LONG-RANGE GRAVITY, STRONG COUPLING EFFECT AND AUXILIARY FIELDS IN THE DGP MODEL

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We consider DGP model with additional auxiliary scalar fields in the action. The use of these fields results in exclusion of the strong coupled mode from the theory. At the same time effective theory on the brane appears to be the same, as in the original DGP model.

The model proposed in [1] (DGP model) possesses a very interesting feature – a modification of gravity at ultra-large scales. This effect is very interesting from the cosmological point of view, namely for possible explanation of the observed acceleration of our Universe [2]. Unfortunately the strong coupling effect was found in the model [3] — it manifests itself through the breakdown of linear approximation at unacceptably small distances. The strong coupled mode is nothing else but the 44-component of metric fluctuations [4]. There were proposed a lot of mechanisms of solution to this problem, such as taking into account the non-linear effects [5] or utilizing a specific regularization [6]. Here we present another mechanism for overcoming the strong coupling effect, which is based on adding an extra terms with auxiliary fields to the original action.

Action of the DGP model has the following form:

$$S_{DGP} = M_*^3 \int R\sqrt{-g}\, d^4x dy + \Omega M_*^3 \int_{y=0} \tilde{R}\sqrt{-\tilde{g}}\, d^4x + \int_{y=0} L_{matter}\sqrt{-\tilde{g}}\, d^4x, \tag{1}$$

where M_* is the five-dimensional Planck mass, $\Omega >> 1/M_*$ and $\tilde{g}_{\mu\nu}$ is the induced metric on the brane, which is located at the point $y = 0$ of the extra dimension, the last term in (1) describes matter on the brane. In addition, let us suppose that the model possesses $y \leftrightarrow -y$ symmetry, which fixes the brane position.

In paper [7] it was shown that solution for the 44-component of metric fluctuations has the following form

$$\Box\, h_{44} = e^{-k|y|}\frac{\hat{\kappa}k}{6}\eta^{\mu\nu}t_{\mu\nu}, \tag{2}$$

where $\hat{\kappa} = \frac{1}{\sqrt{M_*^3}}$; $g_{MN} = \eta_{MN} + \hat{\kappa}h_{MN}$; $M, N = 0,..,4$; $\mu,\nu = 0,..,3$; $\eta_{MN} = diag(-1,1,1,1,1)$ is the flat five-dimensional background metric, h_{MN} denote fluctuations of metric, $t_{\mu\nu}(x)$ is the energy-momentum tensor of matter on the brane, $\Box = \eta^{\mu\nu}\partial_\mu\partial_\nu$ and k is some parameter.

It is evident that such coupling of the field h_{44} to matter, defined by equation (2), makes $\hat{\kappa}h_{44}$ to be much larger than unity if $\Omega >> 1/M_*$ (for example, in the case $M_* \sim 10^{-3}eV$ and if k is chosen to be of the order of M_*) at least at $y = 0$. Linearized approximation breaks down, and this is the origin of the so-called strong coupling effect.

To solve this problem let us add to the action (1) the following terms

$$S_{extra} = M_*^3 \int_{y=0} \varphi(x)\tilde{R}\sqrt{-\tilde{g}}\, d^4x + \int \Phi(x,y)R^2\sqrt{-g}\, d^4xdy\ . \tag{3}$$

The first term in (3) corresponds to the four-dimensional Brans-Dicke theory with Brans-Dicke parameter equal to zero. We also assume that $\varphi(x) \equiv 0$ in the background, otherwise the redefinition of Ω in equation (1) is needed.

From the equation of motion for the field $\Phi(x, y)$ we get equation

$$R = 0, \tag{4}$$

which holds in the bulk.

Linearized equations of motion, corresponding to action (1) and (3), take the form [7]:

1) $\mu\nu$-component

$$\begin{aligned}
& \frac{1}{2}\left(\Box h_{\mu\nu} - \partial_\mu \partial^\rho h_{\rho\nu} - \partial_\nu \partial^\rho h_{\rho\mu} + \partial_4 \partial_4 h_{\mu\nu} + \partial_\mu \partial_\nu h + \partial_\mu \partial_\nu h_{44}\right) + \\
+ \; & \frac{1}{2}\gamma_{\mu\nu}\left(\partial^\rho \partial^\sigma h_{\rho\sigma} - \Box h - \partial_4 \partial_4 h - \Box h_{44}\right) + \\
+ \; & \frac{\Omega}{2}\delta(y)\left[\left(\Box h_{\mu\nu} - \partial_\mu \partial^\rho h_{\rho\nu} - \partial_\nu \partial^\rho h_{\rho\mu} + \partial_\mu \partial_\nu h\right) + \gamma_{\mu\nu}\left(\partial^\rho \partial^\sigma h_{\rho\sigma} - \Box h\right)\right] + \\
+ \; & \frac{1}{\hat{\kappa}}\left(\partial_\mu \partial_\nu \varphi - \eta_{\mu\nu} \Box \varphi\right)\delta(y) = -\frac{\hat{\kappa}}{2} t_{\mu\nu}(x)\delta(y),
\end{aligned} \tag{5}$$

