QUANTUM BLACK HOLE AND HAWKING RADIATION AT MICROSCOPIC MAGNIFYING

V.V. Kiselev [1]

Russian State Research Center "Institute for High Energy Physics", Protvino, Russia

We establish a state of stopping the Hawking radiation by quantum Schwarzschild black hole in the framework of quasi-classical thermal quantization for particles behind the horizon. The mechanism of absorption and radiation by the black hole is presented

1 Introduction

The Hawking radiation of black hole [1, 2] has got a deviation from thermal black-body spectrum of energy. A reason for such the grey-body spectrum is the quantum-mechanical rescattering of particle propagating from horizon to observer at infinity, by the gravitational potential of black hole. Let us illustrate this point by the simplest case of Schwarzschild black hole. The metric is defined by

$$\mathrm{d}s^2 = g_{tt}(r)\,\mathrm{d}^2t - \frac{1}{g_{tt}(r)}\,\mathrm{d}r^2 - r^2\,[\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2], \tag{1}$$

with

$$g_{tt}(r) = 1 - \frac{r_g}{r}, \tag{2}$$

where r_g is the radius of Schwarzschild sphere, i.e. the single horizon of black hole.

For radial geodesics, the action on a trajectory can be represented as

$$S_{HJ} = -E\,t + \mathcal{S}_{HJ}(r), \tag{3}$$

where E is a conserved energy of massive particle, that defines the dimensionless integral of motion

$$A = \frac{m^2}{E^2}. \tag{4}$$

The Hamilton–Jacobi equation reads off

$$\begin{aligned} \frac{1}{m^2}\left(\frac{\partial\mathcal{S}_{HJ}}{\partial r_*}\right)^2 &= \mathcal{E}_A - U(r), \\ \mathcal{E}_A &= \frac{1}{A}, \quad U(r) = g_{tt}(r) = 1 - \frac{r_g}{r}, \end{aligned} \tag{5}$$

where the 'tortoise' coordinate is defined by

$$r_* = \int \frac{\mathrm{d}r}{g_{tt}(r)} = r + r_g \ln\left[\frac{r}{r_g} - 1\right]. \tag{6}$$

The Hamilton–Jacobi equation of (5) states the following: the total energy $\mathcal{E}_A$ is the sum of kinetic term (radial derivatives) and potential term U.

The problem of radial motion is illustrated by Fig. 1, wherein we have depicted the 'potential' and reflection of outgoing waves, which are shown schematically [2].

---

[1] kiselev@th1.ihep.su

[2] The reflected wave has the same energy as the outgoing wave, of course.

Therefore, there is a nonzero coefficient of reflection, which causes the grey-body factors depending on the particle energy. The Hawking radiation originates from vacuum fluctuations in vicinity of horizon. The virtual fluctuations are transformed to observed particles due to action of gravitational field. The pair of particles in fluctuation is separated by the field in the following way: the positive energy particle goes to infinity, while the negative energy particle falls behind the horizon diminishing the total mass of black hole in the exact balance with the radiated energy.

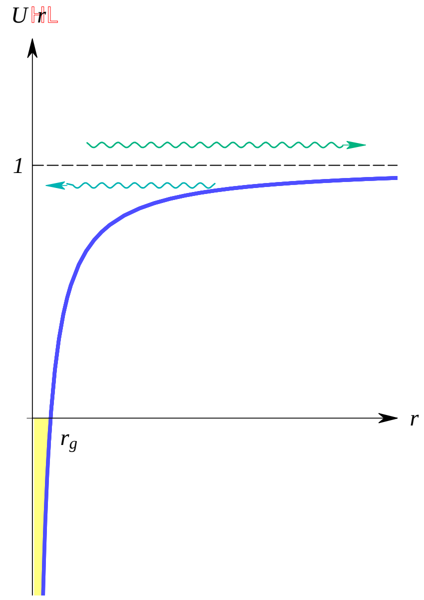

Figure 1. The potential of radial motion and quantum reflection of particle travelling to infinity.

