IS THERE REALLY NO QUANTUM FORM OF EINSTEIN GRAVITY ?

A.Y. Shiekh

International Centre for Theoretical Physics,
Miramare, Trieste, Italy

It is well known that Einstein gravity is non-renormalizable; however this does not preclude the existence of a quantum form.

"Everything should be made as simple as possible, but not simpler."

Albert Einstein

1. Introduction

1.1. Is Einstein gravity incomplete?

Reviewing a subject in less than an hour is hard enough, but reviewing one that does not exist is even harder still; for Einstein gravity has not been quantized [reviewed in Isham, 1981]. We will be taking a rather orthodox approach in reviewing the obstructions to the quantization of traditional Einstein gravity. But there is virtue in sometimes looking back over lost campaigns in life.

Despite the fact that it seems impossible to quantize, this does not exclude the existence of a quantum form of Einstein gravity [Shiekh, 94; 96]; in the same way as prehistoric man had a source of fire (lightning strikes on trees), though not necessarily a means to produce it. It is sometimes all too easy to forget that the world is quantum, and that the classical picture is but a special case, so we are actually working backwards when we try to derive the more general case from a restrictive form, and there is no reason to believe that this is always possible. However, we are compelled to follow this route out of a lack of choice.

1.2. Need gravity be quantize at all?

Is it not possible that although the other forces of nature are seen to be quantized, that perhaps gravity, which is presently not seen as a force at all, need not be quantized [Feynman, 1963; Kibble, 1981]? The concept of force is a classical one that does not even make an appearance in quantum theory, where distribution is governed by the overlap of wave functions. Newtonian gravity certainly carries the notion of force, while the Einstein view is of matter in free fall in a curved space-time. This force free picture of classical gravity might lead one to propose leaving the curved space time unquantized, and to have the quantum fields play out on this arena. It is an appealing scenario, and would present a resolution to the problem of quantization, claiming it is not necessary at all.

However the gravitational field, if left classical, could be used to make measurements on a quantum field, and being classical would not be subject to the Heisenberg uncertainty principle. Thus, if one were to look at a quantum field using gravity as a probe, one would be able to extract information about the quantum field that defied the Heisenberg principle. Such arguments are far from water-tight, but do make a strong case for the quantization of gravity.

Despite the very geometric picture that is usually assigned to Einstein gravity, it can be made to look like the other perturbative field theories, with the graviton a spin two particle. It is this very traditional, particle physics perspective, that we will be following. Unfortunately, the whole structure now takes on a very mathematical structure, and at times it is hard to keep contact with the physics.

Actually, we will be adding a second theme to the quantization that is not strictly necessary, namely renormalization by analytic continuation. In these methods there is no subtraction of infinities, but rather a reinterpretation of the formulae. This is a formulation devoid of infinities, so completely by-passing the problem of interpreting the divergences. At the risk of mixing concepts, the quantization is illustrated using this novel method, which can be shown to be equivalent, in effect, to the older techniques.

1.3. How far can we go without renormalizing?

It might come as somewhat of a surprise, but one can actually go part the way to quantizing without having to deal with renormalization. It was discovered [Donoghue, 1994; 1994] that the one loop correction to the Newtonian potential does not encounter any infinities, and so makes no demands on renormalization. The best way to see this, is to use dimensional analysis to predict the manner of corrections anticipated [Donoghue, 1995]. In configuration space this takes the form:

$$V(r) = -\frac{Gm_1m_2}{r}\left(1 + \alpha\frac{G(m_1+m_2)}{rc^2} + \beta\frac{G\hbar}{r^2c^3} + \ldots\right), \tag{1}$$

which includes the first relativistic correction, and the first quantum correction (also relativistic). These power law corrections come from momentum space (where we normally calculate) via the Fourier transforms:

$$\int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}.\vec{r}} \sqrt{\frac{1}{\vec{q}^2}} = \frac{1}{2\pi^2 r^2} \tag{2}$$

and

$$\int \frac{d^3q}{(2\pi)^3} e^{-i\vec{q}.\vec{r}} \ln\left(\frac{\vec{q}^2}{\mu^2}\right) = -\frac{1}{2\pi^2 r^3}, \tag{3}$$

where we have slipped back into the simplifying habit of picking units where $\hbar = c = 1$.

