SOME REMARKS ON THEORIES IN NONCOMMUTATIVE SPACETIMES

V.A. Petrov*

Institute for High Energy Physics, Protvino, Russia

• In a flow of papers on noncommutative geometry quite a rare attempt to formulate noncommutative axiomatic field theory was undertaken recently in ref. [1]. One more step in this direction is made in ref. [2], where some physical problems related to the scattering amplitude are discussed in the noncommutative framework.

• The common flaw in both papers is the lack of a clear, operational generalization of the basic notion of the Heisenberg field operator for the noncommutative framework. In the case when only some coordinates are noncommutative they are just ignored. The authors concentrate on the residual symmetry (in their approach Lorentz invariance is violated. See for a Lorentz-invariant formulation of the noncommutative theories in ref. [3]), which, in its turn, implies a kind of "residual" locality of the fields. Thus in ref. [1] the authors consider the vacuum explectation of the field commutator, which in the case of noncommutativity of only x_2- and x_3-coordinates results in disappearing of this v.e.v. at $x_0^2 - x_1^2 < 0$ at any values of x_2 and x_3. The fact that noncommuting coordinates have the definite values means actually that theory is considered as a kind of usual, "commutative", field theory but with a (Lorentz non-invariant) nonlocal interaction. This is easily seen in the perturbative framework where the Moyal product is clearly realized as a "delocalization" of the local Lagrangean.

• In the general, axiomatic framework we do not know in what way this nonlocality affects the properties of the Heisenberg field operators $\Phi_\theta(x)$, where θ refers to the primordial commutator

$$[\hat{x}_\mu, \hat{x}_\nu] = i\theta_{\mu\nu}. \tag{1}$$

Making use of the covariance of the tensor $\theta_{\mu\nu}$ under Lorentz transformations one can arrive to the following spectral representation for the commutators of two "θ extended" operators:

$$< \Omega \left| \left[J_\theta\left(\frac{x}{2}\right), J_\theta\left(-\frac{x}{2}\right) \right] \right| \Omega >= \int\limits_0^\infty d\mu^2 \int d\beta \rho(\mu^2, \beta) D(x, \mu^2; \theta, \beta), \tag{2}$$

where

$$D(x, \mu^2|\theta, \beta) = 2Im \int \frac{d\vec{x}}{(2\pi)^3 2\omega_k(\mu)} \delta\left((\theta k_+)^2 + \beta\right)^{i\omega_k x^0 - i\vec{k}\vec{x}} e,$$

$$(\theta k_+)^2 = (\vec{e}\vec{k})^2 - \left([\vec{m} \times \vec{k}]\right)^2 - \vec{e}^2 \cdot \omega_k^2 - 2\omega_k(\vec{e} \cdot (\vec{m} \times \vec{k}]),$$

$$e_i \equiv \theta_{oi}, \quad m_k = \frac{1}{2}\epsilon_{kij}\theta_{ij},$$

while $\rho(\mu^2, \beta)$ is a noncommutative (nonlocal) generalization of the Källen–Lehmann spectral density. At $\theta \to 0$ we are back to the usual spectral representation.

*E-mail: Vladimir.Petrov@mail.ihep.ru

Taking different values of $\vec{e}$ and $\vec{m}$ one arrives at different support properties of the v.e.v. in (1). What is important to keep in mind is that these support properties are not automatically valid for operators, not just v.e.v.'s.

Simple examples of composite operators like

$$: \varphi^2(x) :\to J_\theta(x) = \int \frac{d^4x_1 d^4x_2}{|\det\theta|} e^{i(x-x_1)\theta^{-1}(x-x_2)} : \varphi(x_1)\varphi(x_2) :$$

show that the support properties of the commutators depend in general on the states which make the corresponding matrix element.

• More direct way to account for noncommutativity is to use the operators $\Phi(\hat{x})$, which act on the product Hilbert space $K \otimes H$, where H is a usual Hilbert space for fields in commutative theory, while K is a "coordinate" (rigged) Hilbert space where noncommutative coordinates $\hat{x}_\mu$ act.

In general Eq.(1) allows to diagonalize only one of the four $\hat{x}_\mu$. So one effectively arrives at nolocal fields [4]

$$< x'_\mu|\hat{\Phi}|x_\mu >= \int d^4y\Phi(y)K_\theta(x'_\mu, x_\mu, \theta|y),$$

which are integral transforms of the "old" fields $\Phi(y)$ with exactly known kernels K_θ. In such a formulation one deal with some fuzzy Heisenberg fields, whose properties, however, can be studied at the operator level (in the space H). One can study generalized Wightman functions which are now some integral transforms of the "old" ones.

I have to mention a tempting idea to use the Moyal product for the generalization of the formalism of Wightman functions to the noncommutative case. Such a formal use of the Moyal product for functions at different points is fairly possible (see e.g. [5]). But this leaves intact the "intrinsic" noncommutativity of the field operator. So the trick can be hardly considered as a genuine noncommutative map of the "usual" theory; at least perturbative calculations do not seem to support it.

• The problem which remains is the conctruction of the noncommutative analogue of the S-matrix and derivation of, say, analytic properties of the scattering amplitude. At any rate the ubiqutous presence of the parameters $\theta_{\mu\nu}$ leads to searches for their physical interpretation. This is the subject of our further work.

• With these quite sporadic remarks on the approach which, certainly, deserves much more, I conclude my short talk, and, simultaneously, I announce the end of our Workshop, which, as I could observe, was very productive in the exchange of new (quite often contradictory) ideas.

References

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