In actual analogy with reasons for the appearance of grey-body factors, let us make the following question: Can the rescattering of falling particle lead to its reflection by the black hole? If such the reflection does exist, it could suppress the absorption of particle by the black hole, and, hence, particularly the Hawking radiation, too. Moreover, the total reflection would result in the complete stopping of Hawking radiation.

In order to answer this question we use the quasi-classical thermal approach to the space-time behind the horizon as it was recently developed in [3, 4]. The introduction of such new framework is required by the consideration of trajectories completely confined behind the horizon. Eq. (5) and Fig. 1 clearly show that mentioned trajectories correspond to

$$\mathcal{E}_A \leqslant 0\,,$$

implying imaginary values of energy and time treated as the indication of statistical description. Namely, the period of motion in imaginary time equals the inverse temperature of particles in thermal ensemble existing behind the horizon. The thermal equilibrium demanding the exact periodicity leads to the quasi-classical quantization *a la* the introduction of old Bohr orbits for the particles confined behind the horizon. In the present paper we investigate the ground quantum state of Schwarzschild black hole and its excitations in detail. We show how transitions between the excitations lead to the Hawking radiation with the Gibbs distribution in energy

$$w \sim \mathrm{e}^{-\beta\, E},$$

where $\beta = 1/T$ is the inverse temperature of black hole, and E is the energy of radiation quantum. Then we find the condition, when the Hawking radiation is stopped: the black hole finishes to radiate, if all the excitations have transited to the ground state.

The paper is organized as follows: In section II we remind basic points in classifying the radial trajectories confined behind the horizon and develop the analysis by separating regular contributions described in [3, 4] and so-called "mute" terms, which are analogous to the Dirac sea, since they are cancelled in the total energy and partition function. Then we apply the energy conservation in order to describe excitations. The excitations are formed by energetic particles coupled with their "antipodes", which have an opposite sign of Euclidean energy. In section III we present the mechanism of particle absorption by the black hole and the limit of large quantum numbers for the Hawking radiation, which, by derivation, obeys the Gibbs distribution. The ground state of quantum black hole does not produce any Hawking radiation. Section IV summarizes our conclusions.

2 Quantum levels

The causal geodesics of massive particle confined behind the horizon with time-like intervals can be described in terms of metric for the Schwarzschild black hole [3]

$$\mathrm{d}s^2 = \frac{r_g}{r}\cdot e^{-\frac{r}{r_g}}\cdot(\mathrm{d}\rho^2+\rho^2\mathrm{d}\varphi_E^2) - r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\phi^2), \tag{7}$$

where

$$\begin{cases} t = & -\mathrm{i}\,2r_g\,\varphi_E, & \varphi_E \in & [0,2\pi], \\ r_* = & 2r_g\ln\left[-\dfrac{\rho}{2r_g}\right], & \rho \in & [0,2r_g], \end{cases} \tag{8}$$

and the Euclidean phase φ_E has a period equal to 2π, corresponding to the inverse temperature

$$\beta = 4\pi r_g. \tag{9}$$

The trajectory is determined by

$$\frac{\mathrm{d}\varphi_E}{\mathrm{d}r} = \frac{1}{2r_g}\sqrt{1-\frac{r_c}{r_g}}\,\frac{r}{r_g-r}\sqrt{\frac{r}{r_c-r}}, \tag{10}$$

where the maximal remoteness of particle from the singularity at $r=0$ is

$$r_c = -r_g\,\frac{A}{1-A} \leqslant r_g, \qquad \text{at } A<0. \tag{11}$$

A *regular* cycle is determined by (10), and its value for φ_E is equal to the phase increment during motion from the singularity to r_c and back

$$\Delta_c\varphi_E = 2\int_0^{r_c}\mathrm{d}r\,\frac{\mathrm{d}\varphi_E}{\mathrm{d}r} = \frac{\pi}{2}\left[2-(2+x)\sqrt{1-x}\right], \tag{12}$$

with $x = r_c/r_g$. A new feature we involve is a so-called "mute" solution of (10) defined by

$$r \equiv r_c, \qquad \text{at } r_c < r_g, \tag{13}$$

which is possible because of singularity in (10), since

$$\frac{\mathrm{d}r}{\mathrm{d}\varphi_E} \equiv 0, \qquad \text{at } r \equiv r_c < r_g. \tag{14}$$

