MIXED SYMMETRY TENSOR FIELDS IN MINKOWSKI AND (A)dS SPACES

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We discuss a framework for investigation of mixed symmetry tensor fields in Minkowski and (A)dS spaces and their possible interactions.

Introduction

In four-dimensional flat Minkowski space-time massive particles are characterized by one parameter — spin s and the most simple and economic description of such particles is the one based on completely symmetric (spin)-tensors. But moving to the dimensions greater than four, one faces the fact that representations of appropriate groups require more parameters and as a result in many interesting cases such as supergravity theories, superstrings and (supersymmetric) high spin theories one has to consider different mixed symmetry (spin)-tensors. One of the technical difficulties (besides purely combinatorial ones) one faces working with such fields is that to get analog of gauge invariant "field strengths" one has to build expressions with more and more derivatives (or has to work with non-local terms in the equations of motion or the Lagrangians) [1–4]. This problem appears already for the massless spin two field in gravity, but in this case there exist very elegant solution: one goes to the first order formalism and obtains the description in terms of tetrad $e_\mu{}^a$ and Lorentz connection $\omega_\mu{}^{[ab]}$ which play the roles of the gauge fields and "normal" gauge invariant field strengths $T_{[\mu\nu]}{}^a$ and $R_{[\mu\nu]}{}^{[ab]}$ having nice geometrical interpretation as torsion and curvature. The structure of gauge transformation laws is also appears to be rather complicated, moreover, these transformations often turn out to be reducible. All this make the problem of investigation of possible interactions among such fields a very complicated task [5–10]. In (Anti) de Sitter space the problem becomes even more complicated because particles in (A)dS reveal a number of very peculiar features such as unitary forbidden regions (i.e. not all values of mass and cosmological constant are allowed) and appearance of partially massless theories [11]–[19]. Moreover not all massless fields in flat Minkowski space could be deformed into the (A)dS space without introduction of additional fields [20, 21, 22], making the very definition of mass for such fields problematic.

Our aim here — to give a framework which could be powerful enough to allow careful investigation of all related questions and at the same time simple enough to make such investigations tractable [22, 23, 24]. Two main ingredients of such framework: first order ("tetrad") formalism and gauge invariant description of massive high spin particles.

1. First order formalism

Our first order formalism for mixed symmetry tensor fields is a generalization of well known tetrad formalism for spin 2 in gravity. It is instructive to compare this formalism with another very well known example of first order formulation — for spin 1.

Spin 1

For the description of spin-1 particles one use vector field A_μ and in order to have right number of physical degrees of freedom the theory has to be invariant under the gauge transformations $\delta A_\mu = \partial_\mu \Lambda$. In this, it is easy to construct a field strength $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ which is gauge invariant and contains first derivatives only. Then the second order Lagrangian could be written as $\mathcal{L} = -\frac{1}{4}F_{\mu\nu}{}^2$. Moreover, introducing independent object $F_{\mu\nu}$ one can construct first order Lagrangian: $\mathcal{L} = \frac{1}{4}F_{\mu\nu}{}^2 - F^{\mu\nu}\partial_\mu A_\nu$, which is equivalent to second order one.

Spin 2

The simplest possibility for the description of spin-2 particle is symmetric second rank tensor $h_{\mu\nu}$ with gauge transformations $\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu$. But in this case in order to construct gauge invariant object one has to use second derivatives of $h_{\mu\nu}$:

$$R_{\mu\nu,\alpha\beta} = \partial_\mu \partial_\alpha h_{\nu\beta} - \partial_\nu \partial_\alpha h_{\mu\beta} - \partial_\mu \partial_\beta h_{\nu\alpha} + \partial_\nu \partial_\beta h_{\mu\alpha}.$$

As a result, there is no gauge invariant “field strength” with one derivative and second order Lagrangian can't be written in a form quadratic in some gauge invariant object. Thus, straightforward generalization of first order formalism for the spin-1 field on the spin-2 (and all higher spin) case is not possible.

However, first order formalism for spin-2 particle does exist. This so called tetrad formulation is very well known mainly due to its nice geometrical interpretation, but for our purpose it is instructive to consider this formalism as a transition from second order formulation to first order one. Then it looks like the following steps.

