

ENERGY IN THEORY OF GRAVITY AND ESSENCE OF TIME

Ivanhoe B. Pestov

*Bogoliubov Laboratory of Theoretical Physics, Joint Institute for Nuclear Research,
Dubna, Russia*

In the framework of the field theory it is shown that a time (viewed as a scalar temporal field) is an internal property of the physical system, which defines its causal structure and evolution. A new concept of internal time allows to solve the energy problem in General Relativity and predicts the existence of matter outside the time. It is demonstrated that introduction of the temporal field permits to derive the physical laws of the electromagnetic field(the general covariant four dimensional Maxwell equations for the electric and magnetic fields) from the geometrical equations of this field. It means that the fundamental physical laws are in full correspondence with the essence of time. On this ground, from the geometrical laws of the gravitational field the physical evolution equations of this field are derived. Two characteristic solutions of these equations are obtained (including the Schwarzschild solution).

1. Introduction

In the theory of gravitational field the problems connected with the energy conservation exist in a literal sense since the time of its creation when Einstein set up the problem of including gravity into the framework of the Faraday concept of field. Thorough and deep analysis of the problem of gravity field given by him in the works [1, 2] enables to formulate in what follows the key principles of gravity physics (General Relativity). However, till now in the framework of these principles there is no adequate solution to the energy conservation problem [3, 4]. This topic is still the subject of active research and as this implies, still only incompletely understood. This means that the theory of gravity field is not unique. The fact that the theory is not unique is a bad feature, because if theory is not unique it is clearly missing some essential elements.

In the presented paper we find these elements and give a simple solution of the problem in question, which is based on the connection between the time and energy and necessarily follows from the first principles of General Relativity if one puts them into definite logical sequence. The energy conservation means that the rate of change with time of total energy density of gravity field and all other fields is equal to zero and hence energy density represents the first integral of the system.

New understanding of time presented here will have implications in quantum gravity. In fact, a major conceptual problem in this field is the notion of time and how it should be treated. The importance of this issue was recognized at the beginning of the history of quantum gravity [5], but the problem is still unresolved and has drawn recently an increasing attention (see, for example [6], [7] and [8] and further references therein).

The paper is organized as follows. In §2 we formulate the fundamentals of general covariant theory of time with the guiding idea that the manifold is the main notion in physics and that time itself is a scalar field on the manifold which defines the evolution of the full system of fields being

one of them. The system of interacting fields is considered to be full if the gravitational field is included in it.

All known dynamical laws of nature have the following form: the rate of change with time of certain quantity equals to the results of action of some operator on this quantity. So, a general covariant definition of rate of change with time of any field is one of the main results of the theory of time presented here. It is also a starting point and relevant condition for the consideration of the concept of evolution and of the problem how to write the field equations in the general covariant evolution form.

In §3 the connection of the temporal field with the Einstein gravitational potential is established. This gives the possibility to derive the geometrical laws of the gravitational field, general covariant law of energy conservation and produce the general covariant expression for the energy density of the gravity field. The whole approach relies on the concepts of genuine Riemann geometry and the physical and geometrical sense of Lorentz signature is recognized. To demonstrate a concrete application of general theory, in §4 it is shown how evolution equations for the vectors of electric and magnetic fields in the four dimensional general covariant form can be derived from the geometrical equations for the bivector of electromagnetic field. Through this it is shown that the original Maxwell equations (which as a matter of fact express the fundamental physical law) are in full correspondence with fundamentals of the theory of time. In §5 the notion of the momentum of the gravitational field is introduced and dynamical equations of this field is deduced from the geometrical laws established in §3. In §6 the exact solutions of these equations are considered. One of them describes the gravitational field of pointlike gravitational charge and other one represents the field of the homogenous and isotropic distribution of the gravitational energy. Some concluding remarks are given in §7.

2. Central role of time in gravity theory

In accordance with the principles of gravity physics, in this section the fundamentals of the theory of time are formulated.

According to Einstein, in presence of gravitational field all the systems of coordinates are on equal footing and in general, coordinates have neither physical nor geometrical meaning. Thus, in the gravity theory the coordinates play the same role as the Gauss coordinates in his internal geometry of surfaces from which all buildings of the modern geometry are grown. Hence, one needs to construct the internal theory of physical fields analogous to the Gauss internal geometry of surfaces. In other words, the problem is how the modern differential geometry transforms into the physical geometry.

The fields characterize the events and fill in the geometrical space. In view of what has been said above, this space is a four dimensional smooth manifold because this structure does not distinguish intrinsically between different coordinate systems (the principle of general covariance is naturally included into this notion). Hence it follows that the notion of smooth manifold is the primary issue not only in differential geometry but also in the theoretical physics (dealing with gravitational phenomena. This means that all other definitions, notions and laws should be introduced into the theory through the notion of smooth manifold. Indeed if some notion, definition or law is in agreement with the structure of smooth manifold, then they are general covariant, i.e., do not depend on the choice of coordinate system.

Definition of manifold is considered to be known and we only notice that all information on this topic can be found for example in reference [3] or [9]. For our purposes it is enough to keep in mind that all smooth manifolds can be realized as surfaces in the Euclidean space. In what follows we shall consider only four dimensional manifolds. That is evident from the physical point of view. However there is also a deep purely mathematical reason for this choice. Smooth manifold consists

of topological manifold and differential structure defined on it. It is known [9], that a topological manifold always admits differential structure if and only if its dimension is not larger than four.

It is clear that, in general, manifold should be in some relation with its material content i.e., the fields. In view of this it is very important to know how material content designs manifold. If we take the point of view that manifold apriori is arena for the physical events, then it is natural to select simplest manifold, for example, manifold of special theory of relativity. However, it is evident that nature of gravity field is not compatible with an apriori defined manifold.

It can be shown that manifold as a surface in the Euclidean space is designed by the covariant symmetrical tensor field g_{ij} on the manifold, for which adjoined quadratic differential form (Riemann metric)

$$ds^2 = g_{ij} du^i du^j \quad (1)$$

is positive definite. Thus, covariant positive definite symmetrical tensor field $g_{ij}(u)$ is the necessary element of any general covariant intrinsically self-consistent physical theory. Here we only give the defining system of differential equations

$$g_{ij}(u^1, u^2, u^3, u^4) = \delta_{ab} \frac{\partial F^a}{\partial u^i} \frac{\partial F^b}{\partial u^j}, \quad a, b = 1, \dots, 4 + k, k \geq 0$$

avoiding any detailed consideration. If the functions $g_{ij}(u^1, u^2, u^3, u^4)$ are known in local system of coordinates u^1, u^2, u^3, u^4 , then solving this system of equations we obtain the functions $F^a(u^1, u^2, u^3, u^4)$ and hence the region of manifold, defined by the equations $x^a = F^a(u^1, u^2, u^3, u^4)$, where x^a are the Cartesian coordinates of embedding Euclidean space.

An important conclusion that follows from this consideration is that there is one and only one way for the other fields to design manifold which can be explained as follows. Let u^1, u^2, u^3, u^4 be a local system of coordinates in the vicinity of some point of an abstract manifold. Let us determine in such a vicinity the system of differential equations that connect basis field $g_{ij}(u)$ (genuine Riemann metric) with other ones. Solving this system of equations we find field $g_{ij}(u)$ and by doing so, we design a local manifold of physical system in question. In what follows, a smooth manifold that corresponds to a physical system will be called a physical manifold.