If we ignore the "mute" solution, then the regular number of cycles per period is naively given by

$$\tilde{n} = \frac{2\pi}{\Delta_c\varphi_E}, \tag{15}$$

which we call the "winding number". The examples of geodesics are shown in Fig. 2.

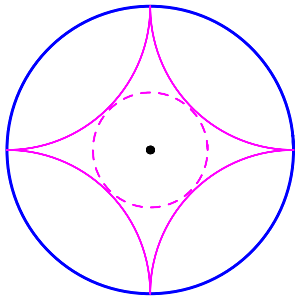

Figure 2. The geodesics confined behind the horizon in polar coordinates $\{\rho, \varphi_E\}$: the winding number equal to 4 is shown by solid arcs, the "mute" trajectory with the same r_c is depicted by the dashed circle. The central point corresponds to the horizon at $r = r_g$, while the solid circle is the singularity.

However, due to the "mute" geodesics the particle can get a mixed regular-mute trajectories as shown in Fig. 3, since both geodesics are tangent at maximal distance from the singularity. We can say that the regular trajectory can adhere to the "mute" path at $r = r_c$. Therefore, one should determine a role of "mute" geodesics.

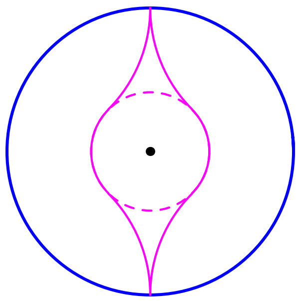

Figure 3. The mixed geodesics containing "mute" parts.

In this way, let us, first, calculate the action on the mute trajectory ($\mathrm{d}r \equiv 0$):

$$S_{\text{mute}} = -\mathcal{E} \oint \mathrm{d}t_E, \tag{16}$$

where the 'Euclidean' time is given by

$$t_E = \frac{\beta}{2\pi}\,\varphi_E, \tag{17}$$

while $\mathcal{E}$ is the 'Euclidean' energy, so that $A = -m^2/\mathcal{E}^2$. Then, the action per period is

$$S_{\text{mute}} = -\beta\,\mathcal{E}. \tag{18}$$

Emphasize, the "mute" action depends on the sign of energy $\mathcal{E}$.

Second, let us exploit the energy conservation. Then, the total energy of black hole is its mass M, which is a real number. The energy $\mathcal{E}$ represents the imaginary contribution to the total energy. Therefore, this contribution should be cancelled by opposite term. So, we **postulate** that for each particle on the "mute" geodesics there is the "anti-mute" particle with opposite energy. Thus, the sum of actions will nullify:

$$S_{\text{mute}} + S_{\text{anti-mute}} \equiv 0. \tag{19}$$

Therefore, the "mute"—"anti-mute" pair does not contribute to the total energy as well as to the partition function: the mute terms are cancelled. Then, we can omit the mute part from the action in a full analogy with the Dirac sea.

However, "mute" pathes can contribute indirectly. Indeed, for the regular term, the increment of action per cycle is given by

$$\Delta_c S = -m\,\pi\,r_g\,x^{3/2}. \tag{20}$$

Note, it is independent on the sign of energy $\mathcal{E}$. Then, the action per period is determined by the increment and the number of cycles corrected by the phase belonging to the mute part,

$$n = \frac{2\pi - \Delta_{\text{mute}}\varphi_E}{\Delta_c\varphi_E}, \tag{21}$$

so that

$$S = -m\,\pi\,r_g\,x_n^{3/2}\,n, \tag{22}$$

where we explicitly mark the maximal remoteness by the natural index of actual winding number in terms of x: $r_c = x_n\,r_g$. The corresponding term of mute path at the trajectory is given by

$$\Delta S_{\text{mute}} = -\beta\,\mathcal{E}\,\frac{\Delta_{\text{mute}}\varphi_E}{2\pi}. \tag{23}$$

