HIGH ENERGY SCATTERING IN THE BRANE WORLD

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The contribution of virtual s-channel Kaluza-Klein (KK) gravitons as well as s-channel reggeized gravitons to high energy scattering of the brane fields is studied in the Randall-Sundrum scenario with a small curvature.

1 Brane world with a small curvature

To explain the hierarchy between electro-weak and Planck scales, a number of theories with extra dimensions (ED's) have been proposed. The Randall-Sundrum (RS) model with one extra spacial dimension [1] describes this hierarchy most economically. The multidimensional gravity appears to be strong, and the fundamental Planck scale can be related with the string scale. It leads to a new phenomenology in the TeV energy region.

The RS model is a realization of the ED theory in a slice of the AdS_5 space-time with the following background warped metric:

$$ds^2 = e^{2\kappa(\pi r - |y|)} \, \eta_{\mu\nu} \, dx^\mu \, dx^\nu + dy^2 \, , \tag{1}$$

where $y = r\theta$ ($-\pi \leqslant \theta \leqslant \pi$), r being the "radius" of ED, and $\eta_{\mu\nu}$ is the Minkowski metric. The parameter κ defines a 5-dimensional scalar curvature of the AdS_5 space.

We will be interested in the so-called RS1 model [1] which has two 3D branes with equal and opposite tensions located at the point $y = \pi r$ (called the *TeV brane*) and point $y = 0$ (referred to as the *Plank brane*). If $k > 0$, then the tension on the TeV brane is negative, whereas the tension on the Planck brane is positive. All the SM fields are constrained to the TeV brane, while the gravity propagates in five dimensions.

Let us note that the warp factor in the metric (1) is equal to 1 on the TeV brane. By calculating the zero mode sector of the effective theory, one then obtains the "hierarchy relation",

$$\bar{M}_{\mathrm{Pl}}^2 = \frac{\bar{M}_5^3}{\kappa} \left(e^{2\pi\kappa r} - 1 \right) \, , \tag{2}$$

with $\bar{M}_5$ being a 5-dimensional Planck scale.

From the point of view of an observer located on the TeV brane, there exists an infinite number of graviton KK excitations with masses

$$m_n = x_n \, \kappa, \qquad n = 1, 2 \ldots \, , \tag{3}$$

where x_n are zeros of the Bessel function $J_1(x)$, with

$$x_n = \pi \left(n + \frac{1}{4} \right) + \mathrm{O} \left(\mathrm{n}^{-1} \right) \, . \tag{4}$$

Note that all zeros of $J_1(x)/x$ are real positive numbers.

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The zero graviton mode, $h_{\mu\nu}^{(0)}$, and massive graviton modes, $h_{\mu\nu}^{(n)}$, are coupled to the energy-momentum tensor of the matter, $T^{\mu\nu}$, as follows:

$$\mathcal{L} = -\frac{1}{\bar{M}_{\rm Pl}} T^{\mu\nu} h_{\mu\nu}^{(0)} - \frac{1}{\Lambda_\pi} T^{\mu\nu} \sum_{n=1}^{\infty} h_{\mu\nu}^{(n)} \,, \tag{5}$$

where

$$\Lambda_\pi = \bar{M}_5 \left(\frac{\bar{M}_5}{\kappa} \right)^{1/2} \tag{6}$$

is a physical scale on the TeV brane.

To get the mass of the lightest KK mode to be $m_1 \sim 1$ TeV, the parameters of the model are usually taken to be $\kappa \sim \bar{M}_5 \sim 1$ TeV. Then one obtains a series of massive graviton resonances in the TeV region which interact rather strongly with the SM particles, since $\Lambda_\pi \sim 1$ TeV.

We will consider a different scenario called *small curvature option* [2]-[4]:

$$\kappa \ll \bar{M}_5 \sim 1 \text{ TeV} \,. \tag{7}$$

In such a scheme, the physical scale is as large as $\Lambda_\pi = 100\,(M_5/\text{TeV})^{3/2}(100 \text{ MeV}/\kappa)^{1/2}$ TeV. Contrary to the case $\kappa \sim \bar{M}_5 \sim 1$ TeV, there exists a series of *very narrow low-mass spin-2 resonances* with an almost *continuous mass distribution*. For such a case, the following inequalities were derived in Ref. [3] [1] :

$$10^{-5} \leqslant \frac{\kappa}{\bar{M}_5} \leqslant 0.1 \,. \tag{8}$$

Notice, in order the hierarchy relation for the warped metric (2) to be satisfied, one has to put $\kappa r \approx 10$.

The present astrophysical bounds significantly restrict the parameter space for a theory with one *compact* ED of the size R_c. Namely, $R_c^{-1} > 4.4 \cdot 10^{-12}$ GeV, and, correspondingly, $\bar{M}_{4+1} > 1.6 \cdot 10^5 \, TeV$ [6], where $\bar{M}_{4+1}$ is a gravity scale in a flat space-time with one compact ED [2].