- Abandon symmetry property of the field: $h_{\mu\nu} \Rightarrow h_\mu{}^a$ (thus introducing additional field components — antisymmetric tensor $h_{[\mu a]}$).
- Change gauge transformations: $\delta h_\mu{}^a = \partial_\mu \xi^a$.
- Now it is easy to construct gauge invariant “field strength” (torsion): $T_{\mu\nu}{}^a = \partial_\mu h_\nu{}^a - \partial_\nu h_\mu{}^a$ containing first derivatives only.
- Then second order Lagrangian could be written as:

  $$\mathcal{L} = \frac{1}{8}T^{\mu\nu,\alpha}T_{\mu\nu,\alpha} + \frac{1}{4}T^{\mu\nu,\alpha}T_{\mu\alpha,\nu} - \frac{1}{2}T^\mu T_\mu.$$

- This Lagrangian is also invariant under the local shifts: $\delta h_{\mu\nu} = \eta_{[\mu\nu]}$. Due to this invariance the additional components $h_{[\mu a]}$ could be gauged away leaving us with correct number of physical degrees of freedom.
- Moreover, this last invariance suggests the introduction of additional field — “Lorentz connection”: $\omega_\mu{}^{ab}$ playing a role of gauge field for local η^{ab} transformations: $\delta\omega_\mu{}^{ab} = \partial_\mu \eta^{ab}$.
- Using such fields one can easily construct a first order Lagrangian:

  $$\mathcal{L}_I = -\frac{1}{2}\omega^{\mu,\alpha\beta}\omega_{\alpha,\mu\beta} + \frac{1}{2}\omega^\mu\omega_\mu - \frac{1}{2}\omega^{\mu,\alpha\beta}T_{\alpha\beta,\mu} - \omega^\mu T_\mu,$$

  which is invariant under ξ^a as well as η^{ab} transformations.
- Algebraic equation of motion for $\omega_\mu{}^{ab}$ gives:

  $$\omega_{\mu,\alpha\beta} = \frac{1}{2}[T_{\mu\alpha,\beta} - T_{\mu\beta,\alpha} - T_{\alpha\beta,\mu}].$$

  Substituting this expression into the first order Lagrangian one obtains exactly the second order Lagrangian given before.

- It is very important that this first order Lagrangian could be rewritten as:

$$\mathcal{L}_I = \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \omega_\mu{}^{ac}\omega_\nu{}^{bc} - \frac{1}{2}\left\{ {}^{\mu\nu\alpha}_{abc} \right\} \omega_\mu{}^{ab}\partial_\nu h_\alpha{}^c,$$

  where $\left\{ {}^{\mu\nu}_{ab} \right\} = \delta_\mu{}^a\delta_\nu{}^b - \delta_\nu{}^a\delta_\mu{}^b$ and so on. Such formulation greatly simplify the investigations of possible interactions for spin-2 field.

Mixed $\Phi_{[\mu\nu],\alpha}$ tensor

As an illustration we consider here the simplest case — mixed $\Phi_{[\mu\nu],\alpha}$ tensor in $d = 4$ space-time, but such formalism could be easily generalized on other mixed tensors in space-time dimensions $d \geq 4$ [23, 25, 26, 27]. Usual description of such field uses third rank tensor $\Phi_{[\mu\nu],\alpha}$ antisymmetric on the first two indices and satisfying the relation $\Phi_{[\mu\nu,\alpha]} = 0$. It has two gauge transformations:

$$\delta\Phi_{\mu\nu,\alpha} = \partial_\mu x_{\nu\alpha} - \partial_\nu x_{\mu\alpha} + 2\partial_\alpha y_{\mu\nu} - \partial_\mu y_{\nu\alpha} + \partial_\nu y_{\mu\alpha},$$

where parameter $x_{\{\alpha\beta\}}$ is symmetric while $y_{[\alpha\beta]}$ — antisymmetric. These gauge transformations are reducible in a sense that if one set

$$x_{\alpha\beta} = 3(\partial_\alpha\xi_\beta + \partial_\beta\xi_\alpha) , \qquad y_{\alpha\beta} = -\partial_\alpha\xi_\beta + \partial_\beta\xi_\alpha ,$$

