CAN BLACK HOLES BE PRODUCED at HIGH-ENERGY COLLIDERS?

V.A. Petrov
Institute for High Energy Physics, 142200, Protvino, Russia

• The present boom of interest to the black holes is very much substantiated by accumulation of astrophysical observations, which can be interpreted in terms of black holes [1]. This, certainly, refers to very large-mass systems that are to be described in the framework of classical (non-quantum) field theory (General Relativity or other classical theories of gravitation).

Quantum properties of the black holes (BH in the following) were addressed some 30 years ago [2] with a conclusion that quantum effects lead to the thermal radiation from the BH with an effective "temperature" inversely proportional to the BH mass. Quick evaporation of the BH with small masses caused some hopes that the "mini-BH" may be relevant in the high-energy collisions of subnuclear particles, and the corresponding events could be observable due to the characteristic event shapes. However expected sizes of such effects are negligible at present or near (LHC) energies because of the small value of the Newton constant $\sim M_{PL}^{-2}$. It is of interest to note that one of the most fervent prophets of the black holes, S.W. Hawking, has recently expressed his deep doubts that the true event horizon ever forms at all (S. Hawking's talk at GR 17, Dublin, 2004).

• New prospects for exploration of the BH effects were opened by the theories with extra-dimensions [3] in which gravitational interactions at small distances could get very strong at the scales of order 1 TeV^{-1}.

The mechanism of the BH production in high-energy collisions is usually presented as follows.

Two energetic partons from the colliding hadrons (protons) with the c.m.s. energy $\sqrt{s}$ — considered as a component of the energy-momentum tensor in the r.h.s. of the Hilbert–Einstein equation — produce the metric perturbation corresponding to a BH of the mass $\sqrt{s}$. This configuration having the Hawking temperature $1/\sqrt{s}$ quickly evaporates giving rise to a specific final multiparticle state. The cross-section of such elementary process is assumed to be purely geometric

$$\hat{\sigma}_{\mathrm{BH}}(s) \sim r_{\mathrm{BH}}^2(s),$$

where

$$r_{\mathrm{BH}}(s) \sim 1\mathrm{TeV}^{-1}(\sqrt{s}/\mathrm{TeV})^{1/(1+\Delta)},$$

is the BH radius and $4+\Delta$ is the dimensionality of space-time. The rate of the BH events can be fantastic: 10 Hz! [4].

At first sight the arguments are quite natural. However more thorough investigation reveals quite serious theoretical problems.

• As we all probably know, theoretical description and, hence, model predictions for observed processes like those which happen in high-energy particle collisions are based on the S-matrix formalism. S-matrix relates idealized asymptotic states at space-temporal infinity. Asymptotic states are necessary for the very interpretation of the field theory in terms of particles. It is well known that the very notion of particle is problematic in generic curved space-times [5].

There are gravitational theories where the flat, pseudoeuclidean space-time is a geometrical basis for any interaction, including the gravitational one [6], and which do not encounter with difficulties mentioned above. However, in these theories there are no black holes at all.

• With a hope that such problems can be resolved in some way in GR we try to see if there are some plausible conditions for the BH formation in violent collisions.

The process would happen when two high-energy particles enter the interaction region and strong gravitational fields are generated due to their high energy. Moreover one assumes that some "trapping surface" would form which finally transforms to a black hole with mass that is a finite fraction of the collision c.m.s. energy [7]. We have to stress that such a "trapping surface" (analogous to the Schwarzschild horizon surface) is not yet obtained as an exact solution of the Hilbert-Einstein equations.

As BH are classical field configurations one can argue that in order to be relevant to the realm of the high energy collisions some conditions of "quasiclassicity" must be respected. In analogy with QED one can ask for a large number of virtual quanta. One also has to have space-time curvature small enough to evade the problems of quantum-gravitational character. In ref. [8] it was shown that these necessary conditions are in general violated [9].

• There is another remark concerning the BH formation. It is known that in the comoving frame the collapse happens in a finite time. On the other hand the distant observer will observe an infinitely long process. From the point of view of the particle detectors in, say, LHC this is namely such a situation.

