

# SINGULAR STRINGS IN THE ROTATING ASTROPHYSICAL SOURCES: A NEW CONJECTURE ON THE QPOS AND JET PHENOMENA

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Stringy and disklike sources of the rotating compact astrophysical objects are considered on the base of the Kerr geometry. It is argued that analyticity of the Kerr solutions may result the appearance of singular strings, which may be the source of two important astrophysical effects: the jets and QPOs phenomena.

## 1. Introduction

Most of the astrophysical works deal with the Kerr solution for description of the rotating compact sources. Description of such sources is usually based on the Kerr solution. The Kerr geometry is a fundamental discovery which is related to many physical systems, from the rotating black holes, neutron stars and galactic nucleus in astrophysics till the structure of spinning particles and fundamental solutions to the low energy string theory.

Indeed, a tendency is now traced to an approaching of the physics of the compact astrophysical objects and the physics of elementary particles. In particular, the well known objects in the physics of elementary particles, such as the bag models, superdense matter, quarks and color superconductivity, penetrate now in astrophysics, demonstrating a Unity of physics.

In observational astrophysics this tendency is confirmed by the problem of quasi-periodic oscillations, QPOs phenomena (see for example [1, 2]).

Black hole binaries exhibit thermal and non-thermal components of X-ray emission which vary widely in intensity. The X-ray spectral and timing studies in the radio, optical and gamma-ray diapasons display the stable low frequency QPOs in the range 0.1-30 Hz and the QPOs in the range 40-450 Hz. Besides, there are observations of the spectral lines which are in a relative stable harmonic relations.

There appears the problem of reconstruction of the models and structure of the compact objects basing on the spectral observations and timing. The spectral analysis becomes one of the main tools of the modern astrophysics resembling the atomic spectroscopy of the beginning of the last century. However, so far there are very large uncertainties for the corresponding physical models and we have a risk here to conjecture a new approach to this problem.

Along with the QPOs problem, there is also the very old problem of the model of jet formations.

The aim of this work is to pay attention to some theoretical effects which are linked to analytic structure of the Kerr geometry and reproduce some features resembling the jet and QPOs phenomena. At this stage we point merely out a qualitative similarity, displaying a potential possible role of these effects, so our conjectures are very far from the real estimations.

The presented model is based on the recent analysis of the *aligned* electromagnetic excitations on the Kerr background which was performed for investigation of the Kerr spinning particle [3]. It was shown that the *aligned* electromagnetic excitations of the Kerr geometry lead to the appearance of two singular stringy structures:

- 1) circular singular string  
and
- 2) axial stringy system.

We conjecture that excitation of the circular string may be the source of QPOs, while the appearance of the axial strings in some cases may reproduce the jet phenomena. So, the QPOs and jet problems may be connected to each other.

It may appear an obvious objection that the electromagnetic excitations on the Kerr background were investigated many times and by many authors. The principal new feature of our treatment is the restriction by the electromagnetic fields which are *aligned* to the Kerr principal null congruence. Physically, the aligned fields are the only ones which do not conflict with the twistorial analyticity of the Kerr background, which means that they can be used for the formation of the self-consistent solutions of the Einstein-Maxwell system with a guarantee that the resulting metric (if exists) will be of the same type as the Kerr solution, i.e. algebraically special metric with a shear free and geodesic principal null congruence. This very natural demand turns out to be very restricted indeed, and leads to unavoidable appearance of the singular axial strings.

## 2. Disklike sources of the Kerr solution

In the papers [4, 5, 6] the smooth rotating disk-like sources of the Kerr and Kerr-Newman solutions were considered in the Kerr-Schild class of metrics.

Note, that in the related previous works it was suggested by Sakharov, Gliner and Markov to replace the singularity of the non-rotating BHs by the de Sitter source. This approach was developed in further by Markov, Mukhanov & Frolow, Israel & Poisson, Dymnikova, Magli et.al (see references in [4, 6]).