For further consideration of the principles of internal field theory we simply note that there is a fundamental difference between physics and geometry. In geometry there is no motion that is tightly connected with the concept of time. Thus, to be logical, we need to introduce time into the theory using its first principles. Since, in general, coordinates have no physical sense, time can be presented as a set of functions of four independent variables (or in more strict manner as a geometrical object on the manifold). It is quite obvious from the logical point of view.

We put forward the idea that the time is a scalar field on the manifold. By this we get a simple answer to the question with long standing history "What is time?" It should be emphasized, that temporal field (together with other fields) designs manifold as it was explained above but it has also another functions which will be considered below.

Temporal field with respect to the coordinate system u^1, u^2, u^3, u^4 in the region U of smooth four dimensional manifold M is denoted as $f(u) = f(u^1, u^2, u^3, u^4)$. If the temporal field is known, then to any two points p and q of manifold one can put in correspondence an interval of time

$$t_{pq} = f(q) - f(p) = \int_p^q \partial_i f du^i. \quad (2)$$

Unlike time, space is not an independent entity. Instead of space we shall consider space cross-sections of the manifold $f^{-1}(t)$, which are defined by the temporal field. For the real number t , space cross-section is defined by the equation

$$f(u^1, u^2, u^3, u^4) = t. \quad (3)$$

One can call the number t "the height" of the space cross-section of manifold. If a point p belongs to the space cross-section $f^{-1}(t_1)$, and a point q to the space cross-section $f^{-1}(t_2)$, then the time interval (2) is equal to the difference of the heights $t_{pq} = t_2 - t_1$. It is clear that $t_{pq} = 0$ if p and q belong to the same space cross-section. Thus, to the every segment of curve one can put in correspondence a time interval.

Given the general covariant definition of time, one should show that it is constructive in all respects. First of all we consider how the temporal field defines the form of physical laws. It is known that the general form of physical laws is very simple and is based on the following recipe: the rate of change with time of a certain quantity is equal to the result of action of some operator on this quantity. To be concrete, let us consider Maxwell equations. We know that the rate of change with time of electrical and magnetic fields enter the dynamical equations of electromagnetic field. Thus, we need to give a general covariant definition of the rate of change with time of any field and in particular this definition should be applicable for the case of electromagnetic field.

This problem has fundamental meaning, since it is impossible to speak about physics when one has no mathematically rigorous definition of evolution. It is clear that correct general covariant definition should be conjugated with simple condition: if the rate of change with time of some field is equal to zero in one coordinate system, then in any other coordinate system the result will be the same.

On the manifold there is only one general covariant operation that can be considered as a basis for the definition of the rate of change with time of any field quantity. This general covariant operation is called derivative in given direction and is defined by the vector field on the manifold and the structure of the manifold itself. Thus, the problem is to connect the temporal field $f(u^1, u^2, u^3, u^4)$ with some vector field $t^i(u^1, u^2, u^3, u^4)$. Since a temporal field is a scalar one, the partial derivatives define covector field $t_i = \partial_i f$. Now, the following definition becomes self-evident: the gradient of temporal field (or the stream of time) is the vector field of the type

$$t^i = (\nabla f)^i = g^{ij} \frac{\partial f}{\partial u^j} = g^{ij} \partial_j f = g^{ij} t_j, \quad (4)$$

where g^{ij} are the contravariant components of the Riemann metric (1). The gradient of the field of time defines the direction of the most rapid increase (decrease) of the field of time. We define now the rate of change with time of some quantity as the derivative in the direction of the gradient of the field of time and denote this operation by the symbol D_t .

Let us find the expression for the rate of change with time of the temporal field itself. We have,

$$D_t f = t^i \partial_i f = g^{ij} \partial_i f \partial_j f.$$

Since $D_t f$ is a general covariant generalization of the evident relation $\frac{d}{dt} t = 1$, the temporal field should obey the fundamental equation

$$(\nabla f)^2 = g^{ij} \frac{\partial f}{\partial u^i} \frac{\partial f}{\partial u^j} = 1. \quad (5)$$

Equation (5) means that the rate of change with time of the temporal field is a constant quantity, the most important constant of the theory. From the geometrical point of view the equation (5) simply shows that the gradient of the temporal field is unit vector field on the manifold with respect to the scalar product that is defined as usual by the metric (1), $(V, W) = g_{ij} V^i W^j$.

It should be noted that the equation $(\nabla f)^2 = 1$ is the main equation of the geometrical optics. In view of this one can consider equation (5) as the equation of 4-optics. This analogy can be useful for consideration of some special problems in the theory of time.

The rate of change with time of the symmetrical tensor field is given by the expression

$$D_t g_{ij} = t^k \frac{\partial g_{ij}}{\partial u^k} + g_{kj} \frac{\partial t^k}{\partial u^i} + g_{ik} \frac{\partial t^k}{\partial u^j} . \quad (6)$$

Similar formulas can be presented for any other geometrical quantities. In mathematical literature the derivative with respect to the given direction is usually called the Lie derivative. Thus, one can say that the rate of change with time of any field is the Lie derivative with respect to the direction of the stream of time.

Consider the notion of time reversal and the invariance with respect to this symmetry that is very important for what follows. It is almost evident that in general covariant form the time reversal invariance means that theory is invariant with respect to the transformations

$$t^i \rightarrow -t^i . \quad (7)$$

It is clear that theory will be time reversal invariant if the gradient of temporal fields will appear in all formulae only as an even number of times, like $t^i t^j$.

Within the scope of special relativity and quantum mechanics, time and energy are tightly connected. It is natural to suppose that in gravity physics the link between time and energy even more deep and energy conservation follows from the invariance of the Lagrangian theory with respect to the transformations

$$f(u) \rightarrow f(u) + a, \quad (8)$$

where a is arbitrary constant. This invariance means that all the space sections of manifold of system in question are equivalent.

Einstein himself put in correspondence to the gravity field symmetrical tensor field \tilde{g}_{ij} , which is characterized by the condition that adjoined quadratic differential form

$$d\tilde{s}^2 = \tilde{g}_{ij} du^i du^j, \quad (9)$$

has the signature of the interval in special relativity. In accordance with the principle of gravity physics discussed above, it is natural to assume that Einstein's interval (9) has a structure that is defined by the form-generating field g_{ij} (Riemann's metric (1)) and temporal field. If disclosed, this structure will give a simple method to introduce temporal field into the equations of gravitational physics. It is quite evident that the metric (1) has an Euclidean signature and hence it has no structure by definition. To be transparent in our consideration, let us remind a simple mathematical construction. One can consider tensor field S_j^i as linear transformation $\bar{V}^i = S_j^i V^j$ in the vector space in question. If the operator S is selfadjoint, that is $(V, SW) = (SV, W)$, then it is always possible to introduce the scalar product associated with operator S via the formula $\langle V, V \rangle = (V, SV)$. It is clear that associated scalar product will be in general indefinite and with respect to the initial scalar product it has a structure. Thus, in general, the connection between the forms (1) and (9) is given by the relation $\tilde{g}_{ij} = g_{ik} S_j^k$. We shall give now simple expression for the operator S_j^i , which defines the Einstein interval and is a simple generalization of time reversal invariance (7) as a fundamental physical principle. To this end, we shall define T -symmetry in the space of the vector field in order to have transformation (7) as a particular case. We say that vector fields \tilde{V}^i and V^i are T -symmetrical, if the sum of this fields is orthogonal to the gradient of temporal field and their difference is collinear to it, $(\tilde{V}^i + V^i)t_i = 0$, $\tilde{V}^i - V^i = \lambda t^i$. We have, $\tilde{V}^i = V^i - 2n^i(V, n) = (\delta_j^i - 2n^i n_j)V^j$, where

