HADRONS AND STRINGS

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We consider hadrons as relativistic straight-line strings with massive spinning point-like quarks at the ends. We describe classical mechanics of this system and then quantize it. As a result we get good description of the observed meson spectra.

1. Introduction

By hadrons here we mean mainly mesons consisting of a quark and an antiquark. But all the ideas of this report can be applied to baryons consisting of a quark and a diquark.

To describe heavy-quark mesons one can use a potential model. But the notion of a potential fails for light quarks. On the other hand quantum chromodynamics suggests that the self-interacting gluon field of a quark and a antiquark forms a narrow tube or a string between them when they are far enough one from the other, what is responsible for their confinement. This idea does not depend on the quark masses and will be considered here.

So we consider a meson as an extended relativistic object formed by a string and point-like particles attached to its ends. First we consider classical mechanics of such a system and then canonically quantize it. We propose to use successive approximations starting from simple configurations of the string.

The simplest configuration is an open straight-line string (massless spinless quarks). It has been shown that it gives good description of the light-quark mesons lying on the leading Regge trajectory [1]. Next approximation considered here is a rigid straight-line string with massive spinning particles at the ends.

More complicated string configurations and quark oscillations may be responsible for the mesons from daughter Regge trajectories which are still not well known from experiment. Of course, successive approximations should not exceed the accuracy of the string model itself which may need a modification at some step. At any case, only those configurations of the string should be considered which admit relativistic quantization.

To make our presentation of the relativistic mechanics of an extended object clearer we start with a point-like spinning particle [2, 3]. Then we introduce a straight-line string and a more general object called rotator, consider its Lagrangian and Hamiltonian mechanics and perform relativistic canonical quantization. The application to massive spinless quarks makes it possible to account for the difference between light-quark and heavy-quark meson trajectories. As a simple illustration we give a formula for the s-quark mass through that of the ρ- and K^*-mesons. Inclusion of the spin of one quark helps to understand why the orbital spin of the simple string model may be replaced by the total spin of the meson in the formula for the Regge trajectory. We conclude by mentioning possible generalizations of the model.

2. Spin and constraints

Since starting from a classical (non-quantized) theory has some advantages, let us briefly introduce a (quasi)classical theory of spinning point-like particle [2, 3]. The angular momentum of non-relativistic particle

$$\vec{L} = [\vec{r}, \vec{p}] \tag{1}$$

with coordinate $\vec{r}$, momentum $\vec{p}$ and canonical Poisson brackets $\{r_i, p_j\} = \delta_{ij}$, obeys the relations

$$\{L_i, L_j\} = \epsilon_{ijk} L_k, \tag{2}$$

which determine the properties of the angular momentum. After replacing classical numbers by quantum operators and Poisson brackets by commutators $\{,\} \to -i[,]_-$, $\vec{L}$ becomes an operator which, because of the operator form of (2) alone can have integer and half-integer eigenvalues, but the relation (1) connecting $\vec{L}$ with <u>two</u> variables selects only integer eigenvalues. To get half-integer spin, we must replace (1) by

$$\vec{S} = \frac{1}{2i}[\vec{\xi}, \vec{\xi}], \tag{3}$$

where $\vec{\xi}$ has canonical Poisson brackets $\{\xi_i, \xi_j\} = -i\delta_{ij}$. Then $\vec{S}$ has the main property of the angular momentum

$$\{S_i, S_j\} = \epsilon_{ijk} S_k. \tag{4}$$

But the definition (3) not to be trivial, ξ_i must be anticommuting (odd) quantities

$$\xi_i \xi_j = -\xi_j \xi_i. \tag{5}$$

Then quantization replaces the Poisson brackets of ξ_i by an anticommutator, $\{,\} \to -i[,]_+$,

$$[\hat{\xi}_i, \hat{\xi}_j]_+ = \delta_{ij}, \tag{6}$$

the solution of which is given by the Pauli matrices

$$\hat{\vec{\xi}} = \frac{1}{\sqrt{2}}\vec{\sigma} \tag{7}$$

and the quantum spin (3) is

$$\hat{\vec{S}} = \frac{1}{2}\vec{\sigma}. \tag{8}$$

In an explicitly invariant relativistic theory a point-like particle can not be described by a 3-vector $\vec{r}(t)$. Instead it must be described by a 4-vector $r^\mu(\tau)$ depending on an invariant evolution parameter τ. Since physically we are interested only in the dependence of $\vec{r}(\tau)$ on $t = r^0(\tau)$, the dependence of both of them on τ is not physical and may be arbitrary changed. The explicitly invariant relativistic theory should not change under the reparametrization $\tau \to f(\tau)$. This implies that the coordinates r^μ and conjugate momentum components p^μ are not independent, but satisfy a constraint condition

