NUMERICAL SPHERICALLY SYMMETRIC STATIC SOLUTION OF THE RTG EQUATIONS OUTSIDE THE MATTER

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There was obtained a numerical external solution for the exact system of the RTG equations with some natural boundary conditions in the static spherically symmetric case. The properties of the solution are discussed.

The Relativistic Theory of Gravity (RTG) is the theory of the self-interacting massive tensor field in the Minkowski space. The exact system of the RTG field equations is in form of [1]

$$\gamma^{\alpha\beta} D_\alpha D_\beta \phi^{\mu\nu} + \frac{1}{r_0^2} \phi^{\mu\nu} = \frac{16\pi G}{c^4} t^{\mu\nu} \, ,$$

$$D_\mu \phi^{\mu\nu} = 0 \, , \tag{1}$$

where $\gamma^{\mu\nu}$ is the metric of the Minkowski space, D_μ is the covariant derivative in the Minkowski space, $\phi^{\mu\nu}$ is the physical gravitational field, $r_0 \equiv \frac{\hbar}{m_g c}$, m_g is the gravitational field mass, $\hbar$ is the Planck constant, c is the speed of light in vacuum, G is the gravitational constant and $t^{\mu\nu} = t_M^{\mu\nu} + t_g^{\mu\nu}$ is the energy-momentum tensor both of matter (except gravitational) fields and the gravitational field itself.

Due to *the geometrization principle* the system (1) may be also represented as the Hilbert-Einstein type system of equations [1]

$$R^{\mu\nu} - \frac{1}{2} g^{\mu\nu} R + \frac{1}{2r_0^2} \left[g^{\mu\nu} + \left(g^{\mu\alpha} g^{\nu\beta} - \frac{1}{2} g^{\mu\nu} g^{\alpha\beta} \right) \gamma_{\alpha\beta} \right] = \frac{16\pi G}{c^4} T^{\mu\nu} \, ,$$

$$D_\mu (\sqrt{-g} g^{\mu\nu}) = 0 \, ,$$

where $g^{\mu\nu}$ is the metric of the effective Riemann space ($\sqrt{-g} g^{\mu\nu} = \sqrt{-\gamma} \phi^{\mu\nu} + \sqrt{-\gamma} \gamma^{\mu\nu}$), $T^{\mu\nu}$ is the tensor of matter in the effective curved space, and in the static spherically symmetric case can be reduced to

$$\frac{dW}{dr} = y \, , \quad \frac{dU}{dr} = U \frac{V(1 + \frac{1}{2r_0^2}(W^2 - r^2)) - y^2 - \frac{W^2}{4r_0^2}(\frac{V}{U} - 1)}{Wy} \, , \tag{2}$$

$$\frac{dV}{dr} = V \frac{V(1 + \frac{1}{2r_0^2}(W^2 - r^2) - \frac{4yr}{W}) + 3y^2 - \frac{W^2}{4r_0^2}(\frac{V}{U} - 1)}{Wy} \, ,$$

$$\frac{dy}{dr} = \frac{V(1 + \frac{1}{2r_0^2}(W^2 - r^2) - \frac{2yr}{W}) + y^2}{W} - \frac{4\pi G}{c^2} VW \rho(r) \, ,$$

where $\rho(r)$ is the density of matter and $U(r)$, $V(r)$, $W(r)$ are the metric coefficients for the interval

$$ds^2 = U(r)(cdt)^2 - V(r)dr^2 - W^2(r)(d\Theta^2 + sin^2\Theta d\phi^2) ,$$

in the effective Riemann space.

Our aim is to find non-trivial solution of (2) which satisfies the following conditions

1) $\lim_{r\to\infty} g^{\mu\nu} = \gamma^{\mu\nu}$,

2) $\lim_{r\to\infty} \phi^{\mu\nu} = \phi^{\mu\nu}_{(\infty)}$, where $\gamma^{\alpha\beta} D_\alpha D_\beta \phi^{\mu\nu}_{(\infty)} + \frac{1}{r_0^2} \phi^{\mu\nu}_{(\infty)} = 0$.