$$n^i = \frac{t^i}{\sqrt{(t, t)}}, \quad (t, t) = g_{ij} t^i t^j .$$

From this formula it follows that the fields t^i and $-t^i$ are T -symmetrical and hence the definition of T -symmetry given in the space of the vector field is correct. From the above consideration it follows

that S is operator of T - symmetry that is $S_j^i = \delta_j^i - 2n^i n_j$ and hence for the Einstein's potential we obtain the following expression

$$\tilde{g}_{ij} = g_{ik}(\delta_j^k - 2n^k n_j) = g_{ij} - 2n_i n_j, \quad (10)$$

which is invariant under the transformations (7). The contravariant components of the tensor field \tilde{g}_{ij} are $\tilde{g}^{ij} = g^{ij} - 2n^i n^j$, $\tilde{g}^{ik} \tilde{g}_{jk} = \delta_j^i$.

Let us give the physical meaning of the Einstein's scalar product associated with T - symmetry. Since

$$\langle V, V \rangle = (V, V) - 2(V, n)^2 = |V|^2(1 - 2 \cos^2 \phi) = -|V|^2 \cos 2\phi,$$

where ϕ is the angle between the vectors V^i and n^i , the Einstein's scalar product is indefinite and can be positive, negative or equal to zero according to the value of the angle ϕ . In particular, $\langle V, V \rangle = 0$, if $\phi = \pi/4$. Thus, the Einstein's scalar product is time reversal invariant and permits to classify all the vectors depending on which angle they form with the gradient of the temporal field. As we see, the temporal field and T -symmetry define the Einstein's form (9) as the metric of the normal hyperbolic type. Hence, the gradient of the temporal field defines the causal structure on the physical manifold and can be identified with it. It is the physical meaning of the Einstein's interval and mechanism of formation of physical pseudo-Euclidian metric as well. It should be noted that in our reasonings we did not use the equation (5). This will become clear later under the consideration of the variational principle.

Now we shall show that the causal structure (the gradient of the temporal field) can be reduced to the canonical form $(0, 0, 0, 1)$ by the suitable coordinate transformation at once in all points of some (may be small) patch of any point on the physical manifold. Local coordinates with respect to which gradient of the temporal field has the form $(0, 0, 0, 1)$ will be called compatible with causal structure or intrinsic coordinates. It should be noted very important significance of the system of coordinates compatible with causal structure since it is similar to the Darboux system of coordinate in the theory of symplectic manifolds which is a geometrical basis for the Hamilton mechanics. What is more, there are transformations of coordinates which conserve the causal structure and these transformations are analogous to the canonical transformations in Hamiltonian mechanics.

Geometrically the stream of time is defined as a congruence of lines (lines of time) on the manifold. Analytically the lines of time are defined as the solutions of the autonomous system of differential equations

$$\frac{du^i}{dt} = g^{ij} \frac{\partial f}{\partial u^j} = g^{ij} \partial_j f = (\nabla f)^i, \quad (i = 1, 2, 3, 4). \quad (11)$$

Let

$$u^i(t) = \varphi^i(u_0^1, u_0^2, u_0^3, u_0^4, t) = \varphi^i(u_0, t) \quad (12)$$

be the solution to equations (11) with initial data $\varphi^i(u_0, t_0) = u_0^i$ so that

$$\partial \varphi^i(u_0, t_0) / \partial u_0^j = \delta_j^i.$$

Substituting $u^i(t) = \varphi^i(u_0, t)$ into the function $f(u^1, u^2, u^3, u^4)$ we obtain $p(t) = f(\varphi(u_0, t))$. Differentiating this function with respect to t , by virtue of (5) and (11), one finds $dp(t)/dt = 1$. It leads to $f(\varphi(u_0, t)) = t - t_0 + f(u_0)$. Suppose that all initial data belong to the space section $f(u_0^1, u_0^2, u_0^3, u_0^4) = t_0$. Rewriting this relation in the parametric form $u_0^i = \psi^i(x^1, x^2, x^3)$, Eqs. (12) can be written as the system of relations

$$u^i = \phi^i(x^1, x^2, x^3, t). \quad (13)$$

The functions (13) have continuous partial derivatives with respect to variable x^1, x^2, x^3, t and their functional determinant is not equal to zero. Hence the functions $\phi^i(x^1, x^2, x^3, t)$ design intrinsic system of coordinate of the dynamical system in question. Now one can show that in such a system of coordinates the covariant and contravariant components of the gradient of the temporal field and some components of the field g_{ij} take a simple numerical value

$$t^i = (0, 0, 0, 1) = t_i, \quad g_{44} = g^{44} = 1, \quad g_{\mu 4} = g^{\mu 4} = 0, \quad (\mu = 1, 2, 3)$$

and hence this coordinate system is compatible with causal structure. Therefore, it follows that in the system of coordinates compatible with causal structure, metric (1) takes the form

$$ds^2 = g_{\mu\nu}(x^1, x^2, x^3, t)dx^\mu dx^\nu + (dt)^2, \quad \mu, \nu = 1, 2, 3, \quad (14)$$

since $t_i = g_{ij}t^j = g_{i4}$.

So, for any point of physical manifold we can indicate a local coordinate neighborhood with the coordinates compatible with causal structure. Covering manifold with such charts we get atlas on the manifold that is compatible with its causal structure. In this atlas the field equations have the most simple form in the sense that all components of the gradient of temporal field take numerical values. We see that causal structure is analogous to the symplectic structure of the Hamiltonian mechanics.

From the above consideration it also follows that variable t , parametrizing the line of time, can be considered as coordinate of time of the physical system in question. This name is justified particularly by the fact that the rate of change with time of any field is equal to the partial derivative with respect to t , i.e., $D_t = \partial/\partial t$ in the system of coordinate compatible with causal structure (see for example (6)). This is very important point since we see that the intrinsic coordinates give the possibility to write all equations in the form analogous to the canonical form of the Hamilton equations.

Furthermore, if we reverse time putting $\tilde{t}^i = -t^i$, then the lines of time will be parametrized by new variable \tilde{t} . From the equations (11) it is not difficult to derive that there is one-to-one and mutually continuous correspondence between the parameters t and \tilde{t} given by the relation $\tilde{t} = -t$. From here it is clear that in the system of coordinate defined by the time reversal, the variable $-t$ will be the coordinate of time. Thus, the general covariant definition of time reversal given by (7) is adjusted with the familiar definition that is connected with the transformation of coordinates. Finally, we note one more important fact related to the intrinsic coordinates. Transformations (8) take the well-known form $t \rightarrow t+a$ in the system of coordinates compatible with the causal structure.

