

SPACE-TIME: EMERGING VS. EXISTING

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The concept of the space-time as emerging in the world phase transition, vs. a priori existing, is put forward. The theory of gravity with two basic symmetries, the global affine one and the general covariance, is developed. Implications for the Universe are indicated.

Introduction

Conventionally, the physical sciences start with the space-time equipped with the metric as the inborn structure. It is proposed in what follows to substitute the metric space-time by the world continuum which possesses only the affine connection. The metric is to emerge spontaneously at the effective level during the world structure formation. Ultimately, this results in the theory of gravity, the Metagravitation, based on two basic symmetries – the global affine one and the general covariance, with the graviton being the tensor Goldstone boson. For more detail, see ref. [1]. This is the physics implementation of the approach to gravity as the nonlinear model $GL(4, R)/SO(1, 3)$ [2, 3]. Generically, the concept developed is much in spirit of the ideas due to E. Schrödinger [4] and I. Prigogin [5].

Affine symmetry

Affine connection Postulate that the predecessor of the space-time is the world continuum equipped only with the affine connection. Let x^μ , $\mu = 0, \dots, 3$ be the world coordinates. In ignorance of the underlying theory, consider all the structures related to the continuum as the background ones. Let $\bar{\psi}^\lambda{}_{\mu\nu}(x)$ be the background affine connection. Let the antisymmetric part of the connection be absent identically. Let P be a fixed but otherwise arbitrary point with the world coordinates X^μ . One can annihilate the connection in this point by adjusting the proper coordinates $\bar{\xi}^\alpha(x, X)$. In the vicinity of P , the connection now becomes

$$\bar{\psi}^\gamma{}_{\alpha\beta}(\bar{\xi}) = \frac{1}{2} \bar{\rho}^\gamma{}_{\alpha\delta\beta}(\bar{\Xi}) (\bar{\xi} - \bar{\Xi})^\delta + \mathcal{O}((\bar{\xi} - \bar{\Xi})^2), \quad (1)$$

with $\bar{\rho}^\gamma{}_{\alpha\delta\beta}(\bar{\Xi})$ being the background curvature tensor in the point P and $\bar{\Xi}^\alpha = \bar{\xi}^\alpha(X, X)$. Let us consider the whole set of the coordinates with the property $\bar{\psi}^\alpha{}_{\beta\gamma}|_P = 0$. The allowed group of transformations of such coordinates is the inhomogeneous general linear group $IGL(4, R)$ (the affine one):

$$(A, a) : \bar{\xi}^\alpha \rightarrow \bar{\xi}'^\alpha = A^\alpha{}_\beta \bar{\xi}^\beta + a^\alpha, \quad (2)$$

with A being an arbitrary nondegenerate matrix. Under these, and only under these transformations, the affine connection in the point P remains to be zero. The group is the global one in the sense that it transforms the P -related coordinates in the global manner, i.e., for all the continuum at once. The respective coordinates will be called the local affine ones. In these coordinates, the continuum in a neighbourhood of the point is approximated by the affinely flat manifold. In particular, the underlying covariant derivative in the affine coordinates in the point P coincides with the partial derivative.

Metarelativity According to the special relativity, the present-day physical laws are invariant relative to the choice of the inertial coordinates, with the symmetry group being the Poincare one. Postulate the principle of the extended relativity, the Metarelativity, stating the invariance relative to the choice of the affine coordinates. The symmetry group extends now to the affine one. The latter is 20-parametric and expand the 10-parameter Poincare group $ISO(1,3)$ by the ten special affine transformations. There being no exact affine symmetry, the latter should be broken to the Poincare symmetry in transition from the underlying to effective level.

Metric Assume that the affine symmetry breaking is achieved due to the spontaneous emergence of the background metric $\bar{\varphi}_{\mu\nu}(x)$ in the world continuum. The metric is assumed to have the Minkowskian signature and to look like

$$\bar{\varphi}_{\alpha\beta}(\bar{\xi}) = \bar{\eta}_{\alpha\beta} - \frac{1}{2}\bar{\rho}_{\gamma\alpha\delta\beta}(\bar{\Xi}) (\bar{\xi} - \bar{\Xi})^\gamma (\bar{\xi} - \bar{\Xi})^\delta + \mathcal{O}((\bar{\xi} - \bar{\Xi})^3). \quad (3)$$

Here one puts $\bar{\eta}_{\alpha\beta} \equiv \bar{\varphi}_{\alpha\beta}(\bar{\Xi})$ and $\bar{\rho}_{\gamma\alpha\delta\beta}(\bar{\Xi}) = \bar{\eta}_{\gamma\delta} \bar{\rho}^{\delta}_{\alpha\beta}(\bar{\Xi})$. The metric (3) is such that the Christoffel connection $\bar{\chi}^\gamma_{\alpha\beta}(\varphi)$, determined by the metric, matches with the affine connection $\bar{\psi}^\gamma_{\alpha\beta}$ in the sense that the connections coincide locally, up to the first derivative: $\bar{\chi}^\gamma_{\alpha\beta} = \bar{\psi}^\gamma_{\alpha\beta} + \mathcal{O}((\bar{\xi} - \bar{\Xi})^2)$. This is reminiscent of the fact that the metric in the Riemannian manifold may be locally approximated, up to the first derivative, by the flat metric. Associated with the background metric, there appears the partition of the amorphous 4-dimensional continuum onto the space and time.