The Kerr-Schild metric has the form

$$g^{\mu\nu} = \eta^{\mu\nu} - 2hk^{\mu}k^{\nu}, \quad (1)$$

where  $k^{\mu}$  is the null vector field  $k_{\mu}k^{\mu} = 0$  which is tangent to the Kerr PNC (principal null congruence).

This is an extremely simple form of metric and one can wonder why this form is able to describe the very complicated Kerr-Newman space-time. To answer this question, first we mention that the function  $h$  has the form<sup>1</sup>

$$h = \frac{Mr - e^2/2}{r^2 + a^2 \cos^2 \theta}, \quad (2)$$

where the oblate coordinates  $r, \theta$  are used on the flat Minkowski background  $\eta^{\mu\nu}$ , so the function  $h$  is singular in the focal points of this system  $r = \cos \theta = 0$ , forming the Kerr singular ring.

The Kerr singular ring is one of the most remarkable peculiarities of the Kerr solution. It is a branch line of space on two sheets: “negative” ( $r < 0$ ) and “positive” ( $r < 0$ ) where the fields change their signs and directions. The vector field  $k^{\mu}$  is the second remarkable structure of the Kerr geometry. This field is lightlike or null,  $k_{\mu}k^{\mu} = 0$ , and forms a twisting principal null congruence (PNC). It represents a vortex lightlike field propagating from the “negative” sheet of the Kerr space onto “positive” one. It is the very important structure. Not only metric, but also vector potential of the Kerr-Newman solution

$$A^{\mu} = \frac{er}{r^2 + a^2 \cos^2 \theta} k^{\mu}, \quad (3)$$

as well as the flow of radiation from the radiative Kerr source  $T_{rad}^{\mu\nu} \sim \Phi(r, \theta)k^{\mu}k^{\nu}$  [7] are determined by this vector field. Its form, shown on the fig.2, shows that the most complicated part of the Kerr metric is concentrated in the form of the field  $k_{\mu}(x)$ .

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<sup>1</sup>Here  $M$  and  $e$  are the mass and charge, and  $a = J/M$  is a density of angular momentum  $J$  per mass unit.

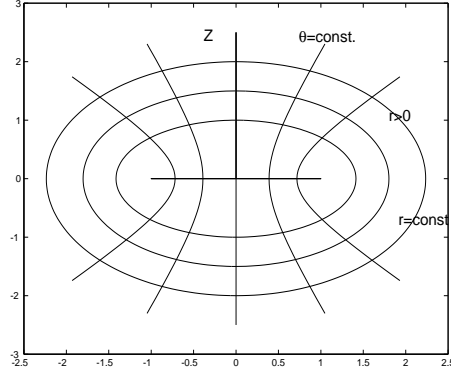


Figure 1: The oblate coordinate system  $r, \theta$ . Coordinate  $r$  cover the space twice: for  $r > 0$  and  $r < 0$ . The focal points are  $r = \cos \theta = 0$ .

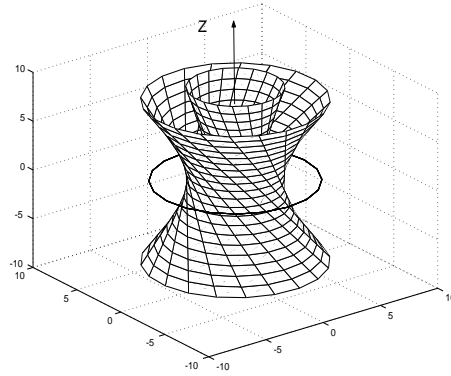


Figure 2: The Kerr singular ring and 3-D section of the Kerr principal null congruence (PNC). Singular ring is a branch line of space, and PNC propagates from “negative” sheet of the Kerr space to “positive” one, covering the space-time twice.

In the case  $e^2 + a^2 \gg m^2$ , corresponding to parameters of elementary particles, the horizons of the Kerr-Newman solution disappear and the Kerr singular ring turns out to be naked. This case is considered in the models of the Kerr spinning particle [4, 6, 3, 8, 9, 10, 11, 14, 13]. To avoid the problems with a twosheeted topology this singularity may be covered by a (disklike) source [4, 6, 11]. The naked Kerr singular ring is also considered in the models of the Kerr spinning particle. In particular as a waveguide providing a circular propagation of an electromagnetic or fermionic wave excitations [3, 9, 10, 16, 12]. It was shown that the Kerr singular ring represents a special type of the folded closed D-string [15]. The value of charge  $e$  is small in astrophysical application, and one can consider as a critical case of the BH formation the condition  $|a/M| < 1$ .