then $\delta\Phi_{\mu\nu,\alpha} = 0$. One can introduce a tensor $T_{[\mu\nu\alpha],\beta}$

$$T_{\mu\nu\alpha,\beta} = \partial_\mu\Phi_{\nu\alpha,\beta} - \partial_\nu\Phi_{\mu\alpha,\beta} + \partial_\alpha\Phi_{\mu\nu,\beta},$$

which is invariant under the $x_{\alpha\beta}$-transformations but not invariant under the $y_{\alpha\beta}$-ones. Then one can write the Lagrangian in the following form:

$$\mathcal{L}_0 = -\frac{1}{6}T^{\mu\nu\alpha,\beta}T_{\mu\nu\alpha,\beta} + \frac{1}{2}T^{\mu\nu}T_{\mu\nu}.$$

There exist one more possibility. Namely, one can introduce another tensor $R_{[\mu\nu],[\alpha\beta]}$

$$R_{\mu\nu,\alpha\beta} = \partial_\alpha\Phi_{\mu\nu,\beta} - \partial_\beta\Phi_{\mu\nu,\alpha} + \partial_\mu\Phi_{\alpha\beta,\nu} - \partial_\nu\Phi_{\alpha\beta,\nu},$$

which is invariant under the $y_{\alpha\beta}$-transformations but not under the $x_{\alpha\beta}$-ones and rewrite the same Lagrangian in a form:

$$\mathcal{L}_0 = -\frac{1}{8}[R^{\mu\nu,\alpha\beta}R_{\mu\nu,\alpha\beta} - 4R^{\mu\nu}R_{\mu\nu} + R^2].$$

Thus, it is not possible to write second order Lagrangian as a square of some gauge invariant quantity because there is no combination of the first derivatives of $\Phi_{[\mu\nu],\alpha}$ that would be invariant under both gauge transformations (that will require two derivatives).

This situation resembles very much the one for spin-2 filed. It turns out possible to construct first order formalism in this case following exactly the same procedure as before. We proceed as follows.

- Abandon the constraint $\Phi_{[\mu\nu,\alpha]} = 0$: $\Phi_{\mu\nu,\alpha} \Rightarrow \Phi_{\mu\nu}{}^a$ (thus introducing additional field components — antisymmetric third rank tensor $\Phi_{[\mu\nu\alpha]}$).
- Change the gauge transformations: $\delta\Phi_{\mu\nu}{}^a = \partial_\mu z_\nu{}^a - \partial_\nu z_\mu{}^a$, where $z_{\mu\nu} = x_{\{\mu\nu\}} + y_{[\mu\nu]}$.
- This allows us to introduce gauge invariant "field strength": $T_{\mu\nu\alpha}{}^a = \partial_{[\mu}\Phi_{\nu\alpha]}{}^a$ containing first derivatives only.

- Second order Lagrangian could be written as:

$$\mathcal{L} = -\frac{1}{6} T^{\mu\nu\alpha,\beta} T_{\mu\nu\alpha,\beta} - \frac{1}{4} T^{\mu\nu\alpha,\beta} T_{\mu\nu\beta,\alpha} + \frac{3}{4} T^{\mu\nu} T_{\mu\nu}.$$

- This Lagrangian is also invariant under local shifts: $\delta\Phi_{\mu\nu,\alpha} = \eta_{[\mu\nu\alpha]}$ so that completely antisymmetric part of $\Phi_{\mu\nu}{}^a$ could be gauged away.

- This last local invariance suggests introduction of an additional field: $\Omega_\mu{}^{abc}$ which will play a role of gauge field for such local shifts: $\delta\Omega_\mu{}^{abc} = \partial_\mu \eta^{abc}$.