Under the affine symmetry, the background metric ceases to be invariant. But it still possesses an invariance subgroup. Namely, without any loss of generality, one can choose among the affine coordinates the particular ones with $\bar{\eta}_{\alpha\beta}$ being in the Minkowski form $\eta = \text{diag}(1, -1, -1, -1)$. The respective coordinates will be called the background inertial ones. They are to be distinguished from the effective inertial coordinates (see later on). Under the affine transformations, one has

$$(A, a) : \eta \rightarrow \eta' = A^{-1T} \eta A^{-1} \neq \eta, \quad (4)$$

whereas the Lorentz transformations $A = \Lambda$ still leave η invariant. It follows that the subgroup of invariance of η is the Poincare group $ISO(1,3) \in IGL(4, R)$. Under the appearance of the metric, the $GL(4, R)$ group is broken spontaneously to the residual Lorentz one

$$GL(4, R) \xrightarrow{M_A} SO(1, 3), \quad (5)$$

with the translation subgroup being intact. For the symmetry breaking scale M_A , one expects a priori $M_A \sim M_{Pl}$, with M_{Pl} being the Planck mass.

Affine Goldstone boson Attach to the point P the auxiliary linear space T , the tangent space in the point. By definition, T is isomorphous to the Minkowski space-time. The tangent space is the structure space of the theory, whereupon the realizations of the physics space-time symmetries, the affine and the Poincare ones, are implemented. Introduce in T the coordinates ξ^α , the counterpart of the background inertial coordinates $\bar{\xi}^\alpha$ in the space-time. By construction, the connection in the tangent space is zero identically. For the connection in the space-time in the the point P to be zero, too, the coordinates are to be related as $\xi^\alpha = \bar{\xi}^\alpha + \mathcal{O}((\bar{\xi} - \bar{\Xi})^3)$.

Due to the spontaneous breaking, $GL(4, R)$ should be realized in the nonlinear manner, with the nonlinearity scale M_A , the Lorentz symmetry being still realized linearly. The spinor representations of the latter correspond conventionally to the matter fields. In this, the finite dimensional spinors appear only at the level of $SO(1,3)$. The broken part $GL(4, R)/SO(1,3)$ should be realized in the Nambu-Goldstone mode. Accompanying the spontaneous emergence of the metric, there should appear the 10-component Goldstone boson which corresponds to the ten generators of the broken affine transformations. The nonlinear realization of the symmetry G spontaneously broken to the

symmetry $H \subset G$ can be built on the quotient space $K = G/H$, the residual subgroup H serving as the classification group. We are interested in the pattern $GL(4, R)/SO(1, 3)$, with the quotient space consisting of all the broken affine transformations. Let $\kappa(\xi) \in K$ be the coset-function on the tangent space. To restrict κ by the quotient space, one should impose on the representative group element some auxiliary condition, eliminating explicitly the extra degrees of freedom. Under the arbitrary affine transformation $\xi \rightarrow \xi' = A\xi + a$, the coset is to transform as

$$(A, a) : \kappa(\xi) \rightarrow \kappa'(\xi') = A\kappa(\xi)\Lambda^{-1}, \quad (6)$$

where $\Lambda(\kappa, A)$ is the appropriate element of the residual group, here the Lorentz one. This makes the transformed group element compatible with the auxiliary condition. In the same time, by the construction, the Minkowskian η is invariant under the nonlinear realization:

$$(A, a) : \eta \rightarrow \eta' = \Lambda^{-1T}\eta\Lambda^{-1} = \eta \quad (7)$$

(in distinction with the linear representation eq. (4)).