In the approach of the papers [4, 6], the core of the Kerr source has the usual Kerr-Schild form of metric. However, the function  $h$  take the more general form

$$h = f(r)/(r^2 + a^2 \cos^2 \theta), \quad (4)$$

where function  $f(r)$  has the order  $\sim r^4$  at  $r = 0$  to suppress the singularity by  $r = 0$ . Note, that for the nonrotating solutions this case  $f(r) = \alpha r^4$  corresponds just to de Sitter core. The external metric is chosen to be the vacuum Kerr solution. Therefore, if a smooth function  $f(r)$  interpolates between  $f_{core} = \alpha r^4$  and  $f_{external} = Mr$  we obtain a smooth source of the Kerr solution.

This matching may be conveniently displayed on the following graphics.

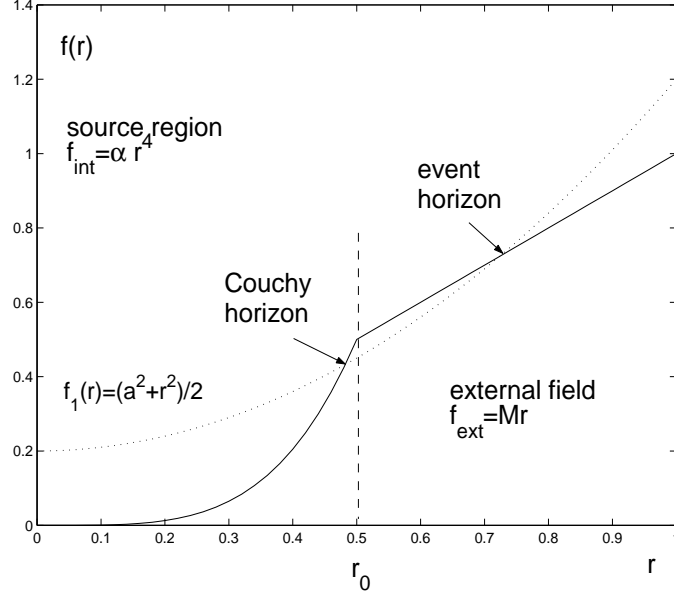


Figure 3: Matching the (rotating) internal “de Sitter” core with the external Kerr-Schild field. The dotted line  $f_1(r) = (r^2 + a^2)/2$  determines graphically the position of horizons as the roots of the equation  $f(r) = f_1(r)$ .

Note, that the condition of smoothness does not depend from parameter  $a$ , so the change of metric due to rotation is only controlled by the denominator of function  $h$  which is determined by the relation of the oblate coordinate system to the Cartesian one

$$\cos \theta = z/r, \quad \sin \theta = \sqrt{\frac{x^2 + y^2}{r^2 + a^2}}. \quad (5)$$

So the sources are represented in the form of the rotating disks with the boundaries  $r = r_0$ . Analysis of the structure of the electromagnetic field near the disk surface has led to the conclusion that the matter of disk must have the superconducting properties. Electromagnetic field near the Kerr singular ring is singular. It means that even if the total charge of the Kerr source is very small the electromagnetic singularity is retained near the ring. It shows that the electromagnetic effects can play very important role in the process of formation of the Kerr disklike source.

**Let us summarize the basic properties of the Kerr disk-like source:**

- the disk is oblate and rigidly rotating,
- the rotation is relativistic,
- the stress-energy tensor has a special condensed vacuum state (de Sitter, flat or anti de Sitter vacua).
- electromagnetic properties of the disk are close to superconductor,
- for the charged sources the strong magnetic and gravitational fields are concentrated on the stringy board of the disk,
- the relation  $J = Ma$ .