- With the use of these fields first order Lagrangian invariant under both local transformations could easily be constructed:

$$\mathcal{L} = \frac{3}{4} \Omega^{\mu,\nu\alpha\beta} \Omega_{\nu,\mu\alpha\beta} - \frac{3}{4} \Omega^{\alpha\beta} \Omega_{\alpha\beta} - \frac{1}{2} \Omega^{\mu,\nu\alpha\beta} T_{\nu\alpha\beta,\mu} + \frac{3}{2} \Omega^{\alpha\beta} T_{\alpha\beta}.$$

- Algebraic equation of motion for $\Omega_\mu{}^{abc}$ gives:

$$\Omega_{\mu,\nu\alpha\beta} = \frac{2}{3} T_{\nu\alpha\beta,\mu} + \frac{1}{3} [T_{\mu\alpha\beta,\nu} + T_{\mu\nu\alpha,\beta} + T_{\mu\beta\nu,\alpha}].$$

  Substituting this expression back into the first order Lagrangian one obtain exactly the same second order Lagrangian as before.

- Moreover, this Lagrangian could be rewritten in a very simple and suggestive form:

$$\mathcal{L} = -\frac{3}{4} \left\{ {}^{\mu\nu}_{ab} \right\} \Omega_\mu{}^{acd} \Omega_\nu{}^{bcd} + \frac{1}{12} \left\{ {}^{\mu\nu\alpha\beta}_{abcd} \right\} \Omega_\mu{}^{abc} T_{\nu\alpha\beta}{}^d.$$

2. Massive particles and gauge invariance

As we have already mentioned particles in (A)dS spaces reveals a number of peculiar features. Let us recall some of them.

- There is no unambiguous definition of mass for general tensor fields in (Anti) de Sitter space.

- There exist unitary forbidden regions i.e. not all combinations of mass-like parameters and cosmological constant lead to the ghost-free theories.

- There are many massless theories in flat Minkowski space that could not be generalized to (A)dS space at all.

- On the other hand in (A)dS there exist so called partially massless theories with the number of physical degrees of freedom different from that of massive or massless theory.

In (A)dS spaces gauge invariant description even for massless particles requires (besides the replacement of usual derivatives by the covariant ones) addition to the Lagrangian and gauge transformation laws mass-like terms proportional to the cosmological constant value. Such a procedure looks very much like the transition from massless to massive particle in gauge invariant description of the later. So it turns out convenient to consider both transition simultaneously. Let us again illustrate an idea on a simple but non-trivial example.

Massive spin-2 particle

Gauge invariant description of massive spin-2 particle requires introduction of two additional (Goldstone) fields: vector A_μ and scalar φ ones. Using first order formalism for all fields we start with the sum of flat space massless Lagrangians:

$$\mathcal{L}_0 = \mathcal{L}_0(\omega_\mu{}^{ab}, h_\mu{}^a) + \mathcal{L}_0(F^{ab}, A_\mu) + \mathcal{L}_0(\pi^a, \varphi),$$
$$\mathcal{L}_0(\omega_\mu{}^{ab}, h_\mu{}^a) = \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} \omega_\mu{}^{ac}\omega_\nu{}^{bc} - \frac{1}{2}\left\{ {}^{\mu\nu\alpha}_{abc} \right\} \omega_\mu{}^{ab}\partial_\nu h_\alpha{}^c,$$
$$\mathcal{L}_0(F^{ab}, A_\mu) = \frac{1}{4}F_{ab}{}^2 - \frac{1}{2}\left\{ {}^{\mu\nu}_{ab} \right\} F^{ab}\partial_\mu A_\nu,$$
$$\mathcal{L}_0(\pi^a, \varphi) = -\frac{1}{2}\pi_a{}^2 + \left\{ {}^{\mu}_{a} \right\} \pi^a\partial_\mu\varphi,$$

which is invariant under the following set of gauge transformations:

$$\delta_0 h_{\mu a} = \partial_\mu\xi_a + \eta_{\mu a}, \qquad \delta_0\omega_\mu{}^{ab} = \partial_\mu\eta^{ab}, \qquad \delta_0 A_\mu = \partial_\mu\Lambda.$$

Working with the first order formalism it is very convenient to use tetrad formulation of the underlying (Anti) de Sitter space. We denote tetrad as $e_\mu{}^a$ and Lorentz covariant derivative as D_μ. (Anti) de Sitter space is a constant curvature space with zero torsion, so we have:

$$D_{[\mu}e_{\nu]}{}^a = 0, \qquad [D_\mu, D_\nu]v^a = R_{\mu\nu}{}^{ab}v_b = -\frac{\Lambda}{3}(e_\mu{}^a e_\nu{}^b - e_\mu{}^b e_\nu{}^a)v_b.$$