By energy conservation, we **postulate** that the same trajectory with the opposite sign of energy should accompany the particle. For such the mirror trajectory, we call the "antipode". Then, the "mute" terms are cancelled again, while the regular contribution of particle–antipode pair is double:

$$S_{\text{pair}} = -2m\,\pi\,r_g\,x_n^{3/2}\,n. \tag{24}$$

This postulate has two items.

i) At the ground level, $\mathcal{E} \equiv 0$, particles and their antipodes are indistinguishable; there are no mute pathes.

ii) Excitations of ground level exist in the form of "particle–antipode" pairs.

For definiteness, we have to fix the value of phase shift $\Delta_{\text{mute}}\varphi_E$. A spectacular way is making use of correspondence principle: at large winding numbers the quasi-classical description has to match with the classical dynamics. In our case this principle implies the following:

The limit of $n \gg 1$ corresponds to $x \to 0$. Then, the phase increment is given by

$$\Delta_c\varphi_E \to \frac{3\pi}{8}\,\frac{1}{x^2}\,, \tag{25}$$

while

$$m = |\mathcal{E}|\sqrt{-A} \to |\mathcal{E}|\,x^{1/2}. \tag{26}$$

Therefore,

$$S_{\text{pair}} = -\beta\,\mathcal{E}_{\text{pair}} \cdot \frac{4}{3}\,\frac{2\pi - \Delta_{\text{mute}}\varphi_E}{2\pi}\,, \tag{27}$$

where the positive energy of particle–antipode pair is defined by

$$\mathcal{E}_{\text{pair}} = 2\,|\mathcal{E}|. \tag{28}$$

As we have demonstrated in [3, 4], the action at such the trajectories with imaginary time contributes to the partition function in the standard way:

$$w = \mathrm{e}^{S}, \tag{29}$$

which should restore the Gibbs distribution in the classical limit of $n \gg 1$,

$$w = \mathrm{e}^{-\beta\,E}.$$

Then, the correspondence principle dictates

$$\Delta_{\text{mute}}\varphi_E = \frac{\pi}{2}, \qquad \text{at} \quad r_c < r_g, \tag{30}$$

where we remind that this prescription is valid for the excited levels, while at the ground level $\Delta_{\text{mute}}\varphi_E \equiv 0$, since there are no "mute" trajectories for massive particles at $r = r_g$ (the interval is exactly equal to zero).

At the moment, we can easily consider two limits in calculating the partition function.

1. If all particles are at the ground level, then the sum of actions is reduced to the sum of particle masses
$$G = \sum S = -\frac{\beta}{2}\sum m = -\frac{1}{2}\,\beta\, M,$$
where M is the black hole mass:
$$M = \sum m.$$
The thermodynamical function G reproduces the correct value for the product of inverse temperature to the free energy $\mathcal{F}$
$$G = -\beta\,\mathcal{F}, \qquad \mathcal{F} = \frac{1}{2}\,M.$$

2. If all particles are highly excited, then the sum of energies for the particle–antipode pairs gives the black hole mass again;
$$M = \int\limits_0^M \mathrm{d}\mathcal{E}_{\text{pair}},$$
where we have introduced the notation of differential for the energy of single pair, which is infinitely small in comparison with the total energy: $\mathcal{E}_{\text{pair}} \mapsto \mathrm{d}\mathcal{E}_{\text{pair}}$, and we insert the integration instead of sum over pairs. In that case, the temperature is determined by the summed energy of other pairs, so that
$$\beta = 8\pi\, M \mapsto \beta(\mathcal{E}_{\text{pair}}) = 8\pi\,\mathcal{E}_{\text{pair}},$$
hence, the G function is obtained by the integration
$$G = -\int\limits_0^M \beta(\mathcal{E}_{\text{pair}})\,\mathrm{d}\mathcal{E}_{\text{pair}} = -4\pi\,\mathcal{E}^2_{\text{pair}}\Big|_0^M = -\frac{1}{2}\,\beta\, M,$$
which again reproduces the correct value.