Fortunately, the above mentioned restriction can not be directly applied to the AdS_5 space-time (see, for instance, [4]). Indeed, the AdS_5 space-time differs significantly from a 5-dimensional flat space-time with one large ED even for very small value of κ (i.e. for the small curvature). To see this, let us consider the hierarchy relation for d-dimensional *flat* space-time:

$$\bar{M}_{\rm Pl}^2 = (2\pi R_c)^d \, \bar{M}_{4+d}^{2+d} \,. \tag{9}$$

For $d = 1$, this relation (9) is a particular case of Eq. (2) in the limit $2\pi\kappa r \ll 1$, with $r = R_c$. However, the condition $2\pi\kappa r \ll 1$ can be only satisfied if the curvature parameter κ is unrealistically small:

$$\kappa \ll \frac{\bar{M}_5^3}{\bar{M}_{\rm Pl}^2} \,. \tag{10}$$

This inequality means that $\kappa \ll 10^{-22}$ eV, for $\bar{M}_5 = 1$ TeV.

2 s-channel virtual gravitons

Let us consider the scattering of two SM fields mediated by massive graviton exchanges in the s-channel,

$$a \, \bar{a} \to G^{(n)} \to b \, \bar{b} \,, \tag{11}$$

[1]For the case $\kappa \sim \bar{M}_5 \sim \bar{M}_{\rm Pl}$, analogous bounds look like $0.01 \leqslant \kappa/\bar{M}_{\rm Pl} \leqslant 0.1$ [5].

[2]Remember that $\bar{M}_5$ denotes the gravity scale in five dimensions with *non-factorizable* metric.

where $a(b) = e^-,\ \gamma,\ q,\ g,\ etc.$ For instance, in hadron collisions virtual graviton effects could be seen in the processes $pp \to 2$ jets $+ X$, $pp \to \gamma\gamma + X$, and Drell-Yan process $pp \to l^+l^- + X$. At linear colliders, the promising reactions are $e^+e^- \to \gamma\gamma$ and $e^+e^- \to f\bar{f}$.

The matrix element of the process (11) will be studied in the following region:

$$\Lambda_\pi \gg \sqrt{s} \sim M_5 \gg \kappa. \tag{12}$$

It looks like

$$\mathcal{M} = \mathcal{A}\,\mathcal{S}\ . \tag{13}$$

The fist factor in Eq. (13) contains the following contraction of tensors:

$$\mathcal{A} = T^a_{\mu\nu}\, P^{\mu\nu\alpha\beta}\, T^b_{\alpha\beta} = T^a_{\mu\nu}\, T^{b\,\mu\nu} - \frac{1}{3}\,(T^a)^\mu_\mu\,(T^b)^\nu_\nu\ , \tag{14}$$

where $P^{\mu\nu\alpha\beta}$ is a tensor part of the graviton propagator, while $T^{a(b)}_{\mu\nu}$ is the energy-momentum tensor of the field $a(b)$.

We will concentrate on the second factor in Eq. (13) which is universal for all types of processes mediated by the s-channel exchanges of the KK gravitons. The relations of all cross sections, which are necessary for studying effects induced by tree-level exchange of the KK gravitons, with the quantity $\mathcal{S}(s)$ can be found in Ref. [2].

The function $\mathcal{S}(s)$ is of the form:

$$\mathcal{S}(s) = \frac{1}{\Lambda_\pi^2} \sum_{n=1}^{\infty} \frac{1}{s - m_n^2 + i\, m_n \Gamma_n}\ . \tag{15}$$

Here Γ_n denotes the total width of the graviton with the KK number n and mass m_n.

This sum is usually estimated in a *zero width* approximation (i.e. assuming $\Gamma_n = 0$ for all n). It such a limit, $\mathcal{S}(s)$ is purely imaginary [2]:

$$\mathrm{Im}\,\mathcal{S}(s) = -\frac{1}{2\bar{M}_5^3\sqrt{s}}\ , \qquad \mathrm{Re}\,\mathcal{S}(s) \simeq 0\ . \tag{16}$$

It means that there is no interference of ED contributions with SM ones.

The width of the massive graviton is indeed very small if its KK-number n is not too large [7]:

$$\frac{\Gamma_n}{m_n} = \eta \left(\frac{m_n}{\Lambda_\pi}\right)^2\ , \tag{17}$$

with $\eta \simeq 0.09$. However, the main contribution to sum (15) comes from the region $n \sim \sqrt{s}/\kappa \gg 1$. So, *nonzero widths* of the gravitons in the RS model with the small curvature *should be taken into account* [4].