Due to non-commutativity of covariant derivatives the Lagrangian no longer invariant under gauge transformations, but this invariance could be restored by adding to Lagrangian:

$$\begin{aligned}\Delta\mathcal{L} &= \frac{m}{\sqrt{2}}[\left\{ {}^{\mu\nu}_{ab} \right\} \omega_\mu{}^{ab}A_\nu + \left\{ {}^{\mu}_{a} \right\} F^{ab}h_\mu{}^b] - \alpha\left\{ {}^{\mu}_{a} \right\} \pi^a A_\mu \\ &\quad + (\frac{m^2}{2} - \frac{\Lambda}{3})\left\{ {}^{\mu\nu}_{ab} \right\} h_\mu{}^a h_\nu{}^b + \frac{m\alpha}{\sqrt{2}}\left\{ {}^{\mu}_{a} \right\} h_\mu{}^a\varphi + m^2\varphi^2,\end{aligned}$$

as well as appropriate terms to gauge transformation laws:

$$\delta_1 h_\mu{}^a = \frac{m}{\sqrt{2}}e_\mu{}^a\Lambda\ , \qquad \delta_1 A_\mu = \frac{m}{\sqrt{2}}\xi_\mu\ , \qquad \delta_1\varphi = \alpha\Lambda,$$

where

$$\alpha^2 = 3m^2 - 2\Lambda.$$

So we have a one parameter family of Lagrangians in general describing a massive spin-2 particle in constant curvature space. Let us analyze this theory in de Sitter and anti de Sitter spaces separately.

De Sitter space

We have unitary forbidden region: $m^2 < 2\Lambda/3$. One can easily check that if we change the sign of scalar field φ kinetic terms then the construction will work for the forbidden region but the theory cease to be unitary, the scalar field being the ghost one. At the boundary of this region $m^2 = 2\Lambda/3 \Rightarrow \alpha = 0 \Rightarrow$ scalar field φ decouples. The rest fields with the Lagrangian

$$\mathcal{L} = \mathcal{L}_0(\omega_\mu{}^{ab}, h_\mu{}^a) + \mathcal{L}_0(F^{ab}, A_\mu) + m\left\{ {}^{\mu\nu}_{ab} \right\} \omega_\mu{}^{ab}A_\nu + m\left\{ {}^{\mu}_{a} \right\} F^{ab}h_\mu{}^b$$

and gauge transformations:

$$\delta h_\mu{}^a = D_\mu\xi^a + e_{\mu b}\eta^{ba} + me_\mu{}^a\Lambda, \quad \delta A_\mu = D_\mu\Lambda + me_{\mu a}\xi^a$$

describes partially massless spin-2 particles (helicities $\pm 2, \pm 1$) which has no direct analog in flat Minkowski space.

Anti de Sitter space

There exists truly massless limit $m \to 0$. In this, the whole system decompose onto two subsystems. One with Lagrangian:

$$\mathcal{L} = \mathcal{L}_0({\omega_\mu}^{ab}, {h_\mu}^a) - \frac{\Lambda}{3} \left\{ {}^{\mu\nu}_{ab} \right\} {h_\mu}^a {h_\nu}^b$$

and gauge transformations:

$$\delta {h_\mu}^a = D_\mu \xi^a + e_{\mu b} \eta^{ba} \qquad \delta {\omega_\mu}^{ab} = D_\mu \eta^{ab} + \frac{\Lambda}{3} ({e_\mu}^a \xi^b - {e_\mu}^b \xi^a)$$

corresponds to massless spin-2 particle. Another one with:

$$\mathcal{L} = \mathcal{L}_0(F^{ab}, A_\mu) + \mathcal{L}_0(\pi^a, \varphi) + m \left\{ {}^{\mu}_{a} \right\} \pi^a A_\mu,$$

