TWO STRINGY SYSTEMS OF THE KERR SPINNING PARTICLE

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A classical spinning particle based on the Kerr-Newman black hole (BH) solution is considered. For parameters of spinning particles $|a| >> m$, the BH horizons disappear and BH image is drastically changed. We show that it turns into a skeleton formed by two coupled stringy systems. One of them is the Kerr singular ring which can be considered as a circular D-string with an orientifold world-sheet. Analyzing the aligned to the Kerr congruence electromagnetic excitations of this string, we obtain the second stringy system which consists of two axial half-infinite chiral D-strings. These axial strings are similar to the Dirac monopole strings but carry the induced chiral traveling pp-waves. Their field structure can be described by the field model suggested by Witten for the cosmic superconducting strings. We discuss a relation of this stringy system to the Dirac equation and argue that this stringy system can play a role of a classical carrier of the wave function.

1. Introduction

The Kerr rotating black hole solution displays some remarkable relations to spinning particles [1–9] For the parameters of elementary particles, $|a| >> m$, and black-hole horizons disappear. This changes drastically the usual black hole image since there appear the rotating sourc in the form of a closed singular ring of the Compton radius $a = J/m$.[1] In the model of the Kerr spinning particle — "microgeon" [3] this ring was considered as a gravitational waveguide leading the traveling electromagnetic (and fermionic) wave excitations. The assumption that the Kerr singular ring represents a closed relativistic string was advanced about thirty years ago [4], which got confirmation on the level of the evidence of Refs. [6, 10, 11]. However, the attempts to show it explicitly ran into obstacles which were related with the very specific motion of the Kerr ring — the lightlike sliding along itself. It could be described as a string containing lightlike modes of only one direction. However, the relativistic string equations do not admit such solutions.

In previous paper [12] we resolved this problem showing that the Kerr ring satisfies all the stringy equations representing a string with an orientifold structure.

In this paper we consider consequences of the electromagnetic excitations of the Kerr circular string and find out an unavoidable appearance of one or two axial half-infinite strings which are topologically coupled to the Kerr ring and similar to the Dirac monopole string. These strings carry the chiral traveling waves induced by the e.m. excitations of the Kerr circular string.

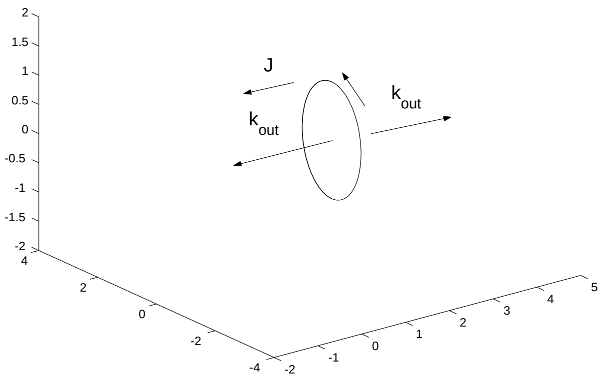
Indeed, the frame of the Kerr spinning particle consists of two topologically coupled stringy systems. The appearance of the axial half-infinite strings looks strange at first sight. Meanwhile, we obtain that it can be a new and very important element of the structure of spinning particles. For the moving particle the excitations of the chiral strings are modulated by de Broglie periodicity and therefore, the axial strings turn out to be the carriers of de Broglie wave.

In the zone which is close to the Kerr string, our treatment is based on the Kerr-Schild formalism [13] and previous paper [15] where the real and complex structures of the Kerr geometry were considered. For the reader convenience we describe briefly the necessary details of these struc-

[1] Here J is angular momentum and m is mass. We use the units $c = \hbar = G = 1$, and signature $(- + ++)$.

tures. Meanwhile, in the far zone, structure of this string is described by the very simple class of pp-wave solutions [17, 18]. The resulting stringy frame turns to be very simple and easy for description. We obtain that these strings belongs to the class of the chiral superconducting strings which have recently paid considerable attention in astrophysics.

Figure 1: Stringy skeleton of the Kerr spinning particle. Circular D-string and the directed outwards two axial half-infinite chiral D-strings.



2. The structure of the Kerr congruence. Microgeon

We use the Kerr-Schild approach to the Kerr geometry [13], which is based on the Kerr-Schild form of the metric

$$g_{\mu\nu} = \eta_{\mu\nu} + 2hk_\mu k_\nu, \tag{1}$$

where $\eta_{\mu\nu}$ is the metric of auxiliary Minkowski space-time, $h = \frac{mr - e^2/2}{r^2 + a^2 \cos^2\theta}$, and k_μ is a twisting null field, which is tangent to the Kerr principal null congruence (PNC) and is determined by the form[2]

$$k_\mu dx^\mu = dt + \frac{z}{r}dz + \frac{r}{r^2+a^2}(xdx + ydy) - \frac{a}{r^2+a^2}(xdy - ydx). \tag{2}$$

The form of the Kerr PNC is shown in **Fig. 1**. It follows from Eq.(1) that the field k^μ is null with respect to $\eta_{\mu\nu}$ as well as with respect to the full metric $g_{\mu\nu}$,

$$k^\mu k_\mu = k^\mu k^\nu g_{\mu\nu} = k^\mu k^\nu \eta_{\mu\nu}. \tag{3}$$

The metric is singular at the ring $r = \cos\theta = 0$, which is the focal region of the oblate spheroidal coordinate system r, θ, ϕ.