3. Geometrical laws of the gravitational field

Consideration made above gives the evident geometrical method of constructing Lagrangians in gravity physics. It is clear that geometrical laws of the gravitational field are defined by the Lagrangian $L = \tilde{R}$, where \tilde{R} is a scalar that is constructed from the Einstein's gravitational potential in the standard way.

Since the Einstein interval is defined by the two fields connected by the equation (5), it is necessary to pay special attention when deriving the equations of gravitation field. A standard method is to incorporate the constraint (5) via a Lagrange multiplier $\varepsilon = \varepsilon(u)$, rewrite the action density for gravity field in the form $L_g = \tilde{R} + \varepsilon(g^{ij}t_it_j - 1)$ and treat the components of the fields g_{ij} and f as independent variables.

Let us consider the action

$$A = \frac{1}{2} \int \tilde{R} \sqrt{g} d^4u + \int L_m(\tilde{g}, F) \sqrt{g} d^4u + \frac{1}{2} \int \varepsilon(g^{ij}t_it_j - 1) \sqrt{g} d^4u, \quad (15)$$

where $g = \text{Det}(g_{ij}) > 0$ and $L_m(\tilde{g}, F)$ is the Lagrangian density of the system of other fields F which incorporates the Einstein's gravitational potential in the conventional form. Such geometrical method of introduction of causal structure into the geometrical equations of fields does not require special explanation but it is not evident how from geometrical laws to derive general covariant dynamical laws (the form of which was defined earlier). This problem will be solved here in the two important cases. It will be demonstrated how from the geometrical laws of the electromagnetic field to derive the Maxwell equations for the electric and magnetic fields in general covariant four-dimensional form (notice that this problem remains unresolved up-to-date). In other case we shall derive general covariant dynamical laws of the gravitational field from the geometrical laws of this field.

Since Einstein's gravitational potential is the function of g_{ij} and f , it is necessary to use the chain rule. We have, $\delta\tilde{R} = \tilde{g}^{ij}\delta\tilde{R}_{ij} + \tilde{R}_{ij}\delta\tilde{g}^{ij}$. Further we denote the Christoffel symbols as Γ_{jk}^i and $\tilde{\Gamma}_{jk}^i$ belonging to the fields g_{ij} and \tilde{g}_{ij} , respectively. In what follows, the covariant derivatives with respect to Γ_{jk}^i ($\tilde{\Gamma}_{jk}^i$) will be denoted as ∇_j ($\tilde{\nabla}_j$). One can show that $\tilde{g}^{ij}\delta\tilde{R}_{ij}$ can be omitted as a perfect differential. Varying now \tilde{g}^{ij} , we get

$$\delta\tilde{g}^{ij} = \delta g^{ij} + P_{kl}^{ij}\delta g^{kl} + Q^{ijk}\partial_k\delta f,$$

where

$$P_{kl}^{ij} = 2n^i n^j n_k n_l - n^i(n_k\delta_l^j + n_l\delta_k^j) - n^j(n_k\delta_l^i + n_l\delta_k^i),$$

$$Q^{ijk} = \frac{2}{\sqrt{(t, t)}}(2n^i n^j n^k - n^i g^{jk} - n^j g^{ik}).$$

A tensor field P_{kl}^{ij} is symmetrical in covariant and contravariant indices. Let $G_{ij} = \tilde{R}_{ij} - \frac{1}{2}\tilde{g}_{ij}\tilde{R}$ be the Einstein's tensor and further we put $\delta L_F = \frac{1}{2}M_{ij}\delta\tilde{g}^{ij}$, and introduce by the standard way the energy-momentum tensor $T_{ij} = M_{ij} - \tilde{g}_{ij}L_F$. Observing that

$$\tilde{g}_{ij} + \tilde{g}_{kl}P_{ij}^{kl} = g_{ij}, \quad g_{ij}Q^{ijk} = n_i n_j Q^{ijk} = 0,$$

it is easy to verify that a total variation of the action can be presented in the following form

$$\delta A = \frac{1}{2} \int (A_{ij})\delta g^{ij} + B\delta f + (g^{ij}t_i t_j - 1)\delta\varepsilon\sqrt{g}d^4u, \quad (16)$$

where

$$A_{ij} = G_{ij} + G_{kl}P_{ij}^{kl} + T_{ij} + T_{kl}P_{ij}^{kl} + \varepsilon t_i t_j - \varepsilon(g^{kl}t_k t_l - 1)g_{ij},$$

$$B = -\nabla_k((G_{ij} + T_{ij} - \varepsilon t_i t_j)Q^{ijk}) - 2\nabla_k(\varepsilon t^k).$$

One can consider tensor P_{ij}^{kl} as operator P acting in the space of symmetrical tensor fields. The characteristic equation of this operator has the form $P^2 + 2P = 0$, and hence $(P + 1)^2 = 1$. Thus, operator $P + 1$ is inverse to itself. Since $t_i t_j + t_k t_l P_{ij}^{kl} = -t_i t_j$, from (16) it follows that in framework of considered here concept of time the geometrical Einstein equations have the form

$$G_{ij} + T_{ij} = \varepsilon\partial_i f\partial_j f, \quad g^{ij}\partial_i f\partial_j f = 1, \quad (17)$$

$$\nabla_k(\varepsilon t^k) = 0. \quad (18)$$

The equations (17) constitute the full system of geometrical equations of the gravitational field. As it is shown above, these equations emerge from the first principles of gravity physics.

Equation (18) expresses the law of energy conservation in gravitational physics which, evidently, is general covariant. To make sure that we indeed deal with conservation of energy, it is sufficient to

figure out that action (15) is invariant with respect to transformation (8) and hence the equation (18) results also from the Noether's theorem. It is also clear that the Lagrange multiplier ε has a physical meaning of energy density of the system in question. From the Eqs. (17) it follows that

$$\varepsilon = G_{ij}t^i t^j + T_{ij}t^i t^j = \varepsilon_g + \varepsilon_m, \quad (19)$$

where $\varepsilon_g = G_{ij}t^i t^j = \frac{1}{2}\tilde{R}_{ij}g^{ij}$ is energy density of the gravitational field and $\varepsilon_m = T_{ij}t^i t^j$ is energy density of other fields. Consider the law of energy conservation from the various points of view. First of all we consider link between (17) and (18). The so-called local energy conservation is written as follows $\tilde{\nabla}_i T^{ij} = 0$, where $T^{ij} = T_{kl}\tilde{g}^{ik}\tilde{g}^{jl}$. These equations are fulfilled on the equations of the fields F that contribute to the energy-momentum tensor. Since $\tilde{\nabla}_i G^{ij} = 0$ identically, from (17) it follows that

$$\tilde{\nabla}_i T^{ij} = \varepsilon t^i (\tilde{\nabla}_i t^j) + t^j \tilde{\nabla}_i (\varepsilon t^i).$$

Since $\tilde{\nabla}_i (\varepsilon t^i) = \nabla_i (\varepsilon t^i)$ and $\tilde{\nabla}_i t^j = 0$, finally we have

$$\tilde{\nabla}_i T^{ij} = t^j \nabla_i (\varepsilon t^i).$$

In view of this the energy conservation law can be treated as the condition of compatibility of the field equations. In this sense, the law of energy conservation is analogous to the law of charge conservation.