Under such conditions the only possible behavior of the effective metric coefficients in the infinity is

$$U(r) = 1 - C\frac{e^{-\frac{r}{r_0}}}{r} + O(e^{-\frac{2r}{r_0}}) , \quad V(r) = 1 + C\frac{e^{-\frac{r}{r_0}}}{r} + O(e^{-\frac{2r}{r_0}}) , \tag{3}$$

$$W(r) = r + C\frac{e^{-\frac{r}{r_0}}}{2} + O(e^{-\frac{2r}{r_0}}) .$$

Here we make an assumption that the asymptotic behavior of the effective metric coefficients is fixed mainly by the source mass, i.e.

$$C \equiv \frac{2G}{c^4} \int t^{00}(\vec{r}) d\vec{r} \approx \int t_M^{00}(\vec{r}) d\vec{r} = \frac{2MG}{c^2} = r_g . \tag{4}$$

There is no gauge symmetry in system (2) and boundary conditions (3), (4) fix a unique solution.

This solution was obtained by numerical integration in *Mathematica 5.0*. But first we did it for the system of Hilbert-Einstein equations in the harmonic coordinates (it can be derived by taking the limit $r_0 \equiv \frac{\hbar}{m_g c} \to \infty$ in (2)) with the boundary conditions taken from the exact Schwarzschild solution in the harmonic coordinates

$$U(r) = \frac{r - \frac{r_g}{2}}{r + \frac{r_g}{2}} , \quad V(r) = \frac{r + \frac{r_g}{2}}{r - \frac{r_g}{2}} , \quad W(r) = r + \frac{r_g}{2} .$$

The numerical solution for the Hilbert-Einstein equations in harmonic coordinates with these boundary conditions (fig. 1) was found in the interval $\frac{1}{2} + 10^{-9} \equiv z_{min} \leq \frac{r}{r_g} \leq 10^3$. One obtains that $U(z_{min}r_g) = 0.9999999973 \cdot 10^{-9}$, $V(z_{min}r_g) = 1.0000000029 \cdot 10^9$, $U(z_{min}r_g)V(z_{min}r_g) = 1.0000000002$, $W(z_{min}r_g) = 1.000000001 r_g$.

The RTG equations were integrated for $10^{-8} \leq \frac{r_g}{r_0} \leq 1$ in the interval $0 \leq r \leq R$ where R had been chosen large enough for the asymptotic formulae (3) to be valid. For some values of $\frac{r_g}{r_0}$ some fragments of the relevant solutions are represented in fig. (2) – (4). These solutions are stable relative to the change of the integration accuracy or the choice of the starting point R. Let us enumerate their main features.

1) The RTG solution does not tend to the General Relativity solution at $r_0 \to \infty$ (in other words, operations of solving the field equations and taking the limit $r_0 \to \infty$ are not permutable).

2) All the metric coefficients in the solution are regular and not equal to zero everywhere in the considered region.

3) If $\frac{r_g}{r_0} < 3 \cdot 10^{-2}$ the solution has a crossover point r_1 such that $U(r) > V(r)$ at $r < r_1$. It means that the light cone of the effective curved space opens wider than the Minkowski space light cone in the region $r < r_1$ ($\lim_{r_0\to\infty} r_1 = \infty$).

4) For any $\frac{r_g}{r_0}$ the effective metric coefficient $V(r)$ has a maximum at some point r_2, i.e. $V(r)$ demonstrates strongly non-Schwarzchild behavior at $r < r_2$ ($\lim_{r_0\to\infty} r_2 = \infty$).

We managed to solve (2) only for $10^{-8} \leq \frac{r_g}{r_0} \leq 1$. To find crucial points r_1 and r_2 of the solution for $\frac{r_g}{r_0} \leq 10^{-8}$ we used the rational interpolation. Now we can obtain some rough estimates of the value of the gravitational field mass if we assume some physical conditions as enumerated below:

1) The Mercury orbit must be located in the region $r > r_2$, $\Rightarrow m_g > 3 \cdot 10^{-58} g$.

2) The light cone of the effective Riemann space for the case of neutron star must not open wider than the light cone of the Minkowski space, i.e. the neutron star radius must be larger than r_1, $\Rightarrow m_g > 10^{-46} g$.

Both the estimates of the lower limit for the gravitational field mass value are in strong contradiction with the upper limit $m_g < 1.3 \cdot 10^{-66} g$ obtained from the analysis of the Universe expansion [1].