One can easily recognize that the mixed situation can be described as a simple combination of two limits above. It is spectacular, that the mass of black hole is given by the sum of both masses for particles at the ground level and energies of particles and antipodes at excited levels.

We show the schematic structure of quantum black hole in Fig. 4.

Finally, let us present a simple analogy for the explanation of phase shift at excited levels and its absence at the ground level. We start from the statement on the periodic motion

$$\oint \mathrm{d}\varphi_E = 2\pi. \tag{31}$$

In order to show the analogy with the quasi-classical quantization we can multiply by the winding number, so that

$$\oint n \cdot \mathrm{d}\varphi_E = 2\pi\, n. \tag{32}$$

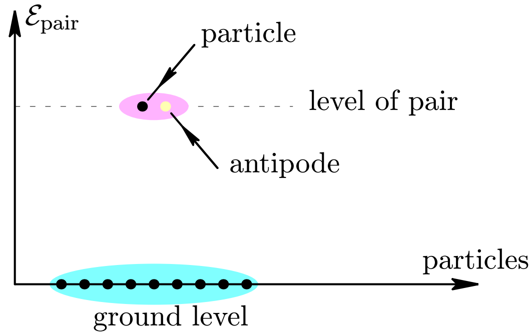

Figure 4. Quantum structure of Schwarzschild black hole.

This relation supports the interpretation of φ_E as the dynamical variable and n as the canonically conjugated momentum of φ_E.

Next, we isolate the regular term, which is given by the integration over the distance,

$$\oint_{\text{reg.}} n \cdot \mathrm{d}\varphi_E = n \cdot 2n \int_0^{r_c} \frac{\mathrm{d}\varphi_E}{\mathrm{d}r}\,\mathrm{d}r = n^2 \cdot \Delta_c\varphi_E\,. \tag{33}$$

This regular term is a complete analogue of complex phase for the wavefunction in the quasi-classical approximation,

$$\oint_{\text{reg.}} \mathcal{P}\mathrm{d}\mathcal{Q}\,,$$

where $\mathcal{Q}$ is a dynamical variable, and $\mathcal{P}$ is its momentum.

Then, Fig. 1 shows that each reflection of trajectory at $r = r_c$ gives the additional phase equal to $-\pi/2$ for the reflected wave with respect to the complex phase of wave falling to $r = r_c$, and the additional phase equal to π for the reflection at $r = 0$, since the potential wall at $r \leqslant 0$ is infinitely high. The quasi-classical quantization by Bohr–Sommerfeld results in

$$\oint_{\text{reg.}} n \cdot \mathrm{d}\varphi_E = 2\pi\, n + n \cdot \left(\frac{\pi}{2} - \pi\right) = n \cdot \frac{3\pi}{2}. \tag{34}$$

Therefore, the true winding number at $r_c < r_g$ is given by

$$n = \frac{3\pi}{2\Delta_c\varphi_E}. \tag{35}$$

The quantization by Bohr–Sommerfeld is quite accurate, if $r_c \ll r_g$ because of the following reason: In fact, we expect that the wavefunction should nullify at $r = r_g$, which is possible, if both rising and falling exponents contribute at $r > r_c$. However, we can neglect the rising exponent, which compensates the tail of falling exponent at $r = r_g$, if $r_c \ll r_g$. Then, the rules applied are justified.

The situation is slightly changed for the ground state, since the wavefunction should nullify at $r = r_g$, which implies the complete reflection of wave at right return point. The corresponding phase between falling and reflected waves is equal to $-\pi$ instead of $-\pi/2$. Then, the regular equation for the winding number at the ground state is restored,

$$n_{\text{ground}} = \frac{2\pi}{\Delta_c\varphi_E}, \tag{36}$$

in a full agreement with the consideration in [3, 4].