Show that rate of change with time of the energy density $D_t(\sqrt{g}\varepsilon)$ equal to zero $D_t(\sqrt{g}\varepsilon) = 0$ and hence this quantity is a first integral of the system in question. We have $D_t(\sqrt{g}\varepsilon) = t^i \partial_i(\sqrt{g}\varepsilon) + \sqrt{g}\varepsilon \partial_i t^i = \partial_i(\sqrt{g}\varepsilon t^i)$. Since $\sqrt{g}\nabla_k(\varepsilon t^k) = \partial_i(\sqrt{g}\varepsilon t^i)$, from the law of energy conservation (18) it follows that energy density is the first integral of the system

$$D_t(\sqrt{g}\varepsilon) = 0.$$

In the system of coordinates, compatible with causal structure, this equation has a more customary form

$$\frac{\partial}{\partial t}(\sqrt{g}\varepsilon) = 0.$$

The Eqs. (17) are geometrical and from the physical point of view it is not evident that they express the fundamental dynamical laws of the nature. This is not a simple exercise to derive from (17) a physical laws of the gravitational field and we first consider more easy (but very important as well) problem. We derive the general covariant Maxwell equations from some natural geometrical equations.

4. Time in the theory of electric and magnetic fields

Let A_i be the vector potential of the electromagnetic field. We define the gauge invariant tensor of electromagnetic field as usual $F_{ij} = \partial_i A_j - \partial_j A_i$ and according to the principle of gravity physics introduce temporal field into the theory of electromagnetic field through the gauge invariant and general covariant Lagrangian

$$L_{em} = \frac{1}{4} F_{ij} F_{kl} \tilde{g}^{ik} \tilde{g}^{jl}.$$

with the energy-momentum tensor

$$T_{ij} = F_{ik} F_{jl} \tilde{g}^{kl} - \tilde{g}_{ij} L_{em}. \quad (20)$$

The equations for the tensor F_{ij} can be written in the form

$$\nabla_i \tilde{F}^{ij} = 0, \quad \nabla_i \tilde{F}^{ij} = 0, \quad (21)$$

where

$$\tilde{F}^{*ij} = \frac{1}{2}e^{ijkl}F_{kl}, \quad \tilde{F}^{ij} = F_{kl}\tilde{g}^{ik}\tilde{g}^{jl},$$

and e^{ijkl} are contravariant components of the Levi-Civita tensor normalized as $e_{1234} = \sqrt{g}$. Now we shall derive from geometrical laws of the electromagnetic field the general covariant four dimensional Maxwell equations for the electric and magnetic fields which in fact express the fundamental physical laws. First of all we formulate the main relations of the vector algebra and vector analysis on the four dimensional physical manifold which is interesting by itself.

A scalar product of two vector fields A^i and B^i is defined by the Riemann metric (1) as earlier. A vector product of two vector fields A^i and B^i we shall construct as follows

$$C^i = [AB]^i = e^{ijkl}t_j A_k B_l.$$

Here the crucial significance of the temporal field should be emphasized. It is evident that $[AB] + [BA] = 0$. By the direct calculation it can be shown that $|[AB]| = |A||B|\sin\varphi$, $[A[BC]] = B(A, C) - C(A, B)$.

Differential operators of the vector analysis on the physical manifold are defined as natural as algebraic ones. For the divergence and gradient we have respectively

$$\text{div } A = \nabla_i A^i = \frac{1}{\sqrt{g}}\partial_i(\sqrt{g}A^i), \quad (\text{grad } \phi)^i = (g^{ij} - t^i t^j)\partial_j \phi = g^{ij}\partial_j \phi - t^i D_t \phi$$

and as a consequence

$$\text{div grad } \phi = \frac{1}{\sqrt{g}}\partial_i(\sqrt{g}g^{ij}\partial_j \phi) - \frac{1}{\sqrt{g}}\partial_i(\sqrt{g}t^i D_t \phi) = \nabla_i \nabla^i \phi - \nabla_i(t^i D_t \phi).$$

A rotor of the vector field A is defined as a vector product of ∂ and A

$$(\text{rot } A)^i = e^{ijkl}t_j \partial_k A_l = \frac{1}{2}e^{ijkl}t_j(\partial_k A_l - \partial_l A_k).$$

It is easy to verify that

$$\text{rot grad } \phi = 0.$$

It is evident that all operations so defined are general covariant.

There is only one direct way to derive from the geometrical Eqs. (21) the fundamental physical laws first formulated by Maxwell. Consider the rate of change with time of the potential of electromagnetic field. By definition, we have

$$D_t A_i = t^k \partial_k A_i + A_k \partial_i t^k = t^k(\partial_k A_i - \partial_i A_k) + \partial_i(t^k A_k) = t^k F_{ki} - \partial_i \phi,$$

where $\phi = -t^k A_k$. Thus, the rate of change with time of the electromagnetic potential can be presented as the difference of two covector fields with one of them having the form

$$E_i = t^k F_{ik}. \tag{22}$$

From this it follows that strength of the electric field is general covariant and gauge invariant quantity that is defined by the equation

$$E_i = t^k F_{ik} = -D_t A_i - \partial_i \phi.$$

Now it is quite clear that general covariant and gauge invariant definition of the magnetic field strength is given by the formula

$$H^i = (\text{rot } A)^i = e^{ijkl}t_j \partial_k A_l = \frac{1}{2}e^{ijkl}t_j(\partial_k A_l - \partial_l A_k).$$

Thus,

$$H_i = t^k {}^*F_{ik}, \quad (23)$$

where ${}^*F_{ij} = g_{ik}g_{jl}{}^*F^{kl}$.

Now it is not difficult to derive the Maxwell equations for the electric and magnetic fields from the geometrical Eqs. (21). Resolving equations (22), (23) over F_{ik} , we obtain

$$F_{ij} = -t_i E_j + t_j E_i - \varepsilon_{ijkl} t^k H^l. \quad (24)$$

Thus, on the physical manifold there is general covariant one-to-one algebraic relation between the electric and magnetic fields and tensor of the electromagnetic field that is defined by the temporal field. Out of (24) we find

$${}^*F^{ij} = -t^i H^j + t^j H^i - e^{ijkl} t_k E_l, \quad \tilde{F}^{ij} = t^i E^j - t^j E^i - e^{ijkl} t_k H_l. \quad (25)$$

Substituting (25) into (21), we shall obtain the Maxwell equations for the strengths of the electric and magnetic fields in the following general covariant and gauge invariant form

$$\frac{1}{\sqrt{g}} D_t(\sqrt{g}H) = -rotE, \quad \frac{1}{\sqrt{g}} D_t(\sqrt{g}E) = rotH. \quad (26)$$

From Eqs. (26) it follows that

$$D_t(\partial_i(\sqrt{g}H^i)) = 0, \quad D_t(\partial_i(\sqrt{g}E^i)) = 0,$$

and therefore the dynamical equations are compatible with constraints.

