Einstein, Hilbert and Equations of Gravitation

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• The history of the equations of gravitational field is of particular interest because they are related to two famous names: Albert Einstein and David Hilbert, and because the circumstances of their invention bear sometimes almost detective character.

One of the early references is the book "Einstein, Hilbert and the Theory of Gravitation" by a renowned historian of science J. Mehra [1], where the great role of Hilbert was showed very clear. Such view was strengthened in 1978 when the correspondence between Einstein and Hilbert was published, from which followed that Hilbert informed Einstein on the gravitational field equations in a letter before his formal publication [2].

The opinion, shared by many physicists, was that it was Hilbert who possessed an undoubtful priority [3].

• However, in 1997 a new sensation shaked just established opinion: the authors of a short article in "Science" [4] argued on the basis of the first proofs of the Hilbert paper on the gravitational equations, digged up from the Hilbert archive, that **Hilbert had no correct, generally covariant equation before Einstein.** Moreover, the authors of [4] transparently alluded that Hilbert "borrowed" some decisive formulae from Einstein! And even that Hilbert tried to hide such an appropriation with help of deliberately wrong dating of his article.

Such an accusation would seriously undermine the image of David Hilbert from the ethical side, and was in a sharp contrast to all what was known about his personality.

• On the other hand the very personality of Einstein is by no means irreproachable. Take, for instance, the case of the relativity theory. So, counter-reaction to paper [4] followed.

One of the first was the book by C.J. Bjerknes [5], well documented and with a rich bibliography, in which the conclusions of paper [4] were contested. This was based on the correspondence between Einstein and Hilbert and an important fact that the proofs of Hilbert's paper — the main evidence of the authors of [4] against Hilbert — were mutilated with some part of proofs cutted off.

This fact was mentioned for the first time, but without a due evaluation, in Ref. [6]. In book [5] it was mentioned that F. Winterberg assumed that the explicit form of the gravitational field could be fairly contained in the cutted off parts of the proofs. This seriously undermined the main argument of the authors [4] against Hilbert[1].

• In ref. [9] the question was considered in detail with analysis of the Hilbert and Einstein papers.

Here I will reproduce only one simple reason we used to reject one of the main accusation of the authors [4] against Hilbert, namely: *"...knowledge of Einstein's result may have been crucial to Hilbert's introduction of the trace term into his field equations".*

Thus, according to the authors of [4], Hilbert's equations

$$\frac{\delta\sqrt{g}H}{\delta g^{\mu\nu}} = 0, \qquad (*)$$

[1]Afterwards this consideration was published [7], and, finally, the book [8] appeared in which D. Wuensch gave a thorough analysis of the mutilated proofs and other relevant documents with a conclusion: Hilbert knew the explicit form of the gravitational equations, they contended in the proofs and the latter were deliberately mutilated in order to falsify the historical truth. — *Note added to proof.*

where H is the sum of the gravitational and material lagrangians, $g^{\mu\nu}$ is the metric tensor with the determinant $-g$, are incomplete and need some ad hoc "introduction" of additional terms ("trace term"). This "discovery" of the authors of paper [4] clearly demonstrates their professional inconsistency: they must be never tried to calculate the variational derivative (*) themselves! Otherwise they would quickly saw that the "crucial" trace term is safely contained in Eq.(*).

Other mistakes of the authors of paper [4] where analysed in our paper [9].

One thing has to be clear. **At the very moment when Hilbert equated the gravitational part of the lagrangean to the scalar Riemannian curvature the whole game was over.**

All the rest was the matter of almost routine calculations, though one has to mention that Hilbert managed to get, in his paper [10] some very important results ("Bianchi identity"). All detailes are given in [9].

So, we conclude that an unfair attack of the authors of ref. [4] against Hilbert's originality in deriving the gravitaional field equations is completely and shamefully failed.

• Now, what about Einstein? In ref. [9], there was admitted that Einstein could derive the gravitational field equations [11] independently of Hilbert.

The main evidence, if to be as loyal as possible to Einstein, is his assertion, made by him in the letter to Hilbert of 18 November 1915: *"The system you furnish agrees — as far as I can — exactly with what I found in the last few weeks and presented to the Academy"* [2].

That is, Einstein, in this letter, acknowledged receiving the Hilbert equations of the gravitational field and informed him that his, Einstein's, equations are essentially the same.

But we know all papers by Einstein presented to the Academy "in the last few weeks", including the paper of the 18 November — they all are still wrong and do not contain the trace term. One has to concede that Einstein had by 18 November 1915 the correct equations but preferred to publish the wrong ones up to 25 November! In principle this is possible, but...

We also have to mention the paper by Einstein of 18 November 1915 [12] where he claimed the successful test of his (wrong!) theory in obtaining the correct result for the Mercury perihelion. F. Winterberg (as cited in ref. [5]) draw the attention to the fact that if Einstein would really follow his equations as they were described in his paper of 18 November, he would obtain the result twice larger than the correct one. Nonetheless, his final result was correct! Further interesting details of this story can be found in [5].

• What is our conclusion? We still keep the opinion expressed in [9]: the gravitational equation has to be named as the "Einstein–Hilbert equation". The reason is that it was Einstein who posed the problem to find out the equation in which the energy-momentum tensor is a source for gravitational potentials [13]. Hilbert had found such an equation. Einstein derived it, quite probably, later in his own way.

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