The above study supports the following representation of wavefunction at $r < r_c < r_g$:

$$\Psi_{r>0} \sim +\cos\left(n\int_0^r \frac{\mathrm{d}\varphi_E}{\mathrm{d}r}\,\mathrm{d}r + \frac{\pi}{2}\right), \tag{37}$$

$$\Psi_{r<r_c} \sim -\cos\left(n\int_{r_c}^r \frac{\mathrm{d}\varphi_E}{\mathrm{d}r}\,\mathrm{d}r + \frac{\pi}{4}\right), \tag{38}$$

which are identical at

$$\frac{1}{2}\,n\Delta_c\varphi_E = \pi + \left(\frac{\pi}{4} - \frac{\pi}{2}\right), \tag{39}$$

that restores the quantization rule. The modification for the ground state is transparent.

This consideration provides us with the evidence for the reasonable treatment of quantum levels in the black hole.

In accordance with (24), (35) and (36), the quantum action per single particle takes discrete values shown in Fig. 5, that slightly corrects naive Fig. 5 of [3] because of modified winding number.

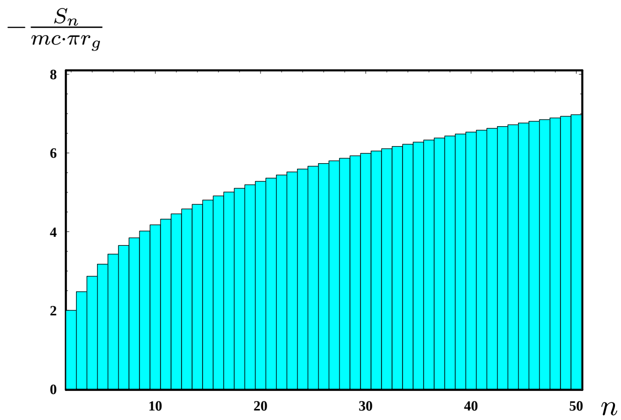

Figure 5. The quantum action.

Thus, the quantum black hole is composed by the particles occupying the ground levels as well as the excited levels. The number of excitations depends on initial conditions before the collapse, which determine the mass of black hole. The spectrum of levels is discrete, though at high energy of excitation one could apply the classical description with the Gibbs distribution.

3 Absorption and Hawking radiation

Let us consider a change of state for the quantum black hole due to absorption of external particle falling behind the horizon. Such the particle has a positive total energy (with respect to observer at $r = \infty$). The basic point is the conservation of energy: the energy of falling particle increases the mass of black hole. We suggest that falling to singularity causes the change of black hole mass and a transition of the particle to a quantum level. The transition depends on the energy of particle.

First, consider the case of $0 < E < m$, i.e. the particle, which binding energy is so large, that the maximal distance from the horizon is finite. Then, the mechanism of absorption is shown in Fig. 6: the falling particle occupies the ground level, while the excited particle–antipode pair gets a transition to the lower level.

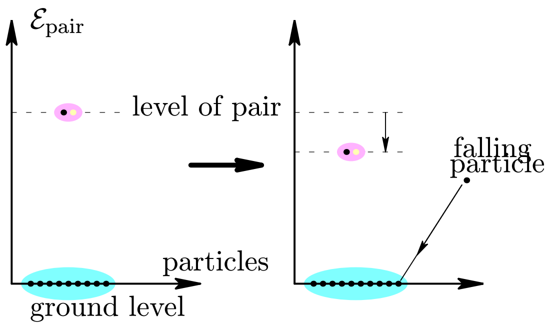

Figure 6. The mechanism of particle absorption at $E < m$.

Due to energy conservation, the shift of level is given by the difference of black hole mass change and the mass of absorbed particle:

$$\mathcal{E} - \mathcal{E}' = \mathrm{d}M - m = -\mathrm{d}\mathcal{E}, \qquad \mathrm{d}M = E. \tag{40}$$

The partition function gets the correct change, of course: the entropy increases by the appropriate value.