To complete this discussion with the Maxwell equations as the main topic, we write the expression for the components of the energy-momentum tensor in terms of electric and magnetic field strength

$$T_{ij} = \frac{1}{2} g_{ij} (|E|^2 + |H|^2) - E_i E_j - H_i H_j - t_i \Pi_j - t_j \Pi_i, \quad (27)$$

where Π_i are covariant components of the Poynting vector

$$\Pi^i = e^{ijkl} t_j E_k H_l, \quad \Pi = [EH].$$

From (27) and definition given above we find the energy density of the electromagnetic field

$$\varepsilon_{em} = \frac{1}{2} (E^2 + H^2)$$

which is a reasonable result. We shall formulate also the energy conservation law starting from the general covariant Maxwell equations. Out of (26) and (27) we get

$$\frac{1}{\sqrt{g}} D_t(\sqrt{g}\varepsilon_{em}) + \nabla_i \Pi^i = -\frac{1}{2} T^{ij} D_t g_{ij}, \quad (28)$$

where $T^{ij} = \tilde{g}^{ik} \tilde{g}^{jl} T_{kl}$. Let us now assume that the electromagnetic field is considered on the background of the physical manifold of some full system of fields. Let it be known also that for gravity field of this system the equation $D_t g_{ij} = 0$ holds valid. In such approximation, when physical manifold is external with respect to the electromagnetic field, from (28) we obtain that the energy density of the electromagnetic field satisfies the equation

$$\frac{1}{\sqrt{g}} D_t(\sqrt{g}\varepsilon_m) + \nabla_i \Pi^i = 0.$$

It is exactly the energy conservation law of the electromagnetic field in the above mentioned approximation.

Thus, it is shown that the principles of the theory of time and gravity physics are in full correspondence with the fundamental physical laws and hence they can be considered as a method to derive new fundamental equations.

5. Physical laws of the gravitational field

Now we have an important experience that allows us to derive physical equations of the gravitational field analogous to the Maxwell equations. The guiding idea is very simple. The equation $div E = 4\pi\rho$ is tightly connected with the charge conservation. Thus, our goal is to find gravitational analog of this equation which should be connected with the energy conservation in a manner similar to the conservation of the electric charge.

To this end we first of all introduce the important notion of the momentum of gravitational field, i.e. quantity which is analogous to $E_i = D_t A_i + \partial_i(t^i A_i)$. Consider a tensor field of the type (1,1)

$$P_j^i = \frac{1}{2}g^{ik}D_t g_{jk} = \frac{1}{2}g^{ik}(\nabla_j t_k + \nabla_k t_j) = g^{ik}\nabla_j t_k = \nabla_j t^i.$$

A tensor field so defined will be called the momentum of the gravitational field or simply the momentum of the field. Give some important relations for the momentum

$$\begin{aligned} t_i P_j^i &= 0, \quad t^j P_j^i = 0, \quad g^{ik}P_k^l g_{lj} = P_j^i. \\ D_t P_j^i &= t^l \nabla_l P_j^i, \quad \nabla_i P_j^k - \nabla_j P_i^k = R_{ijl}{}^k t^l. \\ D_t \Gamma_{jk}^i &= \nabla_j P_k^i + \nabla_k P_j^i - g^{il}g_{km}\nabla_l P_j^m, \quad D_t g = g g^{ik}D_t g_{ik} = 2g P_j^i. \end{aligned}$$

As the following step in the required direction we shall write geometrical equations (17) in other form.

By contraction with \tilde{g}^{ij} we get from (17) that $\tilde{R} = \varepsilon + T$, where $T = T_{ij}\tilde{g}^{ij}$. Using this relation we transform (17) to the following form

$$\tilde{R}_{ij} + T_{ij} - \frac{1}{2}\tilde{g}_{ij}T = \frac{1}{2}g_{ij}\varepsilon. \quad (29)$$

If we start with equations (29), we get by contraction (19) and $\tilde{R} = \varepsilon + T$, and so we can get back to (17). We may use either (17) or (29) as the basic equations.

With (5) we get $\tilde{R}_{ij} = R_{ij} + \nabla_l(t^l D_t g_{ij})$ and hence $g^{ik}\tilde{R}_{jk} = R_j^i + 2D_t P_j^i + 2P_k^k P_j^i$. Introduce the tensor fields

$$B_j^i = h_k^i R_l^k h_j^l + D_t P_j^i + P_k^k P_j^i, \quad N_j^i = h_k^i T_l^k h_j^l - T h_j^i,$$

where $h_j^i = \delta_j^i - t^i t_j$ is projection operator, $h_k^i h_j^k = h_j^i$. We next define the flux vector of the energy of the gravitational field

$$G_i = \nabla_k P_i^k - \partial_i P_k^k + t_i(P_l^k P_k^l + D_t P_k^k)$$

and the flux vector of the energy of other fields

$$\Pi_i = \varepsilon_m t_i - T_{ik}t^k.$$

It is evident that $(t, G) = (t, \Pi) = 0$. With this we can derive from the equations (29) the following system of equations

$$D_t P_j^i + P_k^k P_j^i + B_j^i + N_j^i = \frac{1}{2}\varepsilon h_j^i \quad (30)$$

$$G_i = \Pi_i \quad (31)$$

and vice versa. Our statement is that equations (30) represent in general covariant form the physical laws of the gravitational field similar to the Maxwell equations (26). To emphasize this analogy we write equations (30) in the form

$$\frac{1}{\sqrt{g}}D_t(\sqrt{g}P_j^i) + B_j^i + N_j^i = \frac{1}{2}\varepsilon h_j^i$$

and add that like equation $div E = 0$, the equation (31) may be considered as constraint since from (30) it can be derived that

$$D_t(G_i - \Pi_i) + P_k^i(G_i - \Pi_i) = \frac{1}{\sqrt{g}}D_t(\sqrt{g}(G_i - \Pi_i)) = 0 .$$

The last equation means also that equations (30) and (31) are compatible.

Equation (31) has very simple physical sense that the flux vector of the energy of the gravitational field is exactly equal to the flux vector of energy of other fields . For the energy density of the gravitational field we have $\varepsilon_g = \frac{1}{2}\tilde{R}_{ij}g^{ij} = \frac{1}{2}R + (P_i^i)^2 + D_tP_i^i = T + U$, where

$$T = \frac{1}{2}(P_i^i)^2 - \frac{1}{2}P_j^iP_i^j \quad (32)$$

is a density of kinetic energy and

$$U = \frac{1}{2}R + \frac{1}{2}(P_i^i)^2 + \frac{1}{2}P_j^iP_i^j + D_tP_i^i \quad (33)$$

is a density of potential energy of the gravitational field.