Otherwise, one can abandon any auxiliary condition extending the affine symmetry by the hidden local symmetry $\hat{H} \simeq H$. In the tangent space, we should now distinguish two types of indices: the Lorentz ones, acted on by the local Lorentz transformations $\Lambda(\xi)$, and the affine ones, acted on by the global affine transformations A . Designate the Lorentz indices as a, b , etc, while the affine ones as before α, β , etc. The Lorentz indices are manipulated by means of the Minkowskian η_{ab} (respectively, η^{ab}). The Goldstone field is represented by the arbitrary 4×4 matrix $\hat{\kappa}_a^\alpha$ (respectively, $\hat{\kappa}^{-1\alpha}_a$) which transforms similar to eq. (6) but with arbitrary $\Lambda(\xi)$. The extra Goldstone degrees of freedom are unphysical due to the gauge transformations $\Lambda(\xi)$. This is the linearization of the nonlinear model, with the proper gauge boson being expressed, due to the equation on motion, through $\hat{\kappa}_a^\alpha$ and its derivatives. With this in mind, the abrupt expressions entirely in terms of $\hat{\kappa}_a^\alpha$ and its derivatives are used. The versions differ in the higher orders.

Matter and radiation Put for the matter fields ϕ :

$$\phi(\xi) \rightarrow \phi'(\xi') = \hat{\rho}_\phi(\Lambda)\phi(\xi), \quad (8)$$

with $\hat{\rho}_\phi$ taken in the proper Lorentz representations. As for the gauge bosons, they constitutes one more separate kind of fields, the radiation. By definition, the gauge fields V_α transform under A linearly as the derivative $\partial_\alpha = \partial/\partial\xi^\alpha$. The modified fields $\hat{V}_a = \hat{\kappa}_a^\alpha V_\alpha$ transform as the Lorentz vectors:

$$\hat{V}(\xi) \rightarrow \hat{V}'(\xi') = \Lambda^{-1T}\hat{V}(\xi) \quad (9)$$

and are to be used in the model building.

Nonlinear model To explicitly account for the residual symmetry it is convenient to start with the objects transforming only under the latter symmetry. Clearly, any nontrivial combination of $\hat{\kappa}$ and $\hat{\kappa}^{-1}$ alone transforms explicitly under A . Thus the derivative terms are inevitable. To this end, introduce the Cartan one-form:

$$\hat{\omega} = \eta\hat{\kappa}^{-1}d\hat{\kappa}. \quad (10)$$

The one-form transforms inhomogeneously under the Lorentz group:

$$\hat{\omega}(\xi) \rightarrow \hat{\omega}'(\xi') = \Lambda^{-1T}\hat{\omega}(\xi)\Lambda^{-1} + \Lambda^{-1T}\eta d\Lambda^{-1}. \quad (11)$$

By means of this one-form, one can define the nonlinear derivatives of the matter fields $\hat{D}_a\phi$, the gauge strength \hat{F}_{ab} , as well as the field strength for affine Goldstone boson \hat{R}_{abcd} and its contraction $\hat{R} \equiv \eta^{ab}\eta^{cd}\hat{R}_{abcd}$. The above objects can serve in turn as the building blocks for the nonlinear model $GL(4, R)/SO(1, 3)$ in the tangent space. Postulate the equivalence principle in the sense

that the tangent space Lagrangian should not depend explicitly on the background curvature $\bar{\rho}^d_{abc}$ (cf. eq. (1)). Thus, the Lagrangian may be written as the general Lorentz (and, thus, affine) invariant built of \hat{R} , \hat{F}_{ab} , $\hat{D}_a\phi$ and ϕ . As usually, one restricts himself by the terms containing two derivatives at the most.

Such a Lagrangian being built, one can rewrite it by means of $\hat{\kappa}_a^\alpha$ and $\hat{\kappa}^{-1a}_\alpha$ in the affine terms. This clarifies the geometrical structure of the theory and relates the latter with the gravity. The Lagrangian for the affine Goldstone boson, radiation and matter becomes

$$L = c_g M_A^2 R(\gamma_{\alpha\beta}) + L_r(F_{\alpha\beta}) + L_m(D_\alpha\phi, \phi). \quad (12)$$

Here

$$\gamma_{\alpha\beta} = \hat{\kappa}^{-1a}_\alpha \eta_{ab} \hat{\kappa}^{-1b}_\beta \quad (13)$$

transforms as the affine tensor

$$(A, a) : \gamma_{\alpha\beta} \rightarrow \gamma'_{\alpha\beta} = A^{-1\gamma}_\alpha \gamma_{\gamma\delta} A^{-1\delta}_\beta. \quad (14)$$

It proves that $R(\gamma_{\alpha\beta}) = \hat{R}(\hat{\omega})$ can be expressed as the contraction $R = R^{\alpha\beta}_{\alpha\beta}$ of the tensor $R^{\gamma}_{\alpha\delta\beta} \equiv \eta^{\gamma\gamma'} \hat{\kappa}^{-1a}_\alpha \hat{\kappa}^{-1b}_\beta \hat{\kappa}^{-1d}_\delta \hat{\kappa}^{-1c}_{\gamma'} \hat{R}_{cadb}$, the latter in turn being related with $\gamma_{\alpha\beta}$ as the Riemann-Christoffel curvature tensor with the metric. In this, all the contractions of the affine indices are understood with $\gamma_{\alpha\beta}$ (respectively, $\gamma^{\alpha\beta}$). Similarly, $D_\alpha\phi \equiv \hat{\kappa}^{-1a}_\alpha \hat{D}_a\phi$ looks like the covariant derivative of the matter fields. The gauge strength $F_{\alpha\beta}$ has the usual form containing the partial derivative ∂_α .