Now we note something very important, namely that the form of these terms in momentum space are quite special, in that they are non-analytic. If gravity were somehow renormalizable, then like other renormalizable theories, all the divergent terms must be analytic in order to be accommodated back into the starting Lagrangian. So one has some reason to suspect that the non-analytic contributions with be finite. In practice, one does not worry about the ability to renormalize or not, but simple goes about determining these terms in the hope that they will be entirely divergent free.

A second piece of magic occurs, for on dimensional grounds alone we saw the appearance of the renormalization group parameter μ. This might be expected to undermine the predictive power of the calculation. But we see the disappearance of this factor in the move to configuration space. This is very particular to this calculation and would not be the case for higher order corrections, i.e. when looking at the first quantum correction to the relativistic potential. To go any further, one is compelled discuss renormalization and it's problems in the context of Einstein gravity.

It might be remarked that much of the strain of doing the tensor algebra can be overcome by submitting these parts to the computer; the intermediate results typically reaching into the thousands of terms, even at one loop.

We should perhaps remind ourselves that it is being intrinsically assumed that the complete theory of quantum gravity does not alter this special result. The fact that it does not involve renormalization gives us reason to believe that the act of full quantization will not change things, unless a modification to the classical theory itself is necessary for quantization.

1.4. The problem with traditional quantization (a lightning review)

Those that know the traditional methods and problems with quantizing are not in need of an introductory review, while those that have not tinkered with the guts cannot really be properly shown the approach in an hour. It is with this contradiction in mind that we set out on a lightning tour of the problem.

The normal approach is to start with a classical theory and try to quantize it. In reality the world is quantum, and the classical view is just a special limit. In this sense we are starting from the top and trying to work down to the more fundamental. It is an approach fraught with dangers, but it is the best we can do for now. There is no guarantee that the attempt to derive the more general from the special case will bear fruit, and the fact that Einstein gravity can't be quantized should not be taken to imply that there is no quantum Einstein gravity.

We have been quantizing non-relativistic systems with success for some time now, but relativistic theories seem to demand the use of field theories (to allow for particle creation and destruction). However, the infinite number of degrees of freedom tends to be accompanied by infinite quantities in the theory, and this, very crudely, is the source of the problem when quantizing field theories. However, some field theories are quantizable, despite the presence of these infinities. It turns out that in some very special cases it is possible to re-absorb the infinities into the coupling constants of the original, starting (classical) theory. It is a mathematical technique that is difficult to interpret physically; but despite this difficulty it leads to very good physical predictions for most of the forces of nature (the electric, weak and strong forces). The fact that this so called process of renormalization is only successful for a small class of theories is what makes it predictive. In fact, it is so successful that it has permitted the unification of the electric and weak forces and even has a lesser constrained proposal for also uniting the strong nuclear force. However, the one remaining force, namely gravitation, does not succumb to quantization so easily. This suggests the need for something novel, but should not be taken as reason to totally abandon the past thinking as a complete failure, and so devoid of usefulness. Despite this need for change, there seems to be a proposal for quantizing gravity that is unexpectedly conservative in its lack novelty. In fact, we will be so traditional as to investigate the perturbative quantization of gravity. This means that much of the presentation can be versed in the now rather old language of field theory, and we can embark on a more detailed investigation of the problems obstructing the traditional quantization of Einstein gravity.

1.4.1. Infra-red divergences

A common way to deal with the infra-red divergences of a theory is to give the massless particles a small rest mass, with the intention of eventually taking the mass goes to zero limit. This is not an option for Einstein gravity where the zero mass case and the zero mass limit give differing results [van Nieuwenhuizen, 1973].