For the better understanding of the last constructions we shall consider the general covariant dynamical laws of the gravitational field in the atlas compatible with causal structure. With respect to the coordinate transformations that conserve causal structure the quantity $g_{\mu\nu}(t, x^1, x^2, x^3)$, ($\mu, \nu = 1, 2, 3$) from the metric (14) transforms as the symmetrical tensor and $dl^2 = g_{\mu\nu}(t, x^1, x^2, x^3)dx^\mu dx^\nu$ may be considered as the metrics of the space sections of the physical manifold. For this one parametric set of three dimensional metrics we can consider Christoffel symbols $L_{\nu\sigma}^\mu$ and covariant derivative ∇_μ , Riemannian tensor curvature $S_{\mu\nu\sigma}{}^\tau$, tensor Ricci $S_{\nu\sigma} = S_{\mu\nu\sigma}{}^\mu$ and scalar curvature $S = g^{\mu\nu}S_{\mu\nu} = S_\mu{}^\mu$ as usual. In the atlas compatible with causal structure we have $L_{\nu\sigma}^\mu = \Gamma_{\nu\sigma}^\mu$, $P_i^4 = P_4^i = 0$, $B_i^4 = B_4^i = 0$, ($i = 1, 2, 3, 4$) and what is more important that

$$B_\nu^\mu = S_\nu^\mu, \quad R + (P_i^i)^2 + P_j^iP_i^j + 2D_tP_i^i = S .$$

Thus, out of (32) and (33) we get for the densities of the kinetic and potential energy of the gravitational field following representation

$$T = \frac{1}{2}(P_\sigma^\sigma)^2 - \frac{1}{2}P_\nu^\mu P_\mu^\nu, \quad U = \frac{1}{2}S.$$

Now we write equations (30) and (31) in the atlas compatible with causal structure under assumption that T_{ij} is the tensor of the energy-momentum of the electromagnetic field. In this case

$$N_\nu^\mu = T_\nu^\mu = \frac{1}{2}(E^2 + H^2)\delta_\nu^\mu - E^\mu E_\nu - H^\mu H_\nu$$

is the Maxwell stress energy tensor. Thus, we can write the basic equations (30) and (31) in the following form

$$\dot{P}_\nu^\mu + P_\sigma^\sigma P_\nu^\mu + S_\nu^\mu + T_\nu^\mu = \frac{1}{2}\varepsilon \delta_\nu^\mu, \quad (34)$$

$$\nabla_\nu P_\mu^\nu - \partial_\mu P_\sigma^\sigma = \Pi_\mu = [\mathbf{EH}]_\mu, \quad (35)$$

where dot stands for partial derivative with respect the variable t . From equations (34) and (35) it follows that the Cauchy problem for the gravitational field is not more difficult than for the electromagnetic field.

In the above consideration we derived the law of energy conservation from very general geometrical laws. It is very important from the physical point of view to consider energy conservation on

the basis of Maxwell equations and physical laws of gravitational field (34) and (35). In the atlas compatible with causal structure the Maxwell equations (26) read

$$\dot{E}^\mu + P_\sigma^\sigma E^\mu = e^{\mu\nu\sigma} \partial_\nu H_\sigma, \quad \dot{H}^\mu + P_\sigma^\sigma H^\mu = - e^{\mu\nu\sigma} \partial_\nu E_\sigma.$$

Since $\dot{g} = gg^{\mu\nu} g_{\mu\nu} = 2gP_\sigma^\sigma$, $\frac{\partial}{\partial t}(\sqrt{g}\varepsilon) = \sqrt{g}(\dot{\varepsilon} + P_\sigma^\sigma \varepsilon)$ and one can derive from these equations the formula

$$\dot{\varepsilon}_{em} + P_\sigma^\sigma \varepsilon_{em} = -\nabla_\nu \Pi^\nu - T_\nu^\mu P_\mu^\nu.$$

In the atlas compatible with causal structure we have for the energy density of the gravitational field

$$\dot{\varepsilon}_g = P_\sigma^\sigma \dot{P}_\sigma^\sigma - P_\mu^\nu \dot{P}_\nu^\mu + \frac{1}{2} \dot{S}.$$

From the equations (34) we have the relations

$$\dot{P}_\sigma^\sigma = \frac{1}{2} \varepsilon_{em} - \frac{1}{2} \varepsilon_g - P_\mu^\nu P_\nu^\mu, \quad P_\mu^\nu \dot{P}_\nu^\mu = \frac{1}{2} \varepsilon P_\sigma^\sigma - P_\sigma^\sigma P_\mu^\nu P_\nu^\mu - P_\mu^\nu S_\nu^\mu - P_\mu^\nu T_\nu^\mu.$$

With this one can show that

$$\dot{\varepsilon}_g + P_\sigma^\sigma \varepsilon_g = \frac{1}{2} g^{\mu\nu} \dot{S}_{\mu\nu} + T_\nu^\mu P_\mu^\nu.$$

Since $\frac{1}{2} g^{\mu\nu} \dot{S}_{\mu\nu} = \nabla_\nu G^\nu$, for $\varepsilon = \varepsilon_g + \varepsilon_{em}$ we have

$$\dot{\varepsilon} + P_\sigma^\sigma \varepsilon = \nabla_\nu G^\nu - \nabla_\nu \Pi^\nu.$$

Thus,

$$\frac{\partial}{\partial t}(\sqrt{g}\varepsilon) = \partial_\nu(\sqrt{g}G^\nu) - \partial_\nu(\sqrt{g}\Pi^\nu). \quad (36)$$

We may integrate this relation over a three-dimensional volume V lying in the space section of the physical manifold. For the energy in the volume V we have $W = \int \varepsilon \sqrt{g} dV$. Consider the fluxes of the gravitational $\Phi_g = \int \Omega_\nu d\sigma^\nu$ and the electromagnetic $\Phi_{em} = \int \Pi_\nu d\sigma^\nu$ energy, where integral is taken over the boundary surface of the volume V . Since the right hand side of the equation (36) can be converted by Gauss's theorem, from (36) we have the relation

$$\frac{\partial}{\partial t} W = \Phi_g - \Phi_{em}.$$

Thus, the law of energy conservation means that the fluxes of the gravitational and electromagnetic energy flow in opposite directions and exactly equal to each other. (We know that our sun is a surprisingly powerful source of the radiant energy. Now it is evident that if there is the flow of gravitational energy in the direction to the sun then we can say that our sun is the transducer of the gravitational energy into the electromagnetic one). Now we consider question about exact solutions of the equations (34) and (35).

6. Exact solutions

We have seen above that there is deep analogy between the energy and charge and the gravitational and electromagnetic fields. Here we consider the existence of solutions to the equations (34) and (35) which reproduce the physical situation with pointlike distribution of the gravitational energy, i.e. $\varepsilon_g = m\delta(r)$, where m is a gravitational charge of point source. This physical situation is analogous to the pointlike distribution of density of charge, when $\rho = q\delta(r)$, where q is a electric charge of point source. Thus, we search the solution of the gravitational equations analogous to the Coulomb potential. Starting point is the question about static gravitational field.

Gravitational fields will be called **static** if the rate of change with time of the gravitational potential is equal to zero, $D_t g_{ij} = 0$. In accordance with (6), this condition can be written as

$$D_t g_{ij} = \nabla_i t_j + \nabla_j t_i = 0.$$

It is evident that definition of the static gravitational fields is general covariant. Since $\nabla_i t_j - \nabla_j t_i = 0$, then for a static gravitational field we have $\nabla_i t_j = 0$ and hence $R_{ijk}{}^l t_l = 0$, $\tilde{R}_{ijk}{}^l = R_{ijk}{}^l$. With this it is not difficult to show that a static gravitational field is absent ($R_{ijk}{}^l = 0$.) Thus, the gravitational field generated by the gravitational charge can not be static.