The Kerr singular ring is the branch line of the Kerr space on two folds: positive sheet ($r > 0$) and 'negative' one ($r < 0$). Since for $|a| >> m$ the horizons disappear, there appears the problem of the source of the Kerr solution with the alternative: either to remove this twofoldedness or to give it a physical interpretation. Both approaches have received attention, and it seems that both are valid for different models. The most popular approach was connected with the truncation of the negative sheet of the Kerr space, which leads to the source in the form of a relativistically rotating disk [2] and to the class of the disklike [5] or baglike [9] models of the Kerr spinning particle.

An alternative way is to retain the negative sheet treating it as the sheet of advanced fields. In this case the source of the spinning particle turns out to be the Kerr singular ring with the electromagnetic excitations in the form of traveling waves, which generate spin and mass of the particle. A model of this sort was suggested in 1974 as a model of "microgeon with spin" [3].

[2]The rays of the Kerr PNC are twistors and the Kerr PNC is determined by the Kerr theorem as a quadric in projective twistor space [15].

The Kerr singular ring (**Fig. 2**) was considered as a waveguide providing a circular propagation of an electromagnetic or fermionic wave excitation. Twofoldedness of the Kerr geometry admits the integer and half integer excitations with $n = 2\pi a/\lambda$ wave periods on the Kerr ring of radius a, which turns out to be consistent with the corresponding values of the Kerr parameters $m = J/a$.

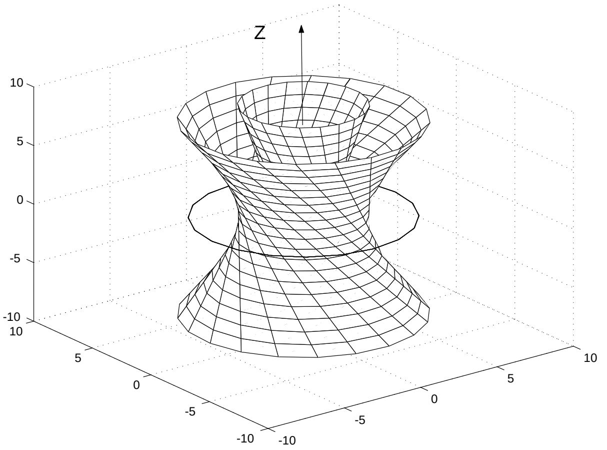


Figure 2: The Kerr singular ring and 3D section of the Kerr principal null congruence. Singular ring is a branch line of space, and PNC propagates from the "negative" sheet of the Kerr space to the "positive " one, covering the space-time twice.

The lightlike structure of the Kerr ring worldsheet is seen from the analysis of the Kerr null congruence near the ring. The lightlike rays of the Kerr PNC are tangent to the ring.

It was recognized long ago [4] that the Kerr singular ring can be considered in the Kerr spinning particle as a string with traveling waves. One of the most convincing evidences obtained by the analysis of the axidilatonic generalization of the Kerr solution (given by Sen [19]) near the Kerr singular ring was given in [6]. It was shown that the fields near the Kerr ring are very similar to the field around a heterotic string.

3. The Kerr Orientifold Worldsheet

One can see that the worldsheet of the Kerr ring satisfies the bosonic string equations and constraints; however, there appear problems with boundary conditions. In this section we recall briefly the analysis given in [12].

The general solution of the string wave equation $(\frac{\partial^2}{\partial\sigma^2} - \frac{\partial^2}{\partial\tau^2})X^\mu = 0$ can be represented as the sum of the 'left' and 'right' modes: $X^\mu(\sigma,\tau) = X^\mu_R(\tau-\sigma) + X^\mu_L(\tau+\sigma)$, and the oscillator expansion is

$$X^\mu_R(\tau-\sigma) = \frac{1}{2}[x^\mu + l^2 p^\mu(\tau-\sigma) + il\sum_{n\neq 0}\frac{1}{n}\alpha^\mu_n e^{-2in(\tau-\sigma)}], \tag{4}$$

$$X^\mu_L(\tau+\sigma) = \frac{1}{2}[x^\mu + l^2 p^\mu(\tau+\sigma) + il\sum_{n\neq 0}\frac{1}{n}\tilde{\alpha}^\mu_n e^{-2in(\tau+\sigma)}], \tag{5}$$

where $l = \sqrt{2\alpha'} = \frac{1}{\sqrt{\pi T}}$, T is tension, x^μ is position of center of mass, and p^μ is momentum of string.

The string constraints $\dot{X}_\mu\dot{X}^\mu + X'_\mu X'^\mu = 0, \qquad \dot{X}_\mu X'^\mu = 0$, are satisfied if the modes are lightlike $[()' \equiv \partial_\sigma()]$,

$$(\partial_\sigma X_{L(R)\mu})(\partial_\sigma X^\mu_{L(R)}) = 0. \tag{6}$$

Setting $2\sigma = a\phi$ one can describe the lightlike worldsheet of the Kerr ring (in the rest frame of the Kerr particle) by the surface

$$X^\mu_L(t,\sigma) = x^\mu + \frac{1}{\pi T}\delta^\mu_0 p^0(t+\sigma) + \frac{a}{2}[(m^\mu + in^\mu)e^{-i2(\tau+\sigma)} + (m^\mu - in^\mu)e^{i2(\tau+\sigma)}], \tag{7}$$

where m^μ and n^μ are two spacelike basis vectors lying in the plane of the Kerr ring. One can see that