2. The Perturbative Quantization of Einstein Gravity

The usual scheme of field quantization is plagued by divergences, but in some special cases those infinities can be consistently ploughed back into the theory to yield a finite end result with a small number of arbitrary constants remaining; these then being obtained from experiment [Ramond, 1990; Collins, 1984]. This is the renown scheme of renormalization, disapproved of by some, but reasonably well defined and yielding results in excellent agreement with nature. For those disturbed by the appearance of infinities, there now exists a finite perturbative version employing analytic continuation (a generalization of the Zeta function, one loop, technique), that goes under the deceptive name of 'operator regularization'. The fact that after renormalization some factors, such as mass and charge, are left undetermined should perhaps not be viewed as a predictive shortcoming, since the fundamental units of nature are relative. That is to say, the choice of reference unit (be it mass, length, time, or charge) is always arbitrary, and then everything else can be stated in terms of these units. In this sense the final theory of everything should not, and cannot, predict all.

The fact that only some theories are renormalizable has the beneficial effect of being selective, and so predictive. This follows the line of reasoning that physics is more than *descriptive*, but *predictive* by virtue of being limited by the requirement of self consistency.

Unfortunately, in the usual sense, general relativity is *not* renormalizable [Veltman, 1976], and we will run quickly over the failure of Einstein gravity to quantize perturbatively, by considering the example of a massive scalar field with gravity. The starting theory in Euclidean space would be characterized by:

$$L = -\sqrt{g}\left(R + \tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \tfrac{1}{2}m^2\phi^2\right). \tag{4}$$

(using units where $16\pi G = 1$, $c = 1$)

One discovers, upon perturbatively quantizing both the matter and gravitational fields, that the infinities cannot be accounted for in the traditional way by altering the coefficients of the terms in the original, starting, Lagrangian. The required counter terms fall 'outside' the original Lagrangian, and so the infinities cannot be reabsorbed.

One natural thought might be to generalize Einstein gravity by extending the starting Lagrangian to accommodate the anticipated counter terms. Here symmetry can be employed as a guide, for by using the most general starting Lagrangian consistent with the original symmetries one arranges that the counter terms (which also retain the symmetry in the absence of a quantum anomaly) fall back within the

Lagrangian. One would not anticipate an anomaly, as these arise from a quantum conflict between two or more symmetries, when one must choose between one or the other. Thus one is lead to the infinitely large starting Lagrangian:

$$L_0 = -\sqrt{g_0}\left(\begin{array}{c} -2\Lambda_0 + R_0 + \frac{1}{2}p_0^2 + \frac{1}{2}m_0^2\phi_0^2 + \frac{1}{4!}\phi_0^4\lambda_0(\phi_0^2) + p_0^2\phi_0^2\kappa_0(\phi_0^2) + R_0\phi_0^2\gamma_0(\phi_0^2) \\ +p_0^4 a_0(p_0^2,\phi_0^2) + R_0 p_0^2 b_0(p_0^2,\phi_0^2) + R_0^2 c_0(p_0^2,\phi_0^2) + R_{0\mu\nu}R_0^{\mu\nu}d_0(p_0^2,\phi_0^2) + ... \end{array}\right), \tag{5}$$

where p_0^2 is shorthand for $g_0^{\mu\nu}\partial_\mu\phi_0\partial_\nu\phi_0$ and not the independent variable of Hamiltonian mechanics. λ_0, κ_0, γ_0, a_0, b_0, c_0, d_0 ... are arbitrary analytic functions, and the second line carries all the higher derivative terms. Strictly this is formal in having neglected gauge fixing and the resulting presence of ghost particles.

The price for having achieved 'formal' renormalization, is that the theory (with its infinite number of arbitrary renormalized parameters) is devoid of predictive content. The failure to quantize has been rephrased from a problem of non-renormalizability to one of non-predictability.

Despite this, after renormalization we are lead to:

$$L = -\sqrt{g}\left(\begin{array}{c} -2\Lambda + R + \frac{1}{2}p^2 + \frac{1}{2}m^2\phi^2 + \frac{1}{4!}\phi^4\lambda(\phi^2) + p^2\phi^2\kappa(\phi^2) + R\phi^2\gamma(\phi^2) \\ +p^4 a(p^2,\phi^2) + Rp^2 b(p^2,\phi^2) + R^2 c(p^2,\phi^2) + R_{\mu\nu}R^{\mu\nu}d(p^2,\phi^2) + ... \end{array}\right). \tag{6}$$

This total loss of predictability is a slight exaggeration, as the people doing the effective field theory of gravity would argue that the renormalised couplings are of a sane size, so that one need only a finite number of terms to describe physics at a given scale. This being the case, there will then only be a finite number of parameters to determine from experiment, and the effective theory has a predictive content, albeit not as strong as one might have hoped for.