References

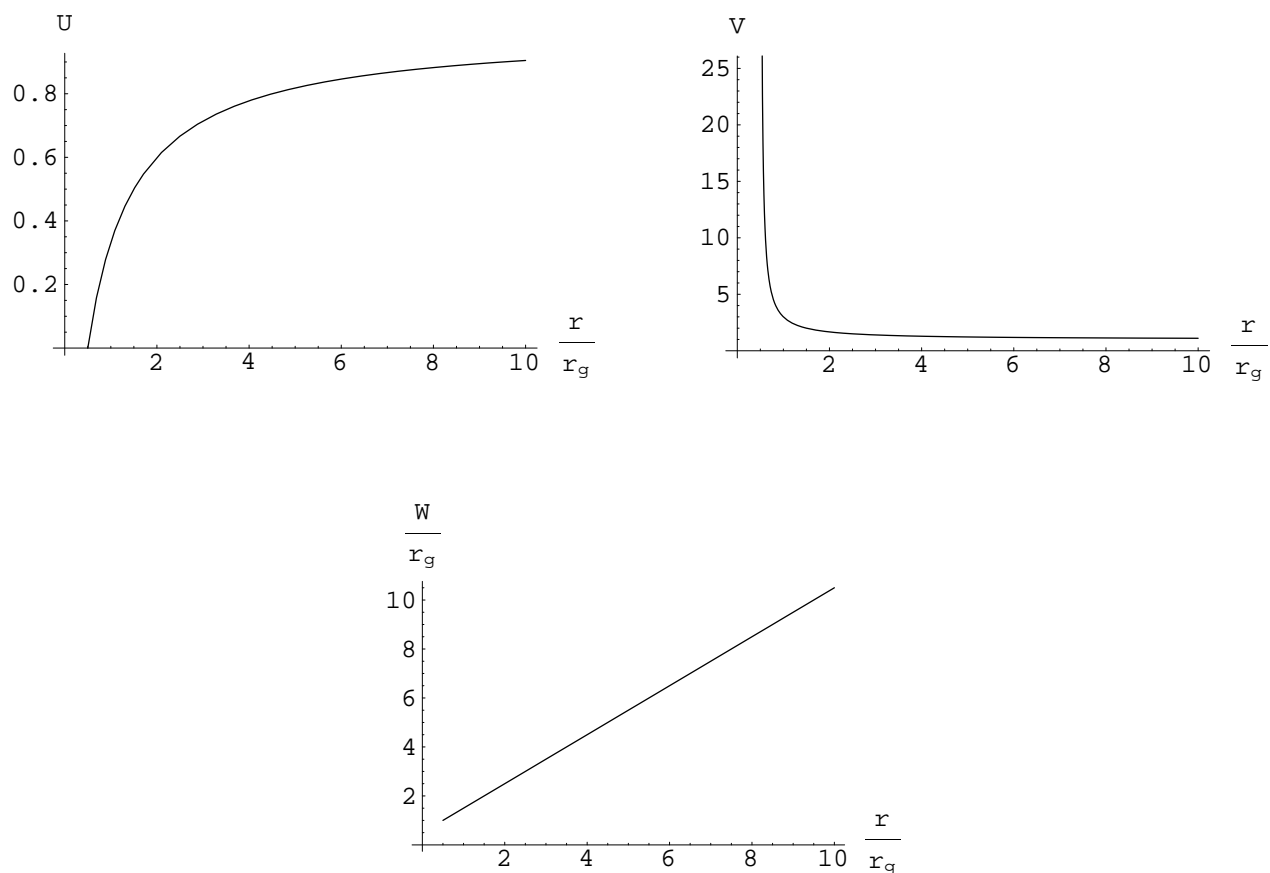[1] A.A. Logunov, **The Theory of Gravity**. Moscow, NAUKA 2001.