$$X_L'^\mu = \frac{1}{\pi T}\delta_0^\mu p^0 + 2a[-m^\mu \sin 2(\tau+\sigma) + n^\mu \cos 2(\tau+\sigma)] \tag{8}$$

will be a light-like vector if one sets $p^0 = 2\pi a T$. It shows that the Kerr worldsheet could be described by modes of one (say "left") null direction. The solution $X(\tau,\sigma) = X_L(\tau+\sigma)$ satisfies the string wave equation and constraints, but there appears the problem with boundary conditions. The closed string boundary condition

$$X^\mu(\tau,\sigma) = X^\mu(\tau,\sigma+\pi) \tag{9}$$

will not be satisfied since the time component $X_L^0(t,\sigma+\pi)$ acquires contribution from the second term in (7), which is usually compensated by this term from the 'right' mode. The familiar boundary conditions for the open strings follow from the condition of cancelling of the surface term $-T\int d\tau[X'_\mu \delta X^\mu|_{\sigma=\pi} - X'_\mu \delta X^\mu|_{\sigma=0}]$ in the string action [20], and are

$$X'^\mu(\tau,0) = X'^\mu(\tau,\pi) = 0, \tag{10}$$

which also demand both types of modes to form standing waves. However, this demand can be weakened to

$$X'^\mu(\tau,0) = X'^\mu(\tau,\pi). \tag{11}$$

It seems that the lightlike oriented string can contain traveling waves of only one direction if we assume that it is open, but has the joined ends. However, the ends $\sigma = 0$, and $\sigma = \pi$ are not joined indeed.

These difficulties can be removed by the formation of the worldsheet orientifold (**Fig. 3**).

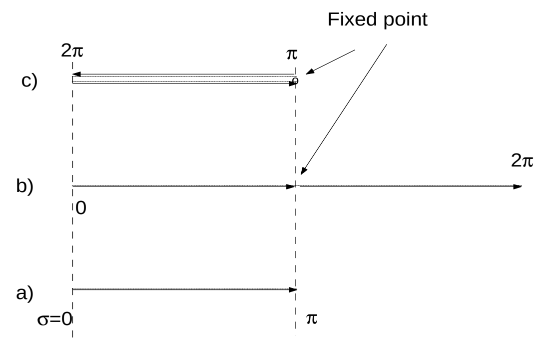


Figure 3: Formation of orientifold: a) the initial string interval; b) extension of the interval and formation of the both side movers; c) formation of the orientifold.

It is well known [20] that the interval of an open string $\sigma \in [0,\pi]$ can be formally extended to $[0, 2\pi]$, setting

$$X_R(\sigma+\pi) = X_L(\sigma), \qquad X_L(\sigma+\pi) = X_R(\sigma). \tag{12}$$

By such an extension, the both types of modes, "right" and "left", will appear in our case since the "left" modes will play the role of "right" ones on the extended piece of interval. If the extension is completed by the changing of orientation on the extended piece, $\sigma' = \pi - \sigma$, with a subsequent identification of σ and σ', then one obtains the closed string on the interval $[0, 2\pi]$ which is folded and takes the form of the initial open string.

Formally, the worldsheet orientifold represents a doubling of the worldsheet with the orientation reversal on the second sheet. The fundamental domain $[0,\pi]$ is extended to $\Sigma = [0, 2\pi]$ with formation of folds at the ends of the interval $[0,\pi]$.

4. Solution of the e.m. field equations

To realize the idea of the Kerr spinning particle as a "microgeon" we have to consider the electromagnetic excitations of the Kerr string which are described by the solutions which are aligned to the Kerr PNC on the Kerr background.

The treatment on this section is based on the Kerr-Schild formalism, and the readers which are not aware of this formalism can omit this part by first reading going to the physical consequences of these solutions.

The aligned field equations for the Einstein-Maxwell system in the Kerr-Schild class were obtained in [13]. Electromagnetic field is given by tetrad components of self-dual tensor

$$\mathcal{F}_{12} = AZ^2 \tag{13}$$

$$\mathcal{F}_{31} = \gamma Z - (AZ),_1 \ . \tag{14}$$

The equations for electromagnetic field are

$$A,_2 - 2Z^{-1}\bar{Z}Y,_3 A = 0, \tag{15}$$

$$\mathcal{D}A + \bar{Z}^{-1}\gamma,_2 - Z^{-1}Y,_3 \gamma = 0. \tag{16}$$

Gravitational field equations yield

$$M,_2 - 3Z^{-1}\bar{Z}Y,_3 M = A\bar{\gamma}\bar{Z}, \tag{17}$$

$$\mathcal{D}M = \frac{1}{2}\gamma\bar{\gamma}, \tag{18}$$

where

$$\mathcal{D} = \partial_3 - Z^{-1}Y,_3 \partial_1 - \bar{Z}^{-1}\bar{Y},_3 \partial_2 \ . \tag{19}$$

Solutions of this system were given in [13] only for stationary case for $\gamma = 0$. Here we consider the oscillating electromagnetic solutions which corresponds to the case $\gamma \neq 0$.

For the sake of simplicity we have to consider the gravitational Kerr-Schild field as stationary, although in the resulting e.m. solutions the axial symmetry is broken, which has to lead to oscillating backgrounds if the back reaction is taken into account.