This model includes also the smooth analogs of the known shell-like models.

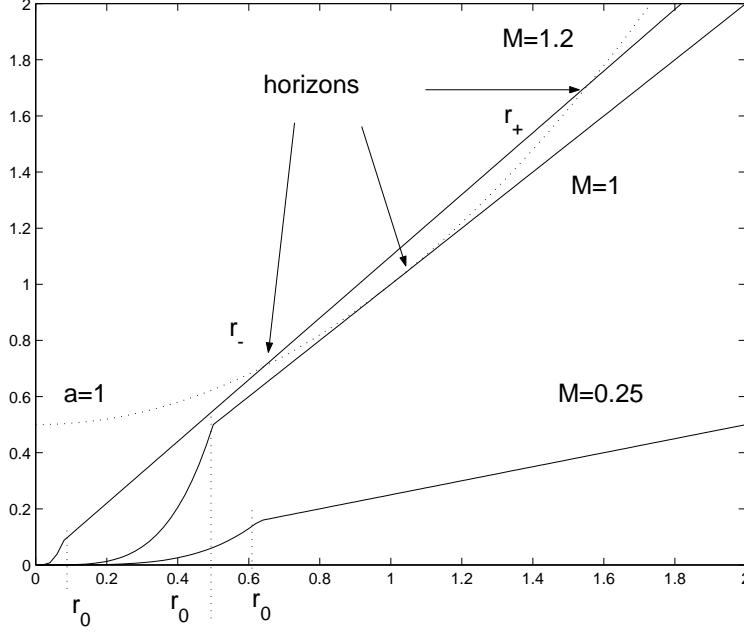


Figure 4: The sources with different masses  $M$  and matter densities  $\rho$ . Sources form the rotating disks with radius  $\sim a$  and thickness  $\sim r_0$  which depends on the matter density  $r_0 = (\frac{3M}{4\pi\rho})^{1/3}$ . The formation of the black hole horizons is shown for  $a^2 < M^2$ .

In the limit of the infinitely thin disk a stringy singularity is formed on the border of disk. This case corresponds to the considered by Israel and Hamity the infinitely thin disklike source.

Twovaluedness of the metric and the field strengths was considered in the field theory as an “Alice” property of the source which was formulated at first for the cosmic “Alice” strings [19, 20]. The “Alice” phenomenon can be connected with the superconducting properties of the sources where the “negative” sheet looks as a mirror image of the “positive” one. This interpretation was also considered for the Kerr source [13]. The “Alice” string is formed on the edge boundary of the thin Kerr disk for  $|a/M| \gg 1$ .

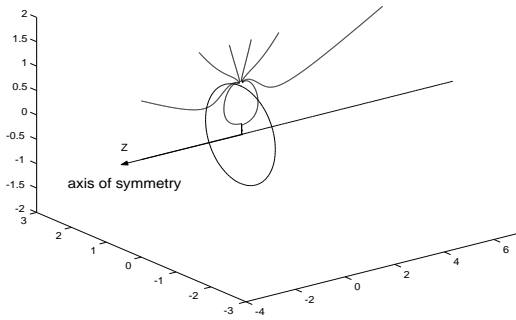


Figure 5: Electric field strengths.

### 3. Stringy structures

In the old paper [9] the Kerr ring was considered as a gravitational waveguide caring the traveling electromagnetic waves which generate the spin and mass of the Kerr spinning particle forming a microgeon with spin. It was conjectured [10] that the Kerr ring represents a closed string, and the traveling waves are the string excitations. It was noted in [12] that in the axidilatonic version of the

Kerr solution the field around this ring is similar to the field around a heterotic string, and recently, it was shown that the Kerr ring is a chiral D-string having an orientifold world-sheet [15].

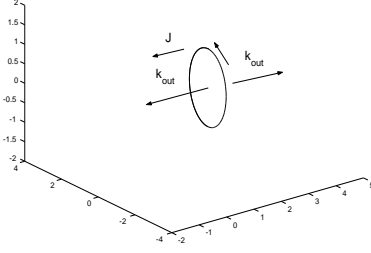


Figure 6: Stringy skeleton of the Kerr spinning particle. Circular string and axial stringy system consisting of two semi-infinite strings of opposite chiralities.