The case of $E > m$, i.e. the particle, which can have a nonzero velocity at infinity, is quite analogous: the excited level gets a transition to a higher one (see Fig. 7). In the limit of extra large energy $E \gg m$, one could neglect the discreteness of levels, so that the falling particle could excite an antipode from the ground level in order to form a pair at large energy as shown in Fig. 8.

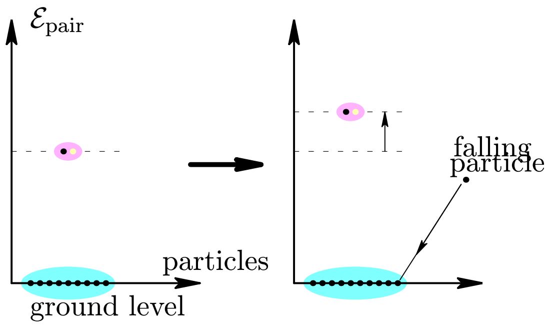

Figure 7. The mechanism of particle absorption at $E > m$.

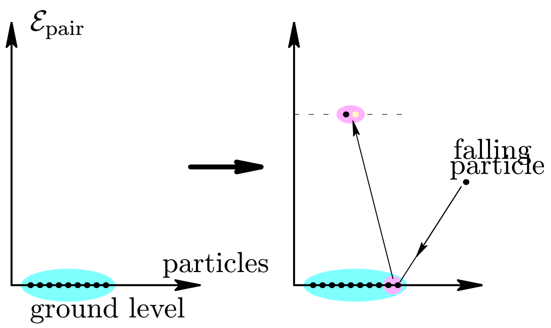

Figure 8. The mechanism of particle absorption at $E \gg m$.

Quantum restrictions to the absorption are quite evident. First, we have the discrete spectrum of absorption, and this fact is especially important at $E \sim m$. Second, the quantum black hole will not absorb a particle with $E < m$, *if the black hole is completely at the ground level*, i.e. if there is no excitations. Then, the quantum black hole can totally reflect the falling particle, if the black hole is at the ground level. Even at $E > m$, the energy could be too low in order to excite a higher level of particle–antipode pair from the ground state. As we have already mentioned in Introduction, such the total reflection should affect the Hawking radiation, too.

Indeed, at $E > m$ we can simply invert the arrows in Figs. 7 and 8 in order to get the process of radiation [3]. Then, the falling particle is inverted to the outgoing particle of Hawking radiation reaching an observer at infinity.

At $E \gg m$, we can easily estimate the probability of radiation, since it is equal to the probability that the particle–antipode pair *was* at the excited level, which is equal to

$$w = \mathrm{e}^{-\beta\,\mathcal{E}_{\mathrm{pair}}}, \tag{41}$$

while the energy of pair is transmitted to the radiated quantum (we neglect the mass of the particle):

$$E \approx \mathcal{E}_{\mathrm{pair}}, \qquad \mathrm{d}M \approx -\mathcal{E}_{\mathrm{pair}}. \tag{42}$$

Therefore, we find the Gibbs distribution for the radiation of single quantum, that leads to the black-body spectrum of radiation, corrected by appropriate grey-body factors because of rescattering on the gravitational potential [4].

The consequence of such quantum mechanism for the Hawking radiation is clear: the radiation is completely stopped, if the quantum black hole is at the ground level!

Highly excited levels correspond to classical description of black hole, since particles move in the very vicinity to the singularity. The ground state is extremely coherent: particles homogeneously occupy all of the space behind the Schwarzschild sphere. Such the coherence could result in the holographic state [6]: the state on the horizon sphere is equivalent to the quantum state of whole black hole. However, the strict statement requires an exact quantum theory of black holes, not the quasi-classical approximation, although we expect that the approximation used presents a rather valid qualitative picture for the quantum black hole.