But perhaps we can go further than leaving the renormalized couplings unknown (though 'small'), for there remain physical criterion to pin down some of these arbitrary factors. Since in general the higher derivative terms lead to acausal classical behavior, their renormalized coefficient might be put down to zero on physical grounds. This still leaves the three arbitrary functions: $\lambda(\phi^2)$, $\kappa(\phi^2)$ and $\gamma(\phi^2)$, associated with the terms ϕ^4, $p^2\phi^2$, and $R\phi^2$ respectively. The last may be abandoned on the grounds of defying the equivalence principle. To see this, begin by considering the first term of the Taylor expansion, namely $R\phi^2$; this has the form of a mass term and so one would be able to make local measurements of mass to determine the curvature, and so contradict the equivalence principle (charged particles, with their non-local fields have this term present with a fixed coefficient). The same line of reasoning applies to the remaining terms, $R\phi^4$, $R\phi^6$, ... etc.

This leaves us the two remaining infinite families of ambiguities with the terms $\phi^4\lambda(\phi^2)$ and $p^2\phi^2\kappa(\phi^2)$. In the limit of flat space in 3+1 dimensions this will reduce to a renormalized theory in the traditional sense if $\lambda(\phi^2) = constant$, and $\kappa(\phi^2) = 0$. So one is lead to proposing that the physical parameters should be:

$$\begin{gathered}
\Lambda = \kappa(\phi^2) = \gamma(\phi^2) = 0, \\
a(p^2, \phi^2) = b(p^2, \phi^2) = c(p^2, \phi^2) = d(p^2, \phi^2) = \ldots = 0, \\
\lambda(\phi^2) = \lambda = scalar\ particle\ self\ coupling\ constant, \\
m = mass\ of\ the\ scalar\ particle
\end{gathered} \tag{7}$$

and so the renormalized theory of quantum gravity for a scalar field should have the form:

$$L = -\sqrt{g}\left(-2\Lambda + R + \tfrac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \tfrac{1}{2}m^2\phi^2 + \tfrac{1}{4!}\lambda\phi^4\right). \tag{8}$$

One might now worry about the renormalization group pulling the coupling constants around. This is an open point to which I feel one of several things might happen:

- *The couplings, set to zero at a low energy scale, might reappear around the Plank scale. Whether the resulting theory then makes sense is a matter for dispute.*

- *Certain coupling constants (beyond those already set to zero) should be related, in order that the beta functions of the zeroed couplings be zero (a fixed point), so ensuring that all their couplings remain at zero. This consistency condition could be the basis of a unification scheme, although its implementation might not be possible within the perturbative formulation.*

This is a highly non-trivial matter that needs looking at more closely.

2.1. Regularization method

On a diverse, but related track, one might wonder which renormalization scheme to choose for implementing the scheme proposed above. In this context analytic continuation [Bollini et al., 1964; Speer, 1968; Salam and Strathdee, 1975; Dowker and Critchley, 1976; Hawking, 1977] is very appealing in being finite, and in this context there is an 'unsung hero' in the guise of operator regularization, which I think deserves a mention [McKeon and Sherry, 1987; McKeon et al., 1987; McKeon et al., 1988; Mann, 1988; Mann et al., 1989; Culumovic et al., 1990; Shiekh, 1990].

In operator regularization one avoids the divergences by using the analytic continuation:

$$\Omega^{-m} = \lim_{\varepsilon \to 0} \frac{d^n}{d\varepsilon^n} \left(\frac{\varepsilon^n}{n!} \Omega^{-\varepsilon - m} \right), \tag{9}$$

where n is chosen large enough to eliminate the infinities (the loop order is sufficient). Actually, operator regularization is a bit of a misnomer, since it need not be applied to an operator and does not just regulate, but also renormalizes all in one. However, under this form of the method *all* theories are finite and predictive (gravity included).