The recent progress in the obtaining the nonstationary solutions of the Kerr-Schild class is connected with introduction of a complex retarded time parameter $\tau = t_0 + i\sigma = \tau|_L$ which is determined as a result of the intersection of the left (L) null plane and the complex world line [15]. The left null planes are the left generators of the complex null cones and play a role of the null cones in the complex retarded-time construction. The τ parameter satisfies to the relations

$$(\tau),_2 = (\tau),_4 = 0 \ . \tag{20}$$

It allows one to represent the equation (15) in the form

$$(AP^2),_2 = 0 \ , \tag{21}$$

and to get the following general solution

$$A = \psi(Y, \tau)/P^2, \tag{22}$$

which has the form obtained in [13]. The only difference is in the extra dependence of the function ψ from the retarded-time parameter τ.

It was shown in [15] that action of operator $\mathcal{D}$ on the variables $Y, \bar{Y}$ and ρ is following

$$\mathcal{D}Y = \mathcal{D}\bar{Y} = 0, \qquad \mathcal{D}\rho = 1 \ , \tag{23}$$

and therefore $\mathcal{D}\rho = \partial\rho/\partial t_0 \mathcal{D}t_0 = P\mathcal{D}t_0 = 1$, that yields

$$\mathcal{D}t_0 = P^{-1}. \tag{24}$$

As a result the equation (16) takes the form

$$\dot{A} = -(\gamma P),_{\bar{Y}} \ , \tag{25}$$

where $\dot{(\ )} \equiv \partial_{t_0}$.

For considered here stationary background $P = 2^{-1/2}(1 + Y\bar{Y})$, and $\dot{P} = 0$. The coordinates Y, and τ are independent from $\bar{Y}$, which allows us to integrate Eq. (25) and we obtain the following general solution

$$\gamma = -P^{-1}\int \dot{A}d\bar{Y} = -P^{-1}\dot{\psi}(Y,\tau)\int P^{-2}d\bar{Y} = \frac{2^{1/2}\dot{\psi}}{P^2 Y} + \phi(Y,\tau)/P, \tag{26}$$

where ϕ is an arbitrary analytic function of Y and τ.

The term γ in $\mathcal{F}_{31} = \gamma Z - (AZ),_1$ describes a part of the null electromagnetic radiation which falls of asymptotically as $1/r$ and propagates along the Kerr principal null congruence e^3. As it was discussed in [15, 16] it describes a loss of mass by radiation with the stress-energy tensor $\kappa T^{(\gamma)}_{\mu\nu} = \frac{1}{2}\gamma\bar{\gamma}e^3_\mu e^3_\nu$ and has to lead to an infrared divergence. However, the Kerr twofoldedness and the structure of the Kerr principal null congruence show us that the loss of mass on the positive sheet of metric is really compensated by an opposite process on the "negative" sheet of the Kerr space where is an in-flow of the radiation. In the microgeon model [15, 12, 16], this field acquires interpretation of the vacuum zero point field $T^{(\gamma)}_{\mu\nu} =< 0|T_{\mu\nu}|0 >$. Similar to the treatment of the zero point field in the Casimir effect one has to regularize stress energy tensor by the subtraction

$$T^{(reg)}_{\mu\nu} = T_{\mu\nu} - < 0|T_{\mu\nu}|0 >, \tag{27}$$

under the condition $T^{(\gamma)\ \mu\nu}{}_{,\mu} = 0$ which is satisfied for the γ term.

Let's now consider in details the second term in (14):

$$(AZ),_1 = (Z/P)^2(\psi,_Y - 2\psi P_Y) + (Z/P^2)\dot{\psi}\tau,_1 + AZ,_1 \,. \tag{28}$$

For stationary case we have relations $Z,_1 = 2ia\bar{Y}(Z/P)^3$ and $\tau,_1 = -2ia\bar{Y}Z/P^2$ (see Appendix). This yields

$$(AZ),_1 = (Z/P)^2(\psi,_Y - 2ia\dot{\psi}\bar{Y}/P^2 - 2\psi P_Y/P) + A2ia\bar{Y}(Z/P)^3. \tag{29}$$

Since $Z/P = 1/(r + ia\cos\theta)$, this expression contains the terms which are singular at the Kerr ring and fall off like r^{-2} and r^{-3}. However, it contains also the factors which depend on coordinate $Y = e^{i\phi}\tan\frac{\theta}{2}$ and can be singular at the z-axis.

These singular factors can be selected in the full expression for the aligned e.m. fields and as a result there appear two half-infinite lines of singularity, z^+ and z^-, which correspond to $\theta = 0$ and $\theta = \pi$ and coincide with corresponding axial lightlike rays of the Kerr principal null congruence. On the "positive" sheet of the Kerr background these two half-rays are directed outward. However, one can see that they are going from the "negative" sheet and appear on the "positive" sheet passing through the Kerr ring (see Fig. 2).