### 3.1. Axial stringy system

Let us consider solutions for traveling waves - electromagnetic excitations of the Kerr circular string. The problem of electromagnetic excitations of the Kerr black hole has been intensively studied as a problem of the quasinormal modes. However, compatibility with the holomorphic structure of the Kerr space-time put an extra demand on the solutions to be aligned to the Kerr PNC, which takes the form  $F^{\mu\nu}k_\mu = 0$ . The aligned wave solutions for electromagnetic fields on the Kerr-Schild background were obtained in the Kerr-Schild formalism [18]. We describe here only the result referring for details to the papers [16, 17]. Similar to the stationary case [18] the general aligned solution is described by two self-dual tetrad components  $\mathcal{F}_{12} = AZ^2$  and  $\mathcal{F}_{31} = \gamma Z - (AZ)_{,1}$ , where function  $A$  has the form

$$A = \psi(Y, \tau)/P^2, \quad (6)$$

$P = 2^{-1/2}(1 + Y\bar{Y})$ , and  $\psi$  is an arbitrary holomorphic function of  $\tau$  which is a complex retarded-time parameter. Function  $Y(x) = e^{i\phi} \tan \frac{\theta}{2}$  is a projection of sphere on a complex plane. It is singular at  $\theta = \pi$ , and one sees that such a singularity will be present in any holomorphic function  $\psi(Y)$ . Therefore, *all the aligned e.m. wave solutions turn out to be singular at some angular direction  $\theta$* . The simplest modes

$$\psi_n = qY^n \exp i\omega_n \tau \equiv q(\tan \frac{\theta}{2})^n \exp i(n\phi + \omega_n \tau) \quad (7)$$

can be numbered by index  $n = \pm 1, \pm 2, \dots$ , which corresponds to the number of the wave lengths along the Kerr ring.

Near the positive  $z^+$  semi-axis we have  $Y \rightarrow 0$  and near the negative  $z^-$  semi-axis  $Y \rightarrow \infty$ .

Omitting the longitudinal components and the radiation field  $\gamma$  one can obtain [16, 17] the form of the leading wave terms

$$\mathcal{F}|_{wave} = f_R d\zeta \wedge du + f_L d\bar{\zeta} \wedge dv, \quad (8)$$

where  $f_R = (AZ)_{,1}$ ;  $f_L = 2Y\psi(Z/P)^2 + Y^2(AZ)_{,1}$  are the factors describing the “left” and “right” waves propagating along the  $z^-$  and  $z^+$  semi-axis correspondingly.

The behavior of function  $Z = P/(r + ia \cos \theta)$  determines a singularity of the waves at the Kerr ring, so the singular waves along the ring induce, via function  $Y$ , singularities at the  $z^\pm$  semi-axis. We are interested in the asymptotical properties of these singularities. Near the  $z^+$  axis  $|Y| \rightarrow 0$ , and by  $r \rightarrow \infty$ , we have  $Y \simeq e^{i\phi} \frac{\rho}{2r}$  where  $\rho$  is the distance from the  $z^+$  axis. Similar, near the  $z^-$  axis  $Y \simeq e^{i\phi} \frac{2r}{\rho}$  and  $|Y| \rightarrow \infty$ . The parameter  $\tau = t - r - ia \cos \theta$  takes near the z-axis the values  $\tau_+ = \tau|_{z^+} = t - z - ia$ ,  $\tau_- = \tau|_{z^-} = t + z + ia$ .

The mode  $n = 0$  describes the stationary electromagnetic field of the Kerr-Newman solution, so it does not contain a modulation of the Kerr circular string and axial singularity is absent.

For  $|n| > 1$  the solutions contain the axial singularities which do not fall off asymptotically, but are increasing that means instability.