This form of the method locates and eliminates the poles (automated minimal subtraction). This is clearly seen by how it treats some terms of the Maclaurin expansion. The taming of 1 yields:

$$\lim_{\varepsilon \to 0} \frac{d}{d\varepsilon} (\varepsilon .1) = 1, \tag{10}$$

while, on the other hand, the taming of $1/\varepsilon$ yields:

$$\lim_{\varepsilon \to 0} \frac{d}{d\varepsilon} \left(\varepsilon . \frac{1}{\varepsilon} \right) = 0. \tag{11}$$

This realized, the general form is easily located, and is given by:

$$\Omega^{-m} = \lim_{\varepsilon \to 0} \frac{d^n}{d\varepsilon^n} \left((1 + \alpha_1 \varepsilon + ... + \alpha_n \varepsilon^n) \frac{\varepsilon^n}{n!} \Omega^{-\varepsilon - m} \right) \tag{12}$$

(the alphas being ambiguous)

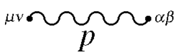
This form is no longer too powerful, and gravity must again be dealt with as before, setting most of the final renormalized parameters to zero on physical grounds

The method of operator regularization has the strength of explicitly maintaining invariances, further even than dimensional regularization, for dimension dependent invariances are not disturbed. It is further blessed with the feature of being finite throughout, as the Zeta function technique [Salam and Strathdee, 1975; Dowker and Critchley, 1976; Hawking, 1975]. But unlike the Zeta function method, it is not limited in applicability to one loop, being valid to all orders.

2.2. The first few Feynman rules

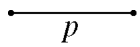
To go about a perturbative calculation we should have the Feynman rules in hand, of which gravity is exceptional in having an infinite number due to the presence of the square root in the Lagrangian which makes it non-polynomial. This does not cause a great problem, because to any finite loop order, only a finite number of Feynman rules are needed. The first few rules we list as:

- The graviton propagator:



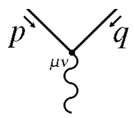
$$\frac{\eta_{\mu\alpha}\eta_{\nu\beta}+\eta_{\mu\beta}\eta_{\nu\alpha}-\eta_{\mu\nu}\eta_{\alpha\beta}}{p^2}. \tag{13}$$

- The scalar propagator:



$$\frac{1}{p^2+m^2}. \tag{14}$$

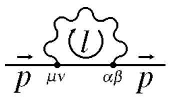
- The first interaction vertex:



$$\tfrac{1}{2}\left(\eta_{\mu\nu}(p.q-m^2)-p_\mu q_\nu-p_\nu q_\mu\right). \tag{15}$$

3. The method in Action

To see some of the assembled machinery in action, we can go about calculating a simple one loop diagram, namely one part of the correction to the scalar propogator:



$$\begin{aligned}=\int_{-\infty}^{\infty}\frac{d^4l}{(2\pi)^4}\ &\left(\frac{1}{l^2+m^2}\right)\left(\frac{\eta_{\mu\alpha}\eta_{\nu\beta}+\eta_{\mu\beta}\eta_{\nu\alpha}-\eta_{\mu\nu}\eta_{\alpha\beta}}{(l+p)^2}\right)\\ &\times\tfrac{1}{2}\left(\eta_{\mu\nu}(p.l-m^2)-p_\mu l_\nu-p_\nu l_\mu\right)\\ &\times\tfrac{1}{2}\left(\eta_{\alpha\beta}(p.l-m^2)-p_\alpha l_\beta-p_\beta l_\alpha\right)\end{aligned} \tag{16}$$

