space-time torsion. This is similar to the fact that charged particles feel the electromagnetic potential, while neutral particles do not feel it.

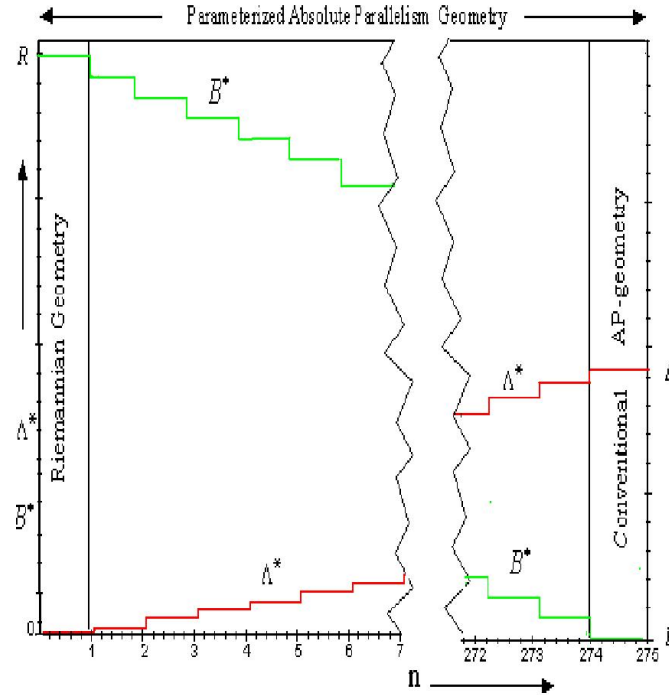


Figure 1: *Quantum Properties of PAP-Geometry.*

The discrepancy, between the experimental results and the theoretical calculations (using Newtonian gravity), of the COW-experiment gives a good indicator for the existence of spin-gravity interaction, on the laboratory scale. The experimental results are found to be lower than the expected theoretical calculations. This discrepancy can be interpreted, qualitatively, by a decrease in the gravitational potential, of the Earth, felt by neutrons (spin one-half particles). The value of this potential, felt by neutrons, is less than the value given by Newton's theory. The application of the new quantum path (4.3), Subsection 6.1, gives good, qualitative and quantitative, agreement with the experimental results. Such agreement give, not only an evidence of the existence of spin-gravity interaction, but also a direct confirmation of equation (4.3).

The application of the linearized form of (4.3), in the case of motion of spinning massless particles coming from SN1987A, Subsection 6.2, gives a good model for the emission times of these particles from this supernova (see Table IV). This may indicate the presence of the spin-gravity interaction on the galactic scale. But more efforts are still needed, both to confirm supernovae mechanisms and for observing more supernovae, to give strong confirmation for the existence of this interaction on the astrophysical scale.

The full path equation (4.3) is applied in the case of cosmology, Subsection 6.3. It is shown that the values of the cosmological parameters will be affected by the spin-gravity interaction, if it exists on the cosmological scale. The values of these parameters will depend on the spin of the particle, from which cosmological information are extracted. It is suggested that, a cosmological parameter measured using two massless particles, with different spins (e.g. photon and neutrino) may confirm the existence of spin-gravity interaction on the cosmological scale. The sensitivity of the apparatus, or experiment, to be used should be better than 0.001

In view of the present work, I will try to give short probable answers to some of the good questions raised by professor V.Petrov in the closing session of the conference:

Q1: What is the appropriate topology/geometry?

A1: A non-symmetric geometry.



Q2: How many dimensions?

A2: So for, in the context of geometrization of physics, we don't need more than four dimensions. Mass and charge appear as constants of integration. There are some attempts to represent other interactions (e.g. electromagnetism) together with gravity in spaces of four dimensions (cf. [7]).

Q3: What are the experimental/observational signature of quantum-geometrical effects?

A3: Concerning the experimental signature, the COW-type experiment is a good media for testing quantum-geometrical effects on the laboratory scale. the discrepancy in the results of this experiment gives a good indicator for the existence of such effects.

Concerning the observational signature, more efforts are needed for observing photons and neutrinos (and probably gravitons, in the future), from supernovae events, in order to detect the existence of such effects on the astrophysical and cosmological scales.

Finally, I would like to thank the organizing committee and Professor V.Petrov for inviting me to participate in the conference and to give this talk.

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