General covariance

General Relativity The preceding construction referred to the tangent space T in the given point P . Accept the so defined Lagrangian as that for the space-time, being valid in the background inertial coordinates in the infinitesimal neighbourhood of the point. After multiplying the Lagrangian by the generally covariant volume element $(-\gamma)^{1/2} d^4\Xi$, with $\gamma = \det\gamma_{\alpha\beta}$, one gets the infinitesimal contribution into the action in the given coordinates.

The relation between the background inertial coordinates and the world ones is achieved by means of the background frame $\bar{e}_\mu^\alpha(X)$. In addition, introduce the effective frame related with the background one as

$$e_\mu^\alpha(X) = \hat{\kappa}^{-1a}_\alpha(X) \bar{e}_\mu^\alpha(X). \quad (15)$$

The effective frame transforms as the Lorentz vector:

$$e_\mu(X) \rightarrow e'_\mu(X) = \Lambda(X) e_\mu(X). \quad (16)$$

Due to the local Lorentz transformations $\Lambda(X)$, one can eliminate six components out of e_μ^α , the latter having thus ten physical components. In this terms, the effective metric in the world coordinates is

$$g_{\mu\nu} \equiv \bar{e}_\mu^\alpha \gamma_{\alpha\beta} \bar{e}_\nu^\beta = e_\mu^a \eta_{ab} e_\nu^b. \quad (17)$$

In other words, the frame e_μ^a defines the effective inertial coordinates. Physically, eq. (15) describes the disorientation of the effective inertial and background inertial frames depending on the distribution of the affine Goldstone boson.

By means of e_μ^a , the tangent space quantities transform in the world coordinates to the usual expressions of the Riemannian geometry containing metric $g_{\mu\nu}$ and the spin-connection $\omega_{ab\mu}$. One gets for the total action:

$$I = \int \left(-\frac{1}{2} M_{Pl}^2 R(g_{\mu\nu}) + L_r(F_{\mu\nu}) + L_m(D_\mu\phi, \phi) \right) (-g)^{1/2} d^4X, \quad (18)$$

with $g = \det g_{\mu\nu}$. In the above, the constants are adjusted so that $c_g M_A^2 = 1/2 M_{Pl}^2 \equiv 1/(16\pi G_N)$, with G_N being the Newton's constant. Thereof, one arrives at the General Relativity (GR) equation for gravity. Formally, the Riemannian geometry is valid at any space-time intervals. Nevertheless, its accuracy worsen at the smaller intervals, requiring more terms for the decomposition in the ratio of energy to the symmetry breaking scale M_A , the property characteristic of the effective theory. Thus, the Planck mass $M_{Pl} \sim M_A$ is a kind of the inverse minimal length in the nature.

Beyond the GR A priori, there are admissible the arbitrary Lagrangians in the tangent space, satisfying the affine symmetry. The theory, generally covariant in the tangent space, results in the theory generally covariant in the space-time. Under extension of the tangent space Lagrangians beyond the general covariance, the theory in the space-time ceases to be generally covariant, too. The general covariance moderates the otherwise arbitrary theories. Relative to the general coordinate transformations, the GR extensions divide into the inequivalent classes, each of which is characterised by the particular set of the background parameter-functions. A priori, no one of the sets is preferable. Which one is suitable (if any), should be determined by observations. Each class consists of the equivalent extensions related by the residual covariance group. Among the inequivalent extensions, there can be implemented the natural hierarchy, according to whether the affine symmetry is explicitly violated or not.

The Metauniverse Suppose that the origin of the Universe lies in the actual transition between the two phases of the world continuum, the affinely connected and metric ones. This transition is thus the “Grand Bang”, the origin of the Universe in line with the very space-time. There is conceivable the appearance, as well as disappearance and coalescence, of the various regions of the metric phase inside the affinely connected one (or v.v.). These metric regions are to be associated with the multiple universes, constituting collectively the Metauniverse. Our Universe being one of a lot, may clarify the famous fine tuning problem.

Conclusion

The concept of the metric space-time as appearing in the processes of the world structure formation, but not as a priori given, is to change drastically the future comprehension of the space-time, gravity and the Universe. There lies a long way ahead to achieve this goal.

References

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