One could argue that big red shifts from the collapsing radiating matter get big in an exponentially fast way and "practically" it takes a finite time to "almost collapse". At the moment the mechanism of the would-be BH formation is too unclear to completely remove such an objection.

• It is clear that at our present stage of knowledge the BH is a classical field configuration. The BH quantum properties are under an active investigation but none conclusive result is achieved.

Coming back to the S-matrix formalism one could have in mind the following.

Usually, in particle physics, one deals, when considering the scattering and production processes, with Fock states representing the asymptotic states of particles with definite masses and momenta.

In principle one can choose any other representation of the CCR and work with states where quantum field has a given functional value:

$$\hat{\varphi}|\varphi(x)>=\varphi(x)|\varphi(x)>,$$

where $\varphi(x)$ is a classical field configuration.

Then one could introduce the amplitude

$$<\varphi|S|p_1,p_2>\ ,$$

which corresponds to the "production" of a field configuration $\varphi(x)$ in the course of the collision of particles p_1 and p_2.

In order to detect such a configuration with detectors that are adapted to "usual" asymptotic states $|k_1,...k_n>$ one can use the amplitude

$$<k_1..k_n|\varphi><\varphi|S|p_1,p_2>\ .$$

An important ingredient is the amplitude $<k_1...k_n|\varphi>$. In simple cases it can be easily computed. For instance, in the case of single neutral scalar field one has [10]:

$$<0|\varphi>\sim \exp\left[-\int d\vec{x}\varphi(\vec{x})\sqrt{m^2-\vec{\nabla}_x^2}\varphi(\vec{x})\right],$$

$$<k_1|\varphi>\sim\tilde{\varphi}(\vec{k}_1)<0|\varphi>,\quad \text{etc.}$$

In our case $\varphi(x)$ is the Schwarzschild solution $g_{\mu\nu}^{Schw}$:

$$g_{\mu\nu}^{Schw}(r)\ \Big|_{\text{Hilbert's gauge}}=-\left(1-\frac{r_{\rm BH}}{r}\right)dt^2+\left(1-\frac{r_{\rm BH}}{r}\right)^{-1}dr^2+r^2d\Omega^2,$$

$$\text{at } \Delta \neq 0 \;\; r_{BH}/r \to (r_{BH}/r)^{1+\Delta}.$$

In other words one has to compute the amplitude

$$< g^{Schw}|S|p_1, p_2 >,$$

where the state $< g^{Schw}|$ can be mixed with other representations. Thus, the amplitude for production of the final state $|k_1..k_n >$ which is a result of the BH "decay" could have the following form:

$$T^{BH}(k_1...k_n|p_1p_2) = \sum_m < k_1...k_2|g^{Schw} >< g^{Schw}|q_1...q_m >< q_1...q_m|T|p_1p_2 > \times$$

$$\times(2\pi)^4\delta\left(\sum_1^m q_j - p_1 - p_2\right).$$

Note that T contains the (quantum) gravitational interactions.

There is, however, a big problem: in such a construction the energy-momentum conservation is not guaranteed, generally. The state $|g^{Schw} >$ is not an eigenstate of the energy-momentum operator.

• Thus, I conclude that in spite of quite exciting and seemingly natural theoretical prospects for observation of copious BH production at the LHC, one encounters, at more close inspection, with lot of theoretical problems, including a proper definition of corresponding amplitudes.

In general, one has to realize that if LHC experiments will not see mini-BHs it will not shake the fundamental basis of our present worldview, just show up some not very well grounded model assumptions.

It does not mean that the very subject is hopeless and devoid of interest. Quite contrary to that!

References

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[9] Note added in proof. In a joint publication by S.B. Giddings and V.S. Rychkov (hep-ph/0409131) it argued that such objections can be lifted if colliding particles are prepared in such a way that the widths of corresponding wavepackets obey to some specific limitations. It is by no means evident that such wavepackets do really correspond to real beams of particles in colliders.

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