The general solution for the aligned electromagnetic fields has the form

$$\mathcal{F} = \mathcal{F}_{31}\ e^3 \wedge e^1 + \mathcal{F}_{12}\ (e^1 \wedge e^2 + e^3 \wedge e^4). \tag{30}$$

In the null Cartesian coordinates the Kerr-Schild null tetrad has the form [3]

$$\begin{aligned} e^1 &= d\zeta - Y dv, \qquad e^2 = d\bar\zeta - \bar Y dv, \\ e^3 &= du + \bar Y d\zeta + Y d\bar\zeta - Y\bar Y dv, \\ e^4 &= dv + he^3. \end{aligned} \tag{32}$$

Evaluating the basis two-forms in the Cartesian coordinates we obtain

$$e^1 \wedge e^2 + e^3 \wedge e^4 = d\zeta \wedge d\bar\zeta + du \wedge dv + Y d\bar\zeta \wedge dv, \tag{33}$$

and

$$e^3 \wedge e^1 = Y\ d\bar\zeta \wedge d\zeta + du \wedge dz - Y du \wedge dv - Y^2\ d\bar\zeta \wedge dv. \tag{34}$$

5. Axial singular waves

The obtained general solution for the aligned electromagnetic fields (30) contains the factors which depend on coordinate $Y = e^{i\phi}\tan\frac{\theta}{2}$ and can be singular at the z-axis.

We will now be interested in the wave terms and omit the terms describing the longitudinal components and the field γ.

The wave terms are proportional to the following basis two-forms
$e^3 \wedge e^1|_{wave} = du \wedge d\zeta + Y^2 dv \wedge d\bar\zeta$
and
$e^1 \wedge e^2 + e^3 \wedge e^4|_{wave} = d\zeta \wedge d\bar\zeta$.

Near the positive half-axis z^+, we have $Y \to 0$ and near the negative half-axis z^-, we have $Y \to \infty$.

Therefore, with the exclusion the γ term, the wave terms of the e.m. field (30) take the form

$$\mathcal{F}|_{wave} = f_R\ d\zeta \wedge du + f_L\ d\bar\zeta \wedge dv, \tag{35}$$

where the factor

$$f_R = (AZ)_{,1} \tag{36}$$

describes the "right" waves propagating along the z^+ half-axis, and the factor

$$f_L = 2Y\psi(Z/P)^2 + Y^2(AZ)_{,1} \tag{37}$$

describes the "left" waves propagating along the z^- half-axis, and some of them are singular at z axis.

[3] In the paper [13] treatment is given in terms of the "in" — going congruence e^3 (advanced fields). Here we need to use the "out" — going congruence. The simplest way to do it retaining the basic relations of the paper [13] is to replace $t \to -t$ in the definition of the null Cartesian coordinates. Therefore, we use here the notations

$$\begin{aligned} 2^{\frac12}\zeta &= x + iy, \qquad 2^{\frac12}\bar\zeta = x - iy, \\ 2^{\frac12}u &= z - t, \qquad 2^{\frac12}v = z + t. \end{aligned} \tag{31}$$

Besides, since $Z/P = (r+ia\cos\theta)^{-1}$, all the terms are also singular at the Kerr ring $r = \cos\theta = 0$. Therefore, the singular excitations of the Kerr ring turn out to be connected with the axial singular waves.

Let us consider the solutions describing traveling waves along the Kerr ring

$$\psi_n(Y,\tau) = qY^n \exp i\omega\tau \equiv q(\tan\frac{\theta}{2})^n \exp i(n\phi + \omega_n\tau). \tag{38}$$

Near the Kerr ring one has $\psi = \exp i(n\phi + \omega t)$, and $|n|$ corresponds to the number of the wave lengths along the Kerr ring. The parameter n has to be integer for the smooth and single-valued solutions, however, as we shell see bellow, the half-integer n can be interesting too.

Meanwhile, by $Y \to 0$ one approaches to the positive z-axis where the solutions may be singular too. Similar, by $Y \to \infty$ one approaches to the negative z-axis, and some of the solutions turns out to be singular there.

When considering asymptotical properties of these singularities by $r \to \infty$, we have $z = r\cos\theta$, and for the distance ρ from the z^+ axis we have the expression $\rho = z\tan\theta \simeq 2r|Y|$ by $Y \to 0$. Therefore, for the asymptotical region near the z^+ axis we have to put $Y = e^{i\phi}\tan\frac{\theta}{2} \simeq e^{i\phi}\frac{\rho}{2r}$, and $|Y| \to 0$, while for the asymptotical region near the z^- axis $Y = e^{i\phi}\tan\frac{\theta}{2} \simeq e^{i\phi}\frac{2r}{\rho}$, and $|Y| \to \infty$.

The parameter $\tau = t - r - ia\cos\theta$ takes near the z-axis the values

$$\tau_+ = \tau|_{z^+} = t - z - ia, \quad \tau_- = \tau|_{z^-} = t + z + ia. \tag{39}$$

It has also to be noted that for $|n| > 1$ the solutions contain the axial singularities which do not fall of asymptotically, but are increasing. Therefore, we shell restrict the treatment by the cases $|n| \le 1$.

The leading wave terms for $|n| \le 1$ are given in the **Appendix**.

The leading singular wave for $n = 1$ is

$$\mathcal{F}_1^- = \frac{4qe^{i2\phi + i\omega_1\tau_-}}{\rho^2}\, d\bar\zeta \wedge dv. \tag{40}$$

It propagates to $z = -\infty$ and has the uniform axial singularity at z^- of order ρ^{-2}.

Meanwhile, the leading singular wave for $n = -1$ is

$$\mathcal{F}_{-1}^+ = -\frac{4qe^{-i2\phi + i\omega_{-1}\tau_+}}{\rho^2}\, d\zeta \wedge du, \tag{41}$$

and has the similar uniform axial singularity at z^+ which propagates to $z = +\infty$.

The waves with $n = 0$ are regular.