The leading singular wave for  $n = 1$ ,

$$\mathcal{F}_1^- = \frac{4qe^{i2\phi+i\omega_1\tau_-}}{\rho^2} d\bar{\zeta} \wedge dv, \quad (9)$$

propagates to  $z = -\infty$  along the  $z^-$  semi-axis.

The leading wave for  $n = -1$ ,

$$\mathcal{F}_{-1}^+ = -\frac{4qe^{-i2\phi+i\omega_{-1}\tau_+}}{\rho^2} d\zeta \wedge du, \quad (10)$$

is singular at  $z^+$  semi-axis and propagates to  $z = +\infty$ . The described singular waves can also be obtained from the potential  $\mathcal{A}^\mu = -\psi(Y, \tau)(Z/P)k^\mu$ . The  $n = \pm 1$  partial solutions  $\mathcal{A}_n^\pm$  represent asymptotically the singular plane-fronted e.m. waves propagating along  $z^+$  or  $z^-$  semi-axis without damping. The corresponding self-consistent solution of the Einstein-Maxwell field equations are described in [16]. They are singular plane-fronted waves having the Kerr-Schild form of metric (1) with a constant vector  $k^\mu$ . For example, the wave propagating along the  $z^+$  axis has  $k^\mu dx^\mu = -2^{1/2}du$ . The Maxwell equations take the form  $\square \mathcal{A} = J = 0$ , where  $\square$  is a flat D'Alembertian, and can easily be integrated leading to the solutions  $\mathcal{A}^+ = [\Phi^+(\zeta) + \Phi^-(\bar{\zeta})]f^+(u)du$ ,  $\mathcal{A}^- = [\Phi^+(\zeta) + \Phi^-(\bar{\zeta})]f^-(v)dv$ , where  $\Phi^\pm$  are arbitrary analytic functions, and functions  $f^\pm$  describe the arbitrary retarded and advanced waves. Therefore, the wave excitations of the Kerr ring lead to the appearance of singular pp-waves which propagate outward along the  $z^+$  and/or  $z^-$  semi-axis.

These axial singular strings are evidences of the axial stringy currents, which are exhibited explicitly when the singularities are regularized. Generalizing the field model to the Witten field model for the cosmic superconducting strings [20], one can show [17] that these singularities are replaced by the chiral superconducting strings, formed by a condensate of the Higgs field, so the resulting currents on the strings are matched with the external gauge field.

The case with two semi-infinite singular strings of opposite chiralities is shown at the Fig.6.

In the cases  $|n| > 1$  singularities cannot be stable since their strength increases with distance. This case may describe some type of jet, see Fig.7.

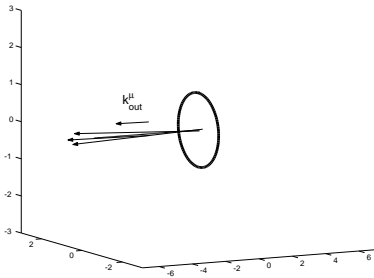


Figure 7: The Kerr circular singularity and a non-stable axial string corresponding to  $|n| > 1$ .

## 4. Conclusion

We have described here two new physical effects which are consequences of the analytic structure of the Kerr geometry and follow from the analysis of the exact solutions for aligned electromagnetic excitations on the Kerr background. These solutions have found application in the models of elementary particles [3], and similar to the Kerr solution itself may find application in astrophysics too.

First of the effects is the possible excitations of the stringy board of the disklike Kerr sources for  $a > M$  with formation of some resonances in a possible harmonic relation.

We should also note that the obtained aligned solutions are indeed independent from the presence (or absence) of the horizon and, consequently, the aligned electromagnetic oscillations do not forbidden for the black holes, too.

The second effect of the aligned solutions is the unavoidable appearance of the axial singularities accompanied by outgoing traveling waves, which may be source of the strong currents leading to formation of astrophysical jets.

The most interesting is the fact that description of the above effects is based on the natural assumption on the analyticity of the Kerr background and does not require implication of some additional assumptions. This talk is based on the collaboration with E. Elizalde, G. Magli and S.R. Hildebrandt. A.B. would also like to thank V. Frolov and F. de Feliche for useful conversations.

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