4 Conclusion

In short, we have described the inner quantum structure of Schwarzschild black hole in the framework of quasi-classical thermal approach. A particle in the thermal ensemble has the ground state as well as excitations formed by a particle–antipode pair. The antipode has the opposite sign of energy with respect to the particle, that follows from the energy conservation. The existence of ground state leads to stopping of Hawking radiation, after all excited states have decayed to the ground level. We have studied the mechanism of particle absorption and emission. The Gibbs distribution for the excitations has been obtained.

We have considered the absorption and radiation of massive particles by hot black hole. What modifications have to be introduced, if we would involve neutral massless particles? We expect that transitions of excited levels for massive particles to lower ones will produce the Hawking radiation

[3] Inverting the process in Fig. 6 could be forbidden, if the particle–antipode pair is excited from the ground state because of the energy balance: loosing the particle of mass m from the ground level should exceed the increase of excitation energy, which could be not possible for some excited levels. However, Fig. 6 presents an alternative mechanism of particle emission: if the quantum black hole has many excitations, particles at the ground level could evaporate by exciting the existing higher levels. However, the emitted particles would fall back to the black hole, since their energies are restricted by the mass, $E < m$.

[4] The Hawking radiation as a tunnel-effect of pair creation by the gravitational field in the quasi-classical approximation was considered in [5].

of such neutral massless particles, until all excitations decay to the ground level, again. Then, the radiation will stop, since no energy can be extracted from the ground quantum state of black hole. In this way, we get two rather dim items: a mechanism of energy transition from massive particles to massless ones, and a sense of quantum orbits for the massless particles inside the black hole.

This work is partially supported by the grant of the president of Russian Federation for scientific schools NSc-1303.2003.2, and the Russian Foundation for Basic Research, grant 04-02-17530.

References

[1] S. W. Hawking, Commun. Math. Phys. **43**, 199 (1975);

J. B. Hartle and S. W. Hawking, Phys. Rev. D **13**, 2188 (1976).

[2] S. W. Hawking, Phys. Rev. D **13**, 191 (1976);

G. W. Gibbons and S. W. Hawking, Phys. Rev. D **15**, 2738 (1977);

S. W. Hawking and G. T. Horowitz, Class. Quant. Grav. **13**, 1487 (1996) [arXiv:gr-qc/9501014].

[3] V. V. Kiselev, arXiv:gr-qc/0412090.

[4] V. V. Kiselev, arXiv:gr-qc/0412097, arXiv:gr-qc/0412110, arXiv:gr-qc/0412117.

[5] E. Keski-Vakkuri and P. Kraus, Phys. Rev. D **54**, 7407 (1996) [arXiv:hep-th/9604151];

M. K. Parikh and F. Wilczek, Phys. Rev. Lett. **85**, 5042 (2000) [arXiv:hep-th/9907001];

M. Angheben, M. Nadalini, L. Vanzo and S. Zerbini, arXiv:hep-th/0503081;

E. C. Vagenas, Phys. Lett. B **559**, 65 (2003) [arXiv:hep-th/0209185];

E. C. Vagenas, Phys. Lett. B **533**, 302 (2002) [arXiv:hep-th/0109108];

E. C. Vagenas, Mod. Phys. Lett. A **17**, 609 (2002) [arXiv:hep-th/0108147];

M. Arzano, A. J. M. Medved and E. C. Vagenas, arXiv:hep-th/0505266;

K. Srinivasan and T. Padmanabhan, Phys. Rev. D **60**, 024007 (1999) [arXiv:gr-qc/9812028];

T. Padmanabhan, Mod. Phys. Lett. A **19**, 2637 (2004) [arXiv:gr-qc/0405072];

T. Padmanabhan, Phys. Rept. **406**, 49 (2005) [arXiv:gr-qc/0311036].

[6] G. 't Hooft, arXiv:gr-qc/9310026;

L. Susskind, J. Math. Phys. **36**, 6377 (1995) [arXiv:hep-th/9409089];

R. Bousso, Rev. Mod. Phys. **74**, 825 (2002) [arXiv:hep-th/0203101].