Expand out the indices to yield:

$$=\int_{-\infty}^{\infty}\frac{d^4l}{(2\pi)^4}\frac{1}{l^2+m^2}\frac{1}{(l+p)^2}\left(p^2l^2+2m^2p\cdot l-2m^4\right) \tag{17}$$

and then introduce the standard Feynman parameter 'trick':

$$\frac{1}{D_1^{a_1}D_2^{a_2}...D_k^{a_k}}=\frac{\Gamma(a_1+a_2+...a_k)}{\Gamma(a_1)\Gamma(a_2)...\Gamma(a_k)}\int_0^1...\int_0^1 dx_1...dx_k\frac{\delta(1-x_1-...x_k)x_1^{a_1-1}...x_k^{a_k-1}}{(D_1x_1+...D_kx_k)^{a_1+...a_k}} \tag{18}$$

to yield:

$$=\int_{-\infty}^{\infty}\frac{d^4l}{(2\pi)^4}\int_0^1 dx\frac{p^2l^2+2m^2p\cdot l-2m^4}{\left[l^2+m^2x+p^2\left(1-x\right)+2l\cdot p\left(1-x\right)\right]^2}. \tag{19}$$

Remove divergences using operator regularization:

$$\Omega^{-m}=\lim_{\varepsilon\to 0}\frac{d^n}{d\varepsilon^n}\left(\left(1+\alpha_1\varepsilon+...+\alpha_n\varepsilon^n\right)\frac{\varepsilon^n}{n!}\Omega^{-\varepsilon-m}\right), \tag{20}$$

n being chosen sufficiently large to cancel the infinities. For the case in hand $n = 1$ is adequate.

$$\Omega^{-2} = \lim_{\varepsilon \to 0} \frac{d}{d\varepsilon} \left((1 + \alpha\varepsilon)\, \varepsilon \Omega^{-\varepsilon - 2} \right). \tag{21}$$

This leads to:

$$= \int_0^1 dx \, \lim_{\varepsilon \to 0} \frac{d}{d\varepsilon} \int_{-\infty}^{\infty} \frac{d^4 l}{(2\pi)^4} \left(\varepsilon\, (1 + \alpha\varepsilon)\, \frac{p^2 l^2 + 2m^2 p \cdot l - 2m^4}{\left[l^2 + m^2 x + p^2\,(1 - x) + 2l \cdot p\,(1 - x)\right]^{\varepsilon + 2}} \right). \tag{22}$$

Then performing the momentum integrations using [Ramond, 1990]:

$$\int_{-\infty}^{\infty} \frac{d^{2\omega} l}{(2\pi)^{2\omega}} \frac{1}{(l^2 + M^2 + 2l \cdot p)^A} = \frac{1}{(4\pi)^\omega \Gamma(A)} \frac{\Gamma(A - \omega)}{(M^2 - p^2)^{A - \omega}}, \tag{23}$$

$$\int_{-\infty}^{\infty} \frac{d^{2\omega} l}{(2\pi)^{2\omega}} \frac{l_\mu}{(l^2 + M^2 + 2l \cdot p)^A} = -\frac{1}{(4\pi)^\omega \Gamma(A)} p_\mu \frac{\Gamma(A - \omega)}{(M^2 - p^2)^{A - \omega}}, \tag{24}$$

$$\int_{-\infty}^{\infty} \frac{d^{2\omega} l}{(2\pi)^{2\omega}} \frac{l_\mu l_\nu}{(l^2 + M^2 + 2l \cdot p)^A} = \frac{1}{(4\pi)^\omega \Gamma(A)} \left[p_\mu p_\nu \frac{\Gamma(A - \omega)}{(M^2 - p^2)^{A - \omega}} + \frac{\delta_{\mu\nu}}{2} \frac{\Gamma(A - \omega - 1)}{(M^2 - p^2)^{A - \omega - 1}} \right] \tag{25}$$

yields the finite expression:

$$= \frac{1}{(4\pi)^2} \int_0^1 dx \, \lim_{\varepsilon \to 0} \frac{d}{d\varepsilon} \left(\frac{\varepsilon\,(1 + \alpha\varepsilon)}{\Gamma(\varepsilon + 2)} \left(\begin{array}{c} \frac{p^4 (1 - x)^2 \Gamma(\varepsilon)}{\left[m^2 x + p^2 x\,(1 - x)\right]^\varepsilon} + 2 \frac{p^2 \Gamma(\varepsilon - 1)}{\left[m^2 x + p^2 x\,(1 - x)\right]^{\varepsilon - 1}} \\ -2 \frac{m^2 p^2 (1 - x) \Gamma(\varepsilon)}{\left[m^2 x + p^2 x\,(1 - x)\right]^\varepsilon} - 2 \frac{m^4 \Gamma(\varepsilon)}{\left[m^2 x + p^2 x\,(1 - x)\right]^\varepsilon} \end{array} \right) \right). \tag{26}$$