$$\delta A_\mu = D_\mu \Lambda \qquad \delta \varphi = -m\Lambda$$

describes massive vector particle with $m^2 = -2\Lambda$

Massive mixed tensor

Tensor $\Phi_{\mu\nu,\alpha}$ has two gauge transformations so for gauge invariant description of massive particle one needs at least two Goldstone fields $h_{\{\mu\nu\}}$ and $B_{[\mu\nu]}$. But these fields have their own gauge transformations. Taking into account reducibility of gauge transformations for $\Phi_{\mu\nu,\alpha}$ one can show that for gauge invariant description one needs one more field: vector A_μ. Thus in first order formalism we ends with four pairs: $({\Omega_\mu}^{abc}, {\Phi_{\mu\nu}}^a)$, $({\omega_\mu}^{ab}, {h_\mu}^a)$, $(C^{abc}, B_{\mu\nu})$ and (F^{ab}, A_μ).

Again we start with the sum of massless flat space Lagrangians:

$$\begin{aligned}
\mathcal{L}_0 &= \mathcal{L}_0({\Omega_\mu}^{abc}, {\Phi_{\mu\nu}}^a) + \mathcal{L}_0({\omega_\mu}^{ab}, {h_\mu}^a) + \mathcal{L}_0(C^{abc}, B_{\mu\nu}) + \mathcal{L}_0(F^{ab}, A_\mu), \\
\mathcal{L}_0(C^{abc}, B_{\mu\nu}) &= -\frac{1}{6} {C_{abc}}^2 + \frac{1}{6} \left\{ {}^{\mu\nu\alpha}_{abc} \right\} C^{abc} \partial_\mu B_{\nu\alpha}
\end{aligned}$$

and corresponding set of gauge transformations:

$$\begin{aligned}
\delta_0 {\Phi_{\mu\nu}}^a &= \partial_\mu {z_\nu}^a - \partial_\nu {z_\mu}^a + {\eta_{\mu\nu}}^a, \qquad \delta_0 {\Omega_\mu}^{abc} = \partial_\mu \eta^{abc}, \\
\delta_0 h_{\mu a} &= \partial_\mu \xi_a + \eta_{\mu a}, \qquad \delta_0 {\omega_\mu}^{ab} = \partial_\mu \eta^{ab}, \\
\delta_0 B_{\mu\nu} &= \partial_\mu \zeta_\nu - \partial_\nu \zeta_\mu, \qquad \delta_0 A_\mu = \partial_\mu \Lambda.
\end{aligned}$$

Then we switch to the (Anti) de Sitter space. To restore gauge invariance we have to add to the Lagrangian:

$$\begin{aligned}
\mathcal{L}_1 = & \frac{\alpha}{4} \left\{ {}^{\mu\nu}_{ab} \right\} {\Omega_\mu}^{abc} {h_\nu}^c + \frac{\alpha}{4} \left\{ {}^{\mu\nu\alpha}_{abc} \right\} {\omega_\mu}^{ab} {\Phi_{\nu\alpha}}^c - \frac{\beta}{4} \left\{ {}^{\mu\nu\alpha}_{abc} \right\} {\Omega_\mu}^{abc} B_{\nu\alpha} + \\
& -\frac{\beta}{4} \left\{ {}^{\mu\nu}_{ab} \right\} C^{abc} {\Phi_{\mu\nu}}^c - \frac{\beta}{\sqrt{2}} \left\{ {}^{\mu\nu}_{ab} \right\} {\omega_\mu}^{ab} A_\nu - \\
& -\frac{\beta}{\sqrt{2}} \left\{ {}^{\mu}_{a} \right\} F^{ab} {h_\mu}^b + \frac{\alpha}{2\sqrt{2}} \left\{ {}^{\mu\nu}_{ab} \right\} F^{ab} B_{\mu\nu}
\end{aligned}$$

and to the gauge transformations:

$$\begin{aligned}
\delta_1 \Phi_{\mu\nu} &= \frac{\alpha}{6} ({e_\mu}^a \xi^b - {e_\mu}^b \xi^a) - \beta ({e_\mu}^a \zeta^b - {e_\mu}^b \zeta^a), \qquad \delta_1 B_{\mu\nu} = \beta z_{\mu\nu}, \\
\delta_1 {h_\mu}^a &= \alpha {z_\mu}^a - \frac{\beta}{\sqrt{2}} {e_\mu}^a \Lambda, \qquad \delta_1 A_\mu = -\frac{\beta}{\sqrt{2}} \xi_\mu + \alpha \sqrt{2} \zeta_\mu,
\end{aligned}$$

$$\alpha^2 - 3\beta^2 = -2\Lambda.$$

Again we obtained one parameter family of Lagrangians. Let us consider to spaces separately.