The following point is the definition of the spherical symmetrical gravitational potential. We use the empirical (non internal) representations about the spherical symmetry. In accordance with this in the atlas compatible with causal structure a potential with spherical symmetry has the form

$$dl^2 = A^2 dr^2 + B^2 (d\theta^2 + \sin^2 \theta d\phi^2),$$

where $A = A(r, t)$, $B = B(r, t)$. For the nonzero components of the Ricci tensor of this one parametric set of metric we find the following expressions

$$S_{11} = \frac{2}{AB}(A'B' - AB''), \quad S_{22} = \frac{B}{A^3}(A'B' - AB'') + 1 - \frac{B'^2}{A^2}$$

and $S_{33} = S_{22} \sin^2 \theta$, where prime stands for derivative with respect to the variable r . Hence, for the potential energy of this configuration we have

$$U = \frac{1}{2}S = \frac{2}{A^3 B}(A'B' - AB'') + \frac{1}{B^2} \left(1 - \frac{B'^2}{A^2} \right).$$

To get $U = 0$ we immediately put $B' = A$. Remind, that we aim to realize the situation with a pointlike distribution of the gravitational energy.

For the nontrivial components of the momentum we have

$$P_1^1 = \frac{\dot{A}}{A}, \quad P_2^2 = P_3^3 = \frac{\dot{B}}{B},$$

where dot denotes partial derivative with respect to the variable t . The flux vector of the energy has only one nonzero component

$$G_1 = \frac{2}{AB}(\dot{A}B' - A\dot{B}')$$

and in accordance with relation $A = B'$ the equation $G_i = 0$ is fulfilled. Thus, we have two equations

$$2B\ddot{B} + \dot{B}^2 = 0, \quad \ddot{A}B^2 + \dot{A}\dot{B}B = \frac{1}{2}A\dot{B}^2.$$

Now we use the other empirical idea about a spherical wave and put

$$A = A(r - t), \quad B = B(r - t).$$

From the equation for B only we find that $B\dot{B}^2 = m$, where m is a constant of integration (gravitational charge) and hence

$$B = m \left[\frac{3}{2} \frac{(r - t)}{m} \right]^{\frac{2}{3}}.$$

Since $A = B'$, then

$$A^{-1} = \left[\frac{3}{2} \frac{(r - t)}{m} \right]^{\frac{1}{3}}$$

and it is not difficult to see that second equation for A and B is fulfilled automatically. Thus, the Einstein gravitational potential can be presented in the following form

$$d\tilde{s}^2 = -dt^2 + \frac{dr^2}{\left[\frac{3}{2}\frac{(r-t)}{m}\right]^{\frac{2}{3}}} + \left[\frac{3}{2}\frac{(r-t)}{m}\right]^{\frac{4}{3}} m^2(d\theta^2 + \sin^2\theta d\phi^2).$$

The solution obtained is known as the Schwarzschild solution [10]. Since for the kinetic energy we have $T = 0$, then we see that this solution really correspond to the situation described above and hence in definite sense it is an analogous of the Coulomb potential.

Now we consider another very simple but important physical situation. The idea about homogenous and isotropic distribution of the gravitational energy may be realized in the atlas compatible with causal structure by the one parametric set of metric

$$dl^2 = a(t)d\sigma^2,$$

where $d\sigma^2$ is the metric of the unit 3d sphere, which in the four dimensional spherical coordinates has the form

$$d\sigma^2 = d\psi^2 + \sin^2\psi(d\theta^2 + \sin^2\theta d\phi^2).$$

Now problem is to find a function $a(t)$. We have

$$S_\nu^\mu = \frac{2}{a^2}\delta_\nu^\mu, \quad P_\nu^\mu = \frac{\dot{a}}{a}\delta_\nu^\mu$$

and hence for the density of the gravitational energy we get

$$\varepsilon_g = \frac{3}{a}(\dot{a}^2 + 1).$$

We know that $\partial(\sqrt{g}\varepsilon_g)/\partial t = 0$. Since $\sqrt{g} = a^3 \sin^2\psi \sin\theta$ then by integrating over the variables ψ, θ, ϕ we get the following equation for $a(t)$ from the law of energy conservation

$$a(\dot{a}^2 + 1) = 2a_0 = \text{const.}$$

With respect to the new variable η , such that $dt = ad\eta$ the solution of the equation in question can be presented in the following form $a = a_0(1 - \cos\eta)$. Thus, we get that emergence and evolution of the clot of gravitational energy is described in parametric form as follows

$$a = a_0(1 - \cos\eta), \quad t = a_0(\eta - \sin\eta).$$

We see that solutions presented here are tightly connected with the gravitational energy. These solutions have singularities and in connection with this we would like to make the following remark.

From the theory of time presented here it follows directly that there is **matter outside the time**. For example, the gravity field and the electromagnetic field exterior to time are described by the equations

$$R_{ij} - \frac{1}{2}g_{ij}R = g_{ij}F^2 - F_{ik}F_{jl}g^{kl},$$

$$\nabla_i F^{ij} = 0, \quad F^{ij} = F_{kl}g^{ik}g^{jl}, \quad F_{ij} = \partial_i A_j - \partial_j A_i,$$

where $F^2 = \frac{1}{4}F_{ij}F^{ij}$. Recall that the metric g_{ij} is positive definite. In this context it is very important to understand the nature of the emergence of time as an objective property of physical systems and as an distinctive order parameter.

7. Conclusion

Let us now sum up the obtained results and focus on some of the problems. It is shown that gravity physics is an internal field theory by nature which does not contain apriori elements and can be characterized as follows. In the theory there is no internal reason that could objectively distinguish one arbitrary coordinate system from another. It is unrolled on the smooth four-manifold that does not exist apriori but is defined by the physical system itself. Manifold is a basic primary notion in physics and representation about space and time is given on this ground. All notions, definitions and laws are formulated in coordinate independent form, i.e., within the framework of the structure of smooth manifold. Einstein's gravitational potentials are determined by the positive definite symmetrical tensor field (Riemann metric) and temporal field. Being a scalar field on the manifold, the temporal field is introduced into the theory in the form that provide, in general, time reversal invariance. The system evolves in its proper time and it is important that we do not have compare intrinsic time with some other times.

Central role of the time in the internal field theory is that time determines causal structure of the field theory and first of all **it characterizes the internal nature of the gravity field as the simplest closed system**. It is established that energy density is the first integral of the closed system of interacting fields. The general covariant definition of the electric and magnetic fields is given and the Maxwell equations for these fields are derived. It is shown how to reveal important internal properties of physical system when coordinates have no physical sense. This means that physical sense of general covariance is recognized and there exists a reparametrization-invariant description. And last but not least, it should be noted that the following problems now become actual: Can gravitational charges be produced on high-energy colliders? Do gravitational charges play essential role on Earth and in space?

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