2) $\mu 4$-component

$$\partial_4(\partial_\mu h - \partial^\nu h_{\mu\nu}) = 0, \tag{6}$$

3) 44-component (it coincides with equation of motion for the field φ)

$$\partial^\mu \partial^\nu h_{\mu\nu} - \Box h = 0, \tag{7}$$

4) auxiliary equation, which is obtained by substituting the equation for 44-component into the contracted equation for $\mu\nu$-component

$$\partial_4 \partial_4 h + \Box h_{44} + \frac{2}{\hat{\kappa}}\Box \varphi \delta(y) = \frac{\hat{\kappa}}{3} t(x)\delta(y). \tag{8}$$

The second term in (3) does not contribute to the equations since it contains R squared, and the main purpose of this term is to make the five-dimensional curvature equal to zero in the bulk.

It is necessary to note that we use the gauge $h_{\mu 4} = 0$ and $h_{44}(x, y) = e^{-k|y|}\phi(x)$ (see [7] for details).

Linearizing equation (4) and taking into account (7), we get

$$\partial_4 \partial_4 h + \Box h_{44} = 0. \tag{9}$$

It means that

$$\Box \varphi = \frac{\hat{\kappa}^2}{6} t \; . \tag{10}$$

Integrating (9) in the limits $(-\infty, \infty)$ and using the physical boundary conditions for the field $h_{\mu\nu}$ (it must vanish at spatial infinity), we get

$$\Box h_{44} = 0. \tag{11}$$

We see, that h_{44}-field does not interact with matter on the brane, i.e. it decouples from the effective theory. This also means that the linear approximation is valid. Moreover, since this field does not interact with matter, we can totally gauge it out [7]. From (6), (7), (9), (11) and physical boundary conditions for the field $h_{\mu\nu}$ it follows that

$$h = 0, \quad \partial^\mu h_{\mu\nu} = 0. \tag{12}$$

Let us explain what happens. The field φ takes over the role of the 44-component of metric fluctuations. Since this field does not exist "inside" the curvature, one does not need to worry about its absolute value – it does not affect the validity of linear approximation. At the same time the field Φ makes h_{44} not interacting with matter on the brane (it is not necessarily so if the field Φ is absent).

Now we are ready to find the equation for the $\mu\nu$-component. It can be easily obtained from (5) and takes the well-known form

$$\Box h_{\mu\nu} + \partial_4\partial_4 h_{\mu\nu} + \Omega\delta(y)\Box h_{\mu\nu} = -\hat{\kappa}\delta(y)\left(t_{\mu\nu} - \frac{1}{3}\left[\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{\Box}\right] t\right). \tag{13}$$

It coincides with that derived in [1] and was examined in a lot of papers, so we will not discuss it here. All the predictions obtained with the help of this equation are valid in this case too.

One can worry about the fact that the field Φ is not defined by the equations of motion. This problem can be easily solved by adding an extra term to the action, for example, one can choose

$$S_\Phi = \frac{\alpha}{3}\int \Phi^3(x,y)\sqrt{-g}\, d^4x dy\,, \tag{14}$$

where α is real and positive. In this case equation (4) will take the following form:

$$R^2 + \alpha\Phi^2 = 0\,, \tag{15}$$

which uniquely determines the field Φ ($\Phi \equiv 0$).

Here we do not consider the problem of the incorrect tensor structure of the graviton, which is analogous to the vDVZ-discontinuity in massive gravity [8]. This problem is inherent to the original DGP model too and was discussed in a lot of papers including [1]. Hence the modification of the DGP model proposed above does not make the model applicable for describing gravity in our world. Nevertheless one can hope that the use of additional fields non-minimally coupled to gravity (not necessarily without kinetic and potential terms) can be quite efficient in some more realistic models, which admit modification of gravity at ultra-large scales and suffer from the problem of strong coupling effect.

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