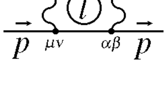
Doing the ε differential using:

$$\lim_{\varepsilon \to 0} \frac{d}{d\varepsilon} \left(\frac{\varepsilon\,(1 + \alpha\varepsilon)}{\Gamma(\varepsilon + 2)} \left(a \frac{\Gamma(\varepsilon)}{\chi^\varepsilon} + b \frac{\Gamma(\varepsilon - 1)}{\chi^{\varepsilon - 1}} \right) \right) = -a + (a - b\chi)\,(\alpha - \ln(\chi)) \tag{27}$$

yields:

$$= \frac{1}{(4\pi)^2} \int_0^1 dx \left(\begin{array}{c} \left((2m^4 + 2m^2 p^2 - p^4) + p^4 x\,(4 - 3x)\right) \left(\ln\left(m^2 x + p^2 x (1 - x)\right) - \alpha\right) \\ +2m^4 + 2m^2 p^2 - p^4 - p^2 x\,(2m^2 - 2p^2 + p^2 x) \end{array} \right). \tag{28}$$

and finally performing the x integration gives rise to the final result in Euclidean space, namely:



$$= \frac{m^4}{(4\pi)^2} \left(\begin{array}{c} \left(3 + 2\frac{p^2}{m^2} + \frac{m^2}{p^2}\right) \ln(1 + p^2/m^2) - 1 - \frac{5}{2}\frac{p^2}{m^2} \\ -\frac{1}{6}\frac{p^4}{m^4} + 2\left(1 + \frac{p^2}{m^2}\right)\left(\ln(m^2/\mu^2) - \alpha\right) \end{array} \right), \tag{29}$$

where there is no actual divergence at $p = 0$, and it should be commented that the use of a computer mathematics package can in general greatly reduced 'calculator' fatigue. The renormalization group factor μ appears on dimensional grounds.

In general the Feynman rules are large and the tensor algebra immense. Much of the simplicity is restored by submitting this part of the complexity to the computer. Even so, the intermediate results can be so extensive that even a super-computer can choke without trivial help. For example, imagine one had the contraction of three tensors:

$$\alpha_{\mu\nu}\beta_{\rho\sigma}\gamma^{\mu\nu\rho\sigma} \tag{30}$$

each of which consists of many terms. Then the computer, in trying to contract out the indices, tends to expand out the entire expression which can easily lead to thousands of terms, that can overpower the computers memory.

The resolution lies in the trivial step of asking the computer to initially expand out only α for example:

$$(\alpha_{1_{\mu\nu}} + \alpha_{2_{\mu\nu}} + \ldots)\beta_{\rho\sigma}\gamma^{\mu\nu\rho\sigma}. \tag{31}$$

In this way the computer is presented with several terms that can each be contracted separately. This seemingly innocuous move can make all the difference between the computer being able to perform the calculation and not. It is details like this that in practice can occupy much of the investigators time.

3.1. Discussion

We are now left with a finite theory that has few arbitrary constants, and so is predictive. Despite the present lack of experimental data to test it against, and regardless of the patch work line of reasoning invoked to arrive at this hypothesis, one might alter perspective and simply be interested in investigating the consequences of such a scheme for its own sake, where many of the arbitrary factors have been set to zero, for whatever reason. At this stage any well behaved, finite theory, is worth investigating; and it is unfortunate that we don't have the guiding hand of mother nature to assist us in this guessing game.

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References

- **C.J. Isham**, *'Quantum Gravity - An Overview'*, in "Quantum Gravity 2: A Second Oxford Symposium", pp. 1-62, eds. C.J. Isham, R. Penrose and D.W. Sciama, (Oxford University Press, Oxford, 1981).

