Advances in Geometrization of Spontaneously Broken Gauge Theories

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1 Functioning of the Superbracket

Grassmann algebras were born [1] out of the structure of *integration over manifolds*; once found, however, they also provided "texture" for *supermanifolds*, including super-vector fields, i.e. generalized tangent and cotangent systems [2], and a $Z(2)$- graded super-Lie bracket for the tangent, including the further option of having an additional ("interior") Z-grading with its $Z(2)$-subgroup also correlated and contributing to the operation of the bracket [3], which would always be super-abelian, otherwise.

In "conventional" Golfand-Likhtman (Poincaré) *supersymmetry* [4], the bracket $Z(2)$ is directly correlated to the *spin-statistics* of the carrier-space components, here relativistic quantum fields with Bose or Fermi statistics. An alternative set of participating elements with the same quantum-statistics-$Z(2)$, but compatible with Lorentz invariance along the carrier-space super-multiplet and no correlation with spin, could be based on alternation between *physical* versus *ghost fields*, as displayed in the BRST constraining finite super-ring [5] guaranteeing off-mass-shell unitarity in Yang-Mills Gauge Theories; in Jean Thierry-Mieg's geometrical interpretation [6] the ghosts consist of *vertical one-forms*, a fact which closes the (virtual) circle. The geometrical methodology has since gained in acceptance, applications and extension [7].

2 Internal Supersymmetry

In a different type of extension, this author introduced in 1979 "Internal Supersymmetry" [8]. The same result was reached [9] independently and almost simultaneously by David Fairlie, who was lead to it from an entirely different angle, that of the method of dimensional reduction. Whatever the origin, however, as indicated by the adjective "internal", the "gauged" supergroup, (namely SU(2/1) here) correlates the superalgebraic Z(2) with a quantum statistics Z(2) in the representation's carrier space but this occurs in a Lorentz invariant way, instead of the $\Delta J = \frac{1}{2}$ jump required by the spin-statistics correlation. We are thus dealing with a mix of physical and ghost fields. In the specific case of an electroweak SU(2/1), the symmetry can be defined as a *deformation* of SU(3) with its adjoint octet representation given by eight *Lorentz-scalar* fields, namely the 4 Feynman-de Witt-Faddeev-Popov *ghosts* for the Yang-Mills gauge fields of the isospin-hypercharge (weak) U(2), plus the 4 K-meson-like Higgs fields selected by the result.

The above definition, however, misses some essential features. If, for instance, one presents the octet in the form of a 3×3 supermatrix, instead of an 8- component supervector, one finds that it might be regarded as a superalgebra acting on a 3-component carrier space surprisingly fitting precisely the physical Hilbert space for any of the lepton flavors, with the super-algebra's Z(2) correlating with the *chiral* Z(2), as applied in the assignment of electroweak quantum numbers to each flavor's fundamental fields in the Standard Model ($\nu_L^0,\ e_L^-,\ e_R^-$) This structure also applies to the quarks, as they fit in the other fundamental representation of SU(2/1), [10] a 4-dimensional representation inherited from the homomorphism between SU(2/1) and Osp(2/2). These 3×3 or 4×4 representations appear to contain important additional features, such as the identification of the symmetry-breaking mass-operator (such as the λ_6 operator, using eightfold-way notation, relating the e_L to e_R). A priori, a supposed invariance of the electroweak system under $SU(2/1) \supset SU(2) \otimes U(1)$ is a natural extension, considering that its maximal even subgroup coincides with the gauge symmetry. However, using chiral Z(2) at first appears inappropriate, as there is no correlated change of statistics under the action of the odd part of the superalgebra, a conceptual difficulty whose removal required a doubling of all multiplets, so that the number of ghost states might become equal to that of the physical states [11, 12].

At this point we should also discuss the number of free parameters in the theory. A priori, the Electroweak unification theory includes the following pieces in its Lagrangian: $L(W, Z\} + \{L(\Phi,\ \bar{\Phi}\}\} + L(leptons) + L\{quarks\} + +L_(\Phi^2) + L_(\Phi^4)$ (the Higgs potential).

The independent parameters are: $(g,\ g')$ or $(g,\ tg\ (\Theta_W)),\ (-\mu^2),\ \lambda)$ and two Yukawa couplings per generation, one for quarks and one for leptons.. Use of SU(2/1) produces $(sin\Theta_W)^2 = .25$ and the coefficient of the quartic (in Φ), $\lambda = (4/3)\ g^2$. Assuming universality in this gauge amounts to assume all Yukawa couplings are of strength g; on the other hand there are as yet no known and proven non-renormalization theorems, so that these last universality statements may be over-optimistic.