In what follows we will show that these singularities form the half-infinite chiral strings, in fact superconducting D-strings. There are several arguments in favor of the system containing a combination of two strings of opposite chirality, $n = \pm 1$.

First, if the solution contains only one half-infinite string, like the Dirac monopole string, it turns out to be asymmetric with respect to the $z^\pm$ half-axis, which leads to a nonstationarity via a recoil.

Then, the symmetric stringy solutions exclude the appearance of monopole charge.

Note also, that the pure chiral strings, containing modes of only one direction, cannot exist and any chiral string has to be connected to some object containing an anti-chiral part. Indeed, the pure chiral excitation depends only on one of the parameters $\tau_\pm = t \pm \sigma$, and as a result the world-sheet is degenerated in a world-line [4]. This is seen in the models of the cosmic chiral strings

[4] This argument was suggested by G. Alekseev.

where the chiral excitations are joined to some mass [21] or are sitting on some string having modes of opposite chirality [22]. In our case the partial pp-wave e.m. excitation has the same chirality as the half-infinite carrier of this excitation (the axial ray of PNC). Therefore, the combination of two $n = \pm 1$ excitations looks very natural and leads to the appearance of a full stringy system with two half-infinite singular D-strings of opposite chirality, "left" and "right", as it is shown at the Fig. 1. The world-sheet of the system containing from two straight chiral strings will be given by

$$x^\mu(t,z) = \frac{1}{2}[(t-z)k^\mu_R + (t+z)k^\mu_L], \tag{42}$$

where the lightlike vectors k^μ are constant and normalized. At the rest frame the timelike components are equal $k^0_R = k^0_L = 1$, and the spacelike components are oppositely directed, $k^a_R + k^a_L = 0, \quad a = 1,2,3$. Therefore, $\dot{x}^\mu = (1,0,0,0)$, and $x'^\mu = (0, k^a)$, and the Nambu-Goto string action

$$S = \alpha'^{-1} \int\int \sqrt{(\dot{x})^2(x')^2 - (\dot{x}x')^2}dtdz \tag{43}$$

can be expressed via k^μ_R and k^μ_L.

To normalize the infinite string we have to perform a renormalization putting $\alpha'^{-1}\int(x')^2dz = m$, which yields the usual action for the center of mass of a pointlike particle

$$S = m\int\sqrt{(\dot{x})^2}dt. \tag{44}$$

For the system of two D-strings in the rest one can use the gauge with $\dot{x}^0 = 1, \quad \dot{x}^a = 0$, where the term $(\dot{x}x')^2$ drops out, and the action takes the form

$$S = \alpha'^{-1}\int dt\int\sqrt{p^a p_a}d\sigma, \tag{45}$$

where

$$p^a = \partial_\sigma x^a = \frac{1}{2}[x'^\mu_R(t+\sigma) - x'^\mu_L(t-\sigma)]. \tag{46}$$

However, one of the most important arguments in favor of the combination of two chiral strings is suggested by analogue to the Dirac equation, which has to be obtained for the Kerr spinning particle if it has a relation to the structure of electron. It is known that in the Weyl basis the Dirac current can be represented as a sum of two lightlike components of opposite chirality

$$J_\mu = e(\bar{\Psi}\gamma_\mu\Psi) = e(\chi^+\sigma_\mu\chi + \phi^+\bar{\sigma}^\mu\phi), \tag{47}$$

where

$$\Psi = \begin{pmatrix} \phi_\alpha \\ \chi^{\dot{\alpha}} \end{pmatrix}, \tag{48}$$

and

$$\bar{\Psi} = (\chi^+, \phi^+) \tag{49}$$

Two real lightlike 4-vectors $k^\mu_L, \quad k^\mu_R$ can be expressed in spinor form

$$k^\mu_L = \phi^+\bar{\sigma}^\mu\phi \qquad k^\mu_R = \chi^+\sigma^\mu\chi, \tag{50}$$

and two extra complex null vectors can be formed

$$m^\mu = \chi^+\sigma^\mu\phi \qquad \bar{m}^\mu = \phi^+\sigma^\mu\chi, \tag{51}$$

which complete the null tetrad. This analogue shows that the Dirac equation can only describe the axial stringy system of the Kerr spinning particle.

Indeed, the solution containing combination of three terms with $n = -1, 0, 1$ represents also especial interest since it yields a smooth e.m. field packed along the Kerr string with one half of the wavelength and gives an electric charge to the solution.

Note, that orientifold structure of the Kerr circular string admits apparently the excitations with $n = \pm 1/2$ too, so far as the negative half-wave can be packed on the covering space turning into positive one on the second sheet of the orientifold. However, the meaning of this case is unclear yet, and it demands a special consideration.

6. Einstein-Maxwell axial pp-wave solutions

The e.m. field given by (35), (36) and (37) can be obtained from the potential

$$\mathcal{A} = -AZe^3 - \chi d\bar{Y}, \tag{52}$$

where $A = \psi/P^2$ is given by (22) and

$$\chi = \int P^{-2}\psi dY, \tag{53}$$

$\bar{Y}$ being kept constant in this integration. The considered wave excitations have the origin from the term

$$\mathcal{A} = P^{-2}\psi_n Ze^3 = qY^n \exp i\omega\tau P^{-2} Ze^3 \tag{54}$$

and acquire the following asymptotical $z^{\pm}$ forms:

For $n = 1; z < 0$

$$\mathcal{A}^- = qYe^{i\omega\tau}(r + ia\cos\theta)^{-1}e^3/P \simeq -2q\frac{e^{i\omega_1\tau_- + i\phi}}{\rho}dv. \tag{55}$$

For $n = -1; z > 0$

$$\mathcal{A}^+ = qY^{-1}e^{i\omega\tau}(r + ia\cos\theta)^{-1}e^3/P \simeq 2q\frac{e^{i\omega_{-1}\tau_+ - i\phi}}{\rho}du\ . \tag{56}$$

Each of the partial solutions represents the singular plane-fronted e.m. wave propagating along z^+ or z^- half-axis without damping. It is easy to point out the corresponding self-consistent solution of the Einstein-Maxwell field equations which belongs to the well known class of pp-waves [17, 18].