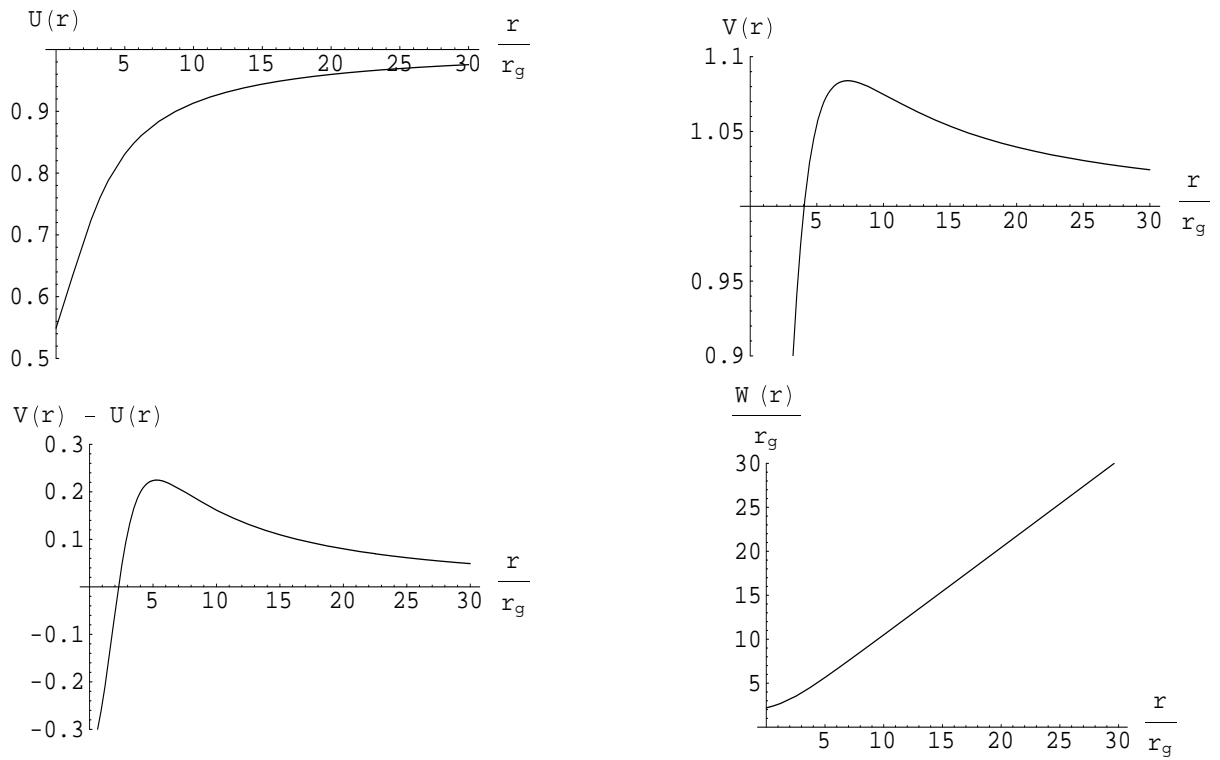Fig. 1. The Schwarzschild solution in the harmonic coordinates.