The analytical expression for $\mathcal{S}(s)$ was derived in Ref. [4]:

$$\mathcal{S}(s) = -\frac{1}{4\bar{M}_5^3\sqrt{s}}\ \frac{\sin 2A + i \sinh 2\varepsilon}{\cos^2 A + \sinh^2 \varepsilon}\ , \tag{18}$$

where

$$A = \frac{\sqrt{s}}{\kappa} + \frac{\pi}{4}\ , \qquad \varepsilon = \frac{\eta}{2}\left(\frac{\sqrt{s}}{\bar{M}_5}\right)^3 . \tag{19}$$

The following inequalities immediately result from (18):

$$-\coth\varepsilon \leqslant \mathrm{Im}\,\tilde{\mathcal{S}}(s) \leqslant -\tanh\varepsilon\ , \tag{20}$$

$$-\frac{1}{1 + 2\sinh^2\varepsilon} \leqslant \mathrm{Re}\,\tilde{\mathcal{S}}(s) \leqslant \frac{1}{1 + 2\sinh^2\varepsilon}\ , \tag{21}$$

where the notation $\tilde{\mathcal{S}}(s) = [2\bar{M}_5^3\sqrt{s}\,]\,\mathcal{S}(s)$ is introduced. The ratio $|\,\mathrm{Re}\,\mathcal{S}(s)/\mathrm{Im}\,\mathcal{S}(s)|$ decreases rapidly with energy, and it varies from 0.85 at $\sqrt{s} = \bar{M}_5$ to 0.08 at $\sqrt{s} = 3\bar{M}_5$. The absolute value of $\mathrm{Im}\,\tilde{\mathcal{S}}(s)$ tends to 1 very quickly when s grows.

Let us stress that the results obtained in zero mass approximation (16) do not agree with the exact expression (18), although the imaginary parts become practically the same in both cases in the trans-Planckian kinematical region, namely, at $\sqrt{s} > 3\bar{M}_5$.

Can one approximates the discrete spectrum by the continuous mass distribution, if the mass splitting Δm_{KK} is "very small"? The quantity Δm_{KK} is dimensional and it should be compared with another dimensional quantities, Γ_n. Actually, we may regard a set of narrow graviton resonances to be a continuous mass spectrum (within relevant interval of n), if only

$$\Delta m_{\mathrm{KK}} < \Gamma_n \tag{22}$$

is satisfied. Let us underline, it is the inequality that allows one *to replace a summation* in KK number n *by integration* over graviton mass m_{KK}.[3]

As was shown in [4], Eq. (22) is equivalent to

$$\sqrt{s} \gtrsim 3\bar{M}_5 \tag{23}$$

(see our comments on the imaginary part of $\mathcal{S}(s)$ after formula (21)).

It is a common belief that in the flat space-time with d compact ED's of the size R_c the mass splitting is so small ($\Delta m_{\mathrm{KK}} = R_c^{-1}$) that the continuous mass approximation is undoubtedly valid. Surprisingly, *it is not a case*. The reason is that the gravitons are extremely narrow resonances, $\Gamma_n \sim m_n^3/\bar{M}_{\mathrm{Pl}}^2$. Accounting for the hierarchy relation for d compact ED's (9) and inequality (22), one finds that only KK gravitons with unrealistically large masses,

$$m_n^3 > \bar{M}_{\mathrm{Pl}}^{2-\frac{2}{d}}\,\bar{M}_{4+d}^{1+\frac{2}{d}}\ , \tag{24}$$

are continuously distributed [4].

3 t-channel reggeized gravitons

Our formula (18) can be also applied to the scattering of the brane particles, induced by t-channel graviton exchanges [7]. In the region $-t \gg \kappa^2$, with t being 4-momentum transfer, the following expression has been obtained:

$$\mathcal{S}(t) = -\frac{1}{2\bar{M}_5^3\sqrt{-t}}\ . \tag{25}$$

Note that $\mathcal{S}(t)$ (25) is *pure real* and it coincides with the imaginary part of $\mathcal{S}(s)$ derived in the zero width approximation (16) up to the replacement $s \to -t$.

In more general approach, one should sum KK-charged *gravi-Reggeons*, i.e. graviton Regge trajectories $\alpha_n(t)$ which are numerated by the KK number n [8, 3]:

$$\alpha_n(t) = 2 + \alpha_g' t - \alpha_g'\, m_n^2, \quad n = 0, 1, \dots\ . \tag{26}$$

In such a case, the amplitude has both real and imaginary parts.

To detect effects induced by low-mass t-channel KK gravitons, it is necessary to look for their contributions to the scattering of the brane fields in the trans-Planckian kinematical region [3]

$$\sqrt{s} \gg -t,\ \bar{M}_5\ . \tag{27}$$

[3] From the point of view of experimental measurements, the mass splitting must be compared with the experimental resolution Δm_{res}. The spectrum looks continuous when $\Delta m_{\mathrm{KK}} < \Delta m_{\mathrm{res}}$, irrespective of Eq. (22).