Anti de Sitter space

One can set $\beta = 0$, in this the whole system decompose onto two disconnected subsystems. One of them

$$\mathcal{L} = \mathcal{L}_0(\Omega_\mu{}^{abc}, \Phi_{\mu\nu}{}^a) + \mathcal{L}_0(\omega_\mu{}^{ab}, h_\mu{}^a) + m[\left\{ {}^{\mu\nu}_{ab} \right\} \Omega_\mu{}^{abc} h_\nu{}^c + \left\{ {}^{\mu\nu\alpha}_{abc} \right\} \omega_\mu{}^{ab} \Phi_{\nu\alpha}{}^c],$$

where $m^2 = -\Lambda/8$ with gauge transformations:

$$\begin{aligned} \delta\Phi_{\mu\nu}{}^a &= D_\mu z_\nu{}^a - D_\nu z_\mu{}^a + \eta_{\mu\nu}{}^a + \frac{2m}{3(d-3)}(e_\mu{}^a \xi_\nu - e_\nu{}^a \xi_\mu), \\ \delta h_\mu{}^a &= D_\mu \xi^a + \eta_\mu{}^a + 4m z_\mu{}^a \end{aligned}$$

and describes partially massless theory.

Another one with:

$$\mathcal{L} = \mathcal{L}_0(C^{abc}, B_{\mu\nu}) + \mathcal{L}_0(F^{ab}, A_\mu) + \frac{M}{4} \left\{ {}^{\mu\nu}_{ab} \right\} F^{ab} B_{\mu\nu},$$

where $M^2 = -4\Lambda$ and gauge transformations:

$$\delta B_{\mu\nu} = D_\mu \zeta_\nu - D_\nu \zeta_\mu, \qquad \delta A_\mu = D_\mu \Lambda + M \zeta_\mu$$

gives a gauge invariant description of massive antisymmetric tensor in (A)dS space.

De Sitter space

Now one can set $\alpha = 0$ and we also obtain two decoupled subsystems. One of them gives us one more example of partially massless theory:

$$\mathcal{L} = \mathcal{L}_0(\Omega_\mu{}^{abc}, \Phi_{\mu\nu}{}^a) + \mathcal{L}_0(C^{abc}, B_{\mu\nu}) + m[\left\{ {}^{\mu\nu\alpha}_{abc} \right\} \Omega_\mu{}^{abc} B_{\nu\alpha} + \left\{ {}^{\mu\nu}_{ab} \right\} C^{abc} \Phi_{\mu\nu}{}^c],$$

where $m^2 = \Lambda/24$ with gauge transformations:

$$\begin{aligned} \delta\Phi_{\mu\nu}{}^a &= D_\mu z_\nu{}^a - D_\nu z_\mu{}^a + \eta_{\mu\nu}{}^a + 4m(e_\mu{}^a \zeta_\nu - e_\nu{}^a \zeta_\mu), \\ \delta B_{\mu\nu} &= D_\mu \zeta_\nu - D_\nu \zeta_\mu - 4m z_{\mu\nu}. \end{aligned}$$

In this, the rest fields $(h_\mu{}^a, A_\mu)$ gives exactly the same partially massless spin-2 theory as in the previous subsection.

Conclusions

- First order formalism with its simple, geometric in nature and very suggestive Lagrangians greatly simplifies investigations with mixed symmetry tensors making calculations at least tractable.

- In spite of large number of fields involved gauge invariant description of massive high spin particles turns out to be very well suited for investigation of unitarity, gauge invariance, (partial) masslessness and so on.

- Partially massless theories (though rather exotic from common flat Minkowski space point of view) can serve as an important laboratory where a number of non-trivial questions could be answered.

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