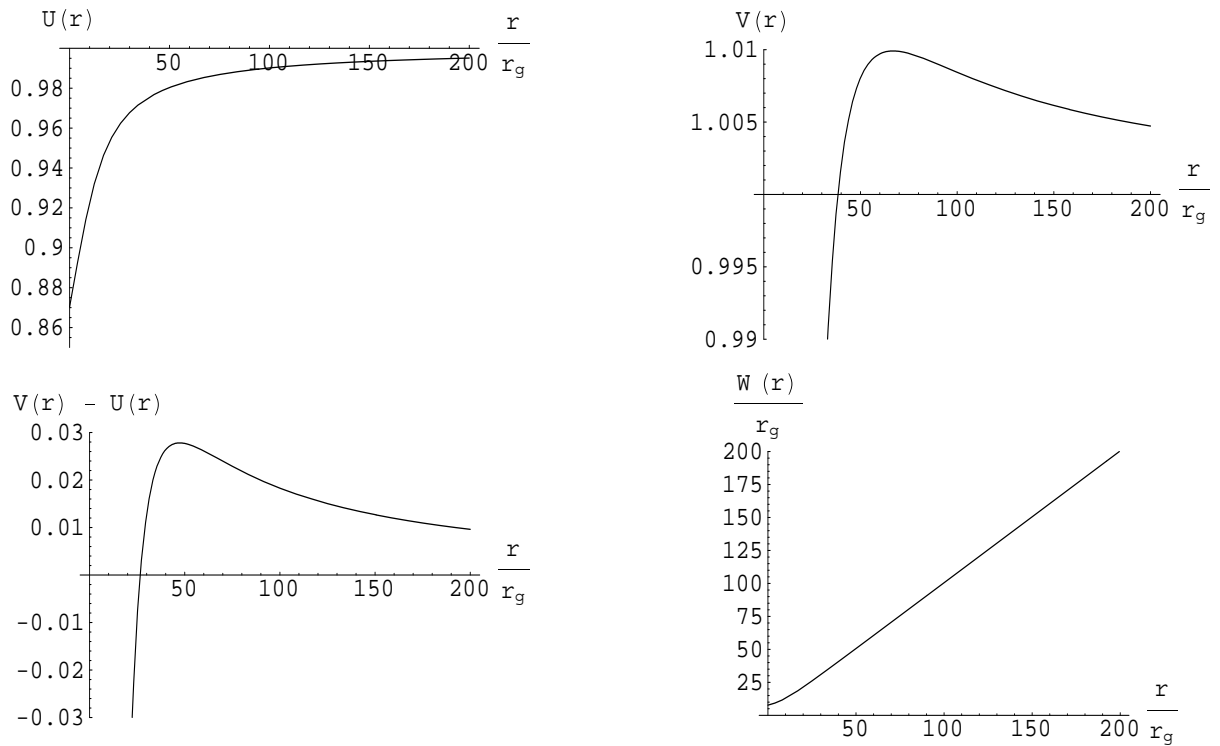Fig. 2. Solution of the RTG equations for $\frac{r_g}{r_0} = 10^{-2}$.

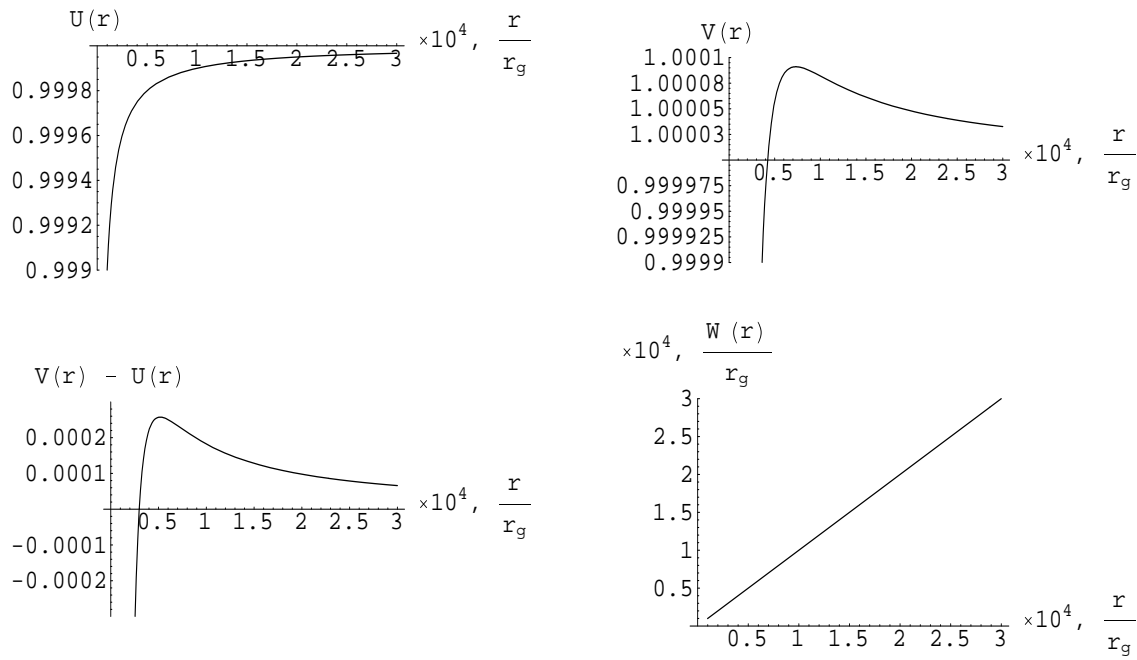Fig. 3. Solution of the RTG equations for $\frac{r_g}{r_0} = 10^{-4}$.

Fig. 4.   Solution of the RTG equations for $\frac{r_g}{r_0} = 10^{-8}$.