In the eikonal approximation, the scattering amplitude in the kinematical region (27) is given by the infinite sum of gravi-Reggeons (reggeized gravitons in the t-channel).

Let us consider the scattering of ultra-high energy cosmic neutrinos off the atmospheric protons in order to compare effects induced by ED with the SM predictions [9]. The gravity contribution to neutrino-proton inelastic cross section,

$$\sigma_{\rm in}^{\nu \rm p}(s) = \int d^2 b \left\{ 1 - \exp \left[-2 {\rm Im}\, \chi_{\nu \rm p}(s,b) \right] \right\} \ , \tag{28}$$

is defined by the eikonal (b is an impact parameter):

$$\chi_{\nu \rm p}(s,b) = \frac{1}{4\pi s} \int\limits_0^\infty q_\perp dq_\perp \, J_0(q_\perp b) \, A_{\nu \rm p}^B(s, -q_\perp^2) \ . \tag{29}$$

The Born amplitude in Eq. (29) is given by [3]:

$$A_{\nu \rm p}^B(s,t) = \frac{\alpha_g' \, s^2}{2\sqrt{\pi}\, \bar{M}_5^3} \sum_i \int dx \, x^2 \, \frac{1}{R_g(sx)} \exp\left[t \, R_g^2(sx) \right] F_i(x,t) \ , \tag{30}$$

where $F_i(x,t)$ is a t-dependent distribution of parton i in momentum fraction x inside the proton. It coincides with a starndard parton distribution, $f_i(x)$, at $t = 0$. The quantity $R_g(s) = \alpha_g' \ln s$ is the gravitational interaction radius, where α_g' is the gravi-Reggeon slope (see Eq. (26)).

The gravitational part of the cross section is presented in Fig. 1 in comparison with the SM prediction, $\sigma_{\rm SM}$, and black hole production cross section, $\sigma_{\rm bh}$. For the latter, a geometrical form, $\sigma_{\rm bh} = \pi R_S^2(s)$, was assumed, where $R_S(s)$ is a 5-dimensional Schwarzschild radius. Thus, gravi-Reggeon interactions can dominate black hole production at $E_\nu > 10^9 - 10^{10}$ GeV, depending on the gravity scale $\bar{M}_5$ and minimal value of the black hole mass, $M_{\rm bh}^{\rm min}$.

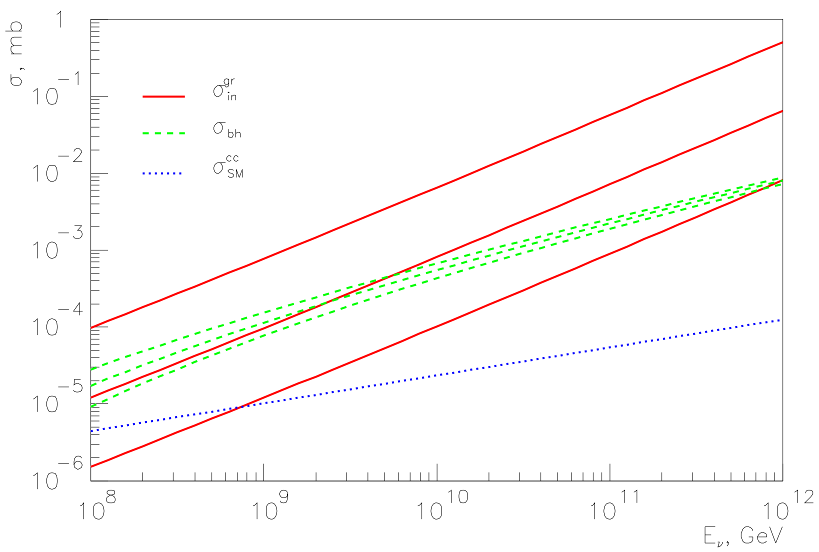

Figure 1. The gravitational inelastic neutrino-proton cross-sections (solid lines) vs. black hole production cross sections (dashed lines) and SM cross section (dotted line). The solid curves correspond to $\bar{M}_5 = 0.25$ TeV, 0.5 TeV, 1 TeV (from the top). The dash lines correspond to $\bar{M}_5 = 0.5$ TeV and $M_{\rm bh}^{\rm min} = 0.5$ TeV, 1 TeV, 2 TeV (from the top).

The interactions of ultra-high energy neutrinos with atmospheric nucleons can be probed in inclined (quasi-horizontal) air showers by the Pierre Auger Observatory [10]. In particular, for the "Waxman-Bahcall" neutrino flux [11], we expect about 5(2) such events per year for $\bar{M}_5 = 1(2)\ TeV$ [12].

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