- **A.Y. Shiekh**, *'The Perturbative Quantization of Gravity'*, in "Problems on High Energy Physics and Field Theory", Proceedings of the XVII workshop 1994, pp. 156-165, Protvino, 1995.
  A.Y. Shiekh, *'Quantizing Orthodox Gravity'*, Can. J. Phys., **74**, 1996, 172-.

- **J.F. Donoghue**, *'Leading Quantum Corrections to the Newtonian Potential'*, Phys. Rev. Lett., **72**, 1994, 2996-.
  J.F. Donoghue, *'General relativity as an effective field theory: The leading quantum corrections'*, Phys. Rev., **D50**, 1994, 3874-.

- **R.P. Feynman**, *"Lectures on Gravitation"* (Addison-Wesley, 1995).
  T.W.B. Kibble, *'Is a Semi-Classical Theory of Gravity Viable?'*, in "Quantum Gravity 2: A Second Oxford Symposium", pp. 63-80, eds. C.J. Isham, R. Penrose and D.W. Sciama, (Oxford University Press, Oxford, 1981).

- **P. Ramond**, *"Field Theory: A Modern Primer"*, 2nd Ed (Addison-Wesley, 1990).
  J. Collins, *"Renormalization"*, (Cambridge University Press, London, 1984).

- **P. van Nieuwenhuizen**, *'Radiation of Massive Gravitation'*, Phys. Rev., **D7**, 1973, 2300-.

- **M.J.G. Veltman**, *'Quantum Theory of Gravitation'*, Les Houches XXVIII, "Methods In Field Theory", pp. 265-327, eds. R. Ballan and J. Zinn-Justin, (North-Holland, Amsterdam, 1976).

- **C. Bollini, J. Giambiagi and A. Domìnguez**, *'Analytic Regularization and the Divergences of Quantum Field Theories'*, Nuovo Cimento **31**, 1964, 550-.
  E. Speer, *'Analytic Renormalization'*, J. Math. Phys. **9**, 1968, 1404-.
  A. Salam and J. Strathdee, *'Transition Electromagnetic Fields in Particle Physics'*, Nucl. Phys. **B90**, 1975, 203-.
  J. Dowker and R. Critchley, *'Effective Lagrangian and energy-momentum tensor in de sitter space'*, Phys. Rev., **D13**, 1976, 3224-.
  S. Hawking, *'Zeta Function Regularization of Path Integrals in Curved Space'*, Commun. Math. Phys., **55**, 1977, 133-.

- **D. McKeon and T. Sherry**, *'Operator Regularization of Green's Functions'*, Phys. Rev. Lett., **59**, 1987, 532-.
  D. McKeon and T. Sherry, *'Operator Regularization and one-loop Green's functions'*, Phys. Rev., **D35**, 1987, 3854-.
  D. McKeon, S. Rajpoot and T. Sherry, *'Operator Regularization with Superfields'*, Phys. Rev., **D35**, 1987, 3873-.
  D. McKeon, S. Samant and T. Sherry, *'Operator regularization beyond lowest order'*, Can. J. Phys., **66**, 1988, 268-.
  R. Mann., *'Zeta function regularization of Quantum Gravity'*, In Proceedings of the cap-nserc Summer Workshop on Field Theory and Critical Phenomena. Edited by G. Kunstatter, H. Lee, F. Khanna and H. Limezawa, World Scientific Pub. Co. Ltd., Singapore, 1988, p. 17-.
  R. Mann, D. McKeon, T. Steele and T. Tarasov, *'Operator Regularization and Quantum Gravity'*, Nucl. Phys., **B311**, 1989, 630-.
  L. Culumovic, M. Leblanc, R. Mann, D. McKeon and T. Sherry, *'Operator regularization and multiloop Green's functions'*, Phys. Rev., **D41**, 1990, 514-.
  A. Shiekh, *'Zeta-function regularization of quantum field theory'*, Can. J. Phys., **68**, 1990, 620-.