3 Quillen's Superconnection

The supermatix solution which we discussed was a precursor to a mathematically strengthened formulation offered by D.Quillen [13] in 1985. Note that in that picture, we have a product of two superspaces, namely the Grassmann superalgebra Ξ corresponding to the parameters with which the superalgebra would be exponentiated to produce a group element and the defining supermatrix Υ. The superconnection still having to fulfill the role of a gauge-potential (or connection..) for the supergroup, it has to be *odd* in the total (product) $Z(2)$. Thus, whereas the Υ-*even* subgroup submatrices will get the Yang-Mills potential's Ξ-(*odd*) one-forms as coefficients, the Higgs fields, in themselves Ξ-*even* 0-forms, multiply the Υ-*odd* part of the supermatrix. This was precisely the structure originally adopted intuitively by DB Fairlie and this author. The adaptation of SU(2 /1) to this formalism was thus straightforward [14]. Further work has produced the extension to include $SU(3)_{color}$ and generations [15, 16].

4 Noncommutative Geometry

This approach [17] is an extension of geometry, inspired by systems in which one learns about the properties of a manifold by studying the functions which can exist on it. In NCG they form a Unital algebra of functions, $\mathcal{A}$ (noncommutative). Other important components are a "Dirac operator" $\mathcal{D}$ and a Hilbert Space $\mathcal{H}$, also an involution $(*)$. The set $(\mathcal{A},\ \mathcal{D},\ \mathcal{H},^*)$ defines a geometry. NCG extends the notions of distance, etc to spaces which are not manifolds. One relevant example is the model for the electroweak case, developed by A. Connes and J. Lott [18]. It consists in a Bundle whose base-space is a direct product $Z(2) \otimes M = M^L \oplus M^R$, where $Z(2)$ is a 2-points space, the points representing L and R, namely the two chiralities, M is spacetime, so that in the product we have two spacetimes with only one chirality each. The group in the Principal Bundle is thus $SU(2)_L \otimes U(1)$ on both Chiral Spacetimes. The physics occurs in an Associated Vector Bundle with, in the lepton case, for example,

$$\begin{gathered}(\nu_L^0(x),\ e_L^-(x)),\\ e_R^-(x),\\ \forall x \in M_L,\ e_R^-(x) = 0,\\ \forall y \in M_R,\ \nu_L^0(y) = 0,\ e_L^-(y) = 0.\end{gathered}$$

Parallel transport within either M_L or M_R is achieved by the usual covariant derivative, but in order to move, e.g. in the above set up, over the path $\nu_L^0(x) \longleftrightarrow e_R^-(y)$ we use the $SU(2)$ connection in $\partial_\mu - W_\mu^-(x)$ to go $\nu_L^0(x) \longleftrightarrow e_L^-(x')$, where x' is the "image" of y in M_L after which we have to perform a discrete "jump" $M_L \longleftarrow M_R$.

The bridge over the abyss between the two chiralized spacetimes is supplied in the form of a *matrix derivative* [19], in this case bridging $e_L^-(x') \longleftarrow e_R^-(y)$ accompanied by its connection, the $\Phi(x)$. Although the argument is somewhat more profound, the result can be described in simple terms: the total covariant derivative will roughly be $\partial_\mu W_\mu(x) + \Delta - -\Phi(x)$ and the curvature $F = \partial W[W,W] + \Phi, \Phi + \Delta\Phi$ so that the Lagrangian $F \wedge {}^*F$ now automatically includes the negative sign quadratic term of the Higgs potential, arising from the squaring of $\Delta\Phi$ [20], aside from the inclusion of the quartic term through the squaring of the symmetric bracket $\Phi,\ \Phi$ provided by the symmetric bracket in supergroup unification.

One theory which has all along applied these concepts as they developed is the Electroweak SU(2/1) Unification [8,9] mentioned above. It has since been amended in its prediction for the Higgs mass (originally set at $2\sqrt{2}M(W) = 240$ GeV, reevaluated [21] and set at $2M(W) = 170$ GeV and after [22] a linearized approximation of renormalization effects, expected at $M(\Phi) = 130+/-6$ GeV. This author has since launched a second example of the *matrix-derivative* version of a gauged supergroup, namely applying the hyperexceptional superalgebra $\overline{P}(4,R) \supset \overline{S}L(4,R) \supset \overline{S}O(3,1) = SL(2,C)$ to obtain the (Riemannian) Einstein geometry from a Metric-Affine gauge theory [23]. $P(4,R)$ is a 31-dimensional superalgebra with the 15 generators of $sl(4,R)$ in the even subalgebra and the 16 generators of $gl(4,R)$ split between the supermatrix' upper right corner (10 symmetric 4×4 matrices) and lower left corner (6 antisymmetric 4×4 matrices). The overline bar denotes a double-covering. The matrix-derivative coincides with the Minkowski metric in the upper right corner.

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