The metric has the Kerr-Schild form

$$g_{\mu\nu} = \eta_{\mu\nu} + 2hk_\mu k_\nu, \tag{57}$$

where function h determines the Ricci tensor

$$R^{\mu\nu} = -k^\mu k^\nu \Box h, \tag{58}$$

$k^\mu = e^{3\mu}/P$ is the normalized principal null direction (in particular, for the z^+ axis $k^\mu dx^\mu = -2^{1/2}du$), and $\Box$ is a flat D'Alembertian

$$\Box = 2\partial_\zeta\partial_{\bar{\zeta}} + 2\partial_u\partial_v\ . \tag{59}$$

The Maxwell equations take the form

$$\Box\mathcal{A} = J = 0 \tag{60}$$

and can easily be integrated leading to the solutions

$$\mathcal{A}^+ = [\Phi^+(\zeta) + \Phi^-(\bar{\zeta})] f^+(u,v) du, \tag{61}$$
$$\mathcal{A}^- = [\Phi^+(\zeta) + \Phi^-(\bar{\zeta})] f^-(u,v) dv, \tag{62}$$

where $\Phi^\pm$ are arbitrary analytic functions, and functions $f^\pm$ describe the arbitrary retarded and advanced waves. In our case we have the retarded-time parameter $\tau = t - r - ia\cos\theta$ which takes at the z^+ axis the values $\tau \simeq -2^{1/2}u - ia$ and at the z^- axis the values $\tau \simeq 2^{1/2}v + ia$. Therefore, we have

$$f^+ = f^+(u), \quad f^- = f^-(v). \tag{63}$$

The corresponding energy-momentum tensor will be

$$T^{\mu\nu} = \frac{1}{8\pi} |\mathcal{F}^+_{-1}|^2 k^\mu k^n, \tag{64}$$

where for z^+ wave $k_\mu dx^\mu = -2^{1/2} du$ and for z^- wave $k_\mu dx^\mu = 2^{1/2} dv$.

The Einstein equations $R^{\mu\nu} = -8\pi T^{\mu\nu}$ take the simple asymptotic form

$$\Box h = |\mathcal{F}^+_{-1}|^2 = 16 q^2 e^{-2a\omega} \rho^{-4}. \tag{65}$$

This equation can easily be integrated and yields the singular solution

$$h = 8 q^2 e^{-2a\omega} \rho^{-2}. \tag{66}$$

Therefore, the wave excitations of the Kerr ring lead to the appearance of singular pp-waves which propagate outward along the z^+ and/or z^- half-axis.

These axial singularities are evidences of the axial stringy currents, which are exhibited explicitly when we try to regularize the singularities [23] on the base of the Witten field model for the cosmic superconducting strings [14].

The resulting excitations have the Compton wave length which is determined by the size of the Kerr circular string. However, for the moving systems the excitations of the axial stringy system are modulated by de Broglie periodicity.

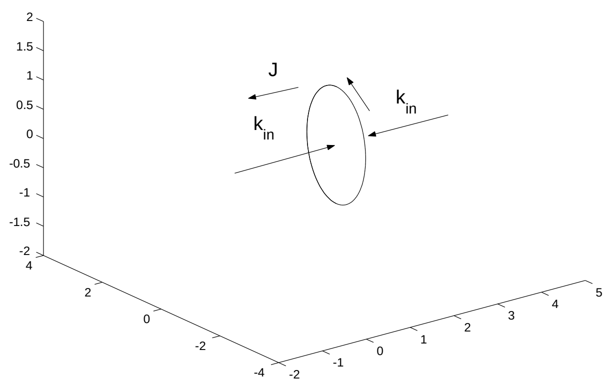


Figure 4: Schematic description of the Kerr antiparticle. Two axial singular strings are directed "inward".

One can see here a striking similarity with the well known elements and methods of the signal transmission in the systems of radio engineering and in the radar systems. In fact, the chiral axial string resembles a typical system for the signal transmission containing a carrier frequency which is modulated by the signal — the carrier of an information. One sees that the Kerr circular string can also be considered as a generator of the carrier frequency, and plays a role of the antenna. Basing on the principle that the fine description of a quantum system has to absorb maximally the known classical information on this system, one can conjecture that the above strikingly simple structure can have a relation to the structure of spinning particles.

In conclusion, one can conjecture which changes could correspond to the Kerr anti-particle. It has to be the change of the PNC direction, as it is shown in **Fig. 4**. It yields a natural picture of annihilation as it is shown in the **Fig. 5**. It was discussed in [24] that the size of the Kerr circular string for the massless Kerr spinning particle has to grow to infinity and disappear. As a result there retains only a single chiral string.

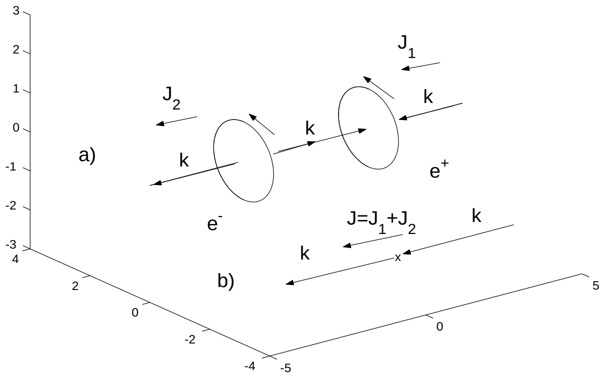


Figure 5: (a) annihilation of the Kerr particle and antiparticle and (b) formation of the lightlike particle.

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Appendix

The leading wave terms for $|n| \le 1$ are the following:

$$\mathcal{F}^-_1 = \frac{4qe^{i2\phi + i\omega_1\tau_-}}{\rho^2}\, d\bar\zeta \wedge dv, \qquad \mathcal{F}^+_{-1} = -\frac{4qe^{-i2\phi + i\omega_{-1}\tau_+}}{\rho^2}\, d\zeta \wedge du, \tag{67}$$

For $n = 1$.
At z^- half-axis

$$\mathcal{F}^-_1 = 4q\frac{e^{i2\phi} + i\omega_1\tau_-}{\rho^2}\, d\bar\zeta \wedge dv - \frac{qe^{i\omega_1\tau_-}}{r^2}\, d\zeta \wedge du, \tag{68}$$

and at z^+ half-axis

$$\mathcal{F}_1^+ = \frac{3qe^{i2\phi+i\omega_1\tau_+}\sin^2\theta}{4r^2}\; d\bar{\zeta}\wedge dv + \frac{qe^{i\omega_1\tau_+}}{r^2}\; d\zeta\wedge du. \tag{69}$$

For $n = 0$.
At z^- half-axis

$$\mathcal{F}_0^- = q\frac{(1+2a\omega)e^{i\phi+i\omega_0\tau_-}\sin\theta}{r^2}\; d\bar{\zeta}\wedge dv - q\frac{e^{-i\phi+i\omega_0\tau_-}\sin\theta}{r^2}\; d\zeta\wedge du, \tag{70}$$

and at z^+ half-axis

$$\mathcal{F}_0^+ = q\frac{e^{i\phi+i\omega_0\tau_+}\sin\theta}{r^2}\; d\bar{\zeta}\wedge dv + q\frac{(2a\omega-1)e^{-i\phi+i\omega_0\tau_+}\sin\theta}{r^2}\; d\zeta\wedge du. \tag{71}$$

For $n = -1$.
At z^- half-axis

$$\mathcal{F}_{-1}^- = -\frac{qe^{i\omega_{-1}\tau_-}}{r^2}\; d\bar{\zeta}\wedge dv - \frac{3qe^{-i2\phi+i\omega_{-1}\tau_-}\sin^2\theta}{4r^2}\; d\zeta\wedge du. \tag{72}$$

and at z^+ half-axis

$$\mathcal{F}_{-1}^+ = \frac{qe^{i\omega_{-1}\tau_+}}{r^2}\; d\bar{\zeta}\wedge dv - \frac{4qe^{-i2\phi+i\omega_{-1}\tau_+}}{\rho^2}\; d\zeta\wedge du. \tag{73}$$

Similar, for $n = 1/2$.
At z^- half-axis

$$\mathcal{F}_{1/2}^- = q\frac{2^{1/2}e^{i3\phi/2} + i\omega_{1/2}\tau_-}{\rho^{3/2}r^{1/2}}\; d\bar{\zeta}\wedge dv - \frac{3qe^{-i\phi/2+i\omega_{1/2}\tau_-}\sin^{1/2}\theta}{2^{3/2}r^2}\; d\zeta\wedge du, \tag{74}$$

and at z^+ half-axis

$$\mathcal{F}_{1/2}^+ = \frac{5qe^{i3\phi/2+i\omega_{1/2}\tau_+}\sin^{3/2}\theta}{2^{1/2}\;4\;r^2}\; d\bar{\zeta}\wedge dv + \frac{qe^{-i\phi/2+i\omega_{1/2}\tau_+}}{2^{1/2}\rho^{1/2}r^{3/2}}\; d\zeta\wedge du. \tag{75}$$

For $n = -1/2$.
At z^- half-axis

$$\mathcal{F}_{-1/2}^- = \frac{-qe^{i\phi/2+i\omega_{-1/2}\tau_-}}{2^{1/2}\rho^{1/2}r^{3/2}}\; d\bar{\zeta}\wedge dv - \frac{5qe^{-i3\phi/2+i\omega_{-1/2}\tau_-}\sin^{3/2}}{2^{1/2}\;4\;r^2}\; d\zeta\wedge du, \tag{76}$$

and at z^+ half-axis

$$\mathcal{F}_{-1/2}^+ = \frac{3qe^{i\phi/2+i\omega_{-1/2}\tau_+}\sin^{1/2}\theta}{2^{3/2}r^2}\; d\bar{\zeta}\wedge dv - \frac{2^{1/2}qe^{-i3\phi/2+i\omega_{-1/2}\tau_+}}{\rho^{3/2}r^{1/2}}\; d\zeta\wedge du. \tag{77}$$