$$\varphi(p) = 0, \tag{9}$$

where for a free spinless particle with mass m the constraint function $\varphi(p) = p^2 - m^2$. The Hamiltonian of such a system is simply a linear combination of the constraint functions [4]. In our case the Hamiltonian is

$$H = c\varphi(p), \tag{10}$$

where c is arbitrary or fixed by a gauge condition. The τ-evolution of the dynamical variable $X(= r, p)$ is given by the Poisson brackets of X and H

$$\dot{X} = \{X, H\}, \tag{11}$$

where

$$\{p^\mu, r^\nu\} = g^{\mu\nu}. \tag{12}$$

In (10)-(12) all r^μ and p^ν are considered as independent and only after calculating the brackets in (11) one should use the constraint condition (9).

In quantum theory the wave function of the particle satisfies the equation

$$\varphi(\hat{p})\psi = 0, \tag{13}$$

where $\varphi(\hat{p})$ in the constraint function of the operator arguments.

Let us illustrate this scheme by a free point-like relativistic particle with mass m and spin 1/2. Instead of $\vec{\xi}(t)$ we must use a 4-(axial)vector $\xi^\mu(\tau)$. To exclude its unphysical component one introduces a new unphysical dynamical variable $\xi^5(\tau)$ and a Lagrange multiplier λ (entering the Lagrangian without τ derivative). ξ^5 and λ are axial scalars. All the spin variables ξ^μ, ξ^5 and λ anticommute and have the Poisson brackets

$$\{\xi^\mu, \xi^\nu\} = ig^{\mu\nu}, \;\; \{\xi^5, \xi^5\} = -i. \tag{14}$$

The Lagrangian has the form

$$\mathcal{L} = \mathcal{L}(\dot{r}, \xi, \xi^5, \dot{\xi}, \dot{\xi}^5, \lambda) = \mathcal{L}_0(\dot{r}, \xi, \xi^5, \lambda) + \mathcal{L}_1(\dot{\xi}, \dot{\xi}^5), \tag{15}$$

where, in accordance with (14),

$$\mathcal{L}_1(\dot{\xi}) = \frac{1}{2i}(\xi\dot{\xi} - \xi^5\dot{\xi}^5). \tag{16}$$

The $\dot{r}$-dependent part $\mathcal{L}_0$ is determined by the reparametrization and Poincaré invariance

$$\mathcal{L}_0 = -\sqrt{\dot{r}^2}F(u), \tag{17}$$

where u are reparametrization-invariant axial scalars

$$u^0 = v\xi, \quad u^5 = \xi^5, \quad v = \frac{\dot{r}}{\sqrt{\dot{r}^2}}. \tag{18}$$

Since $(u^0)^2 = 0, (u^5)^2 = 0, (u\lambda)^* = \lambda u = -u\lambda$ and F must be a real even scalar function, its most general form is

$$F(u) = m + i(F_0 u^0 + F_5 u^5)\lambda, \tag{19}$$

where m, F_0 and F_5 are constants. We could add to (19) a term icu^0u^5, but since λ is arbitrary we can eliminate this term by a redefinition of λ ($\lambda \to \lambda + cu^0/F_5$). The momentum of the particle, conjugate to r, is

$$p^\mu = -\frac{\partial\mathcal{L}}{\partial\dot{r}_\mu} = v^\mu F + (\xi^\mu - (\xi v)v^\mu)iF_0\lambda. \tag{20}$$

Let us put

$$p_1 = p - (\xi - (\xi v)v)iF_0\lambda. \tag{21}$$

Then

$$p_1 = vF \tag{22}$$

and since $v^2 = 1$ we have

$$p_1^2 = F^2, \quad v = \frac{p_1}{\sqrt{p_1^2}}. \tag{23}$$

The first eq.(23) gives us a constraint function

$$\varphi = p_1^2 - F^2, \tag{24}$$

and the second one permits to express the velocity variable v through momentum p. As a result, the constraint function φ is

$$\varphi = p^2 - m^2 - 2mi(F_0\frac{p\xi}{\sqrt{p^2}} + F_5\xi^5)\lambda. \tag{25}$$

Up to an arbitrary factor this is the Hamiltonian of our particle. Taking the variation of it with respect to λ we get a spin constraint

$$\varphi_1 = F_0\frac{p\xi}{\sqrt{p^2}} + F_5\xi^5, \quad \varphi_1 = 0. \tag{26}$$

It is important that this constraint must conserve

$$\dot{\varphi}_1 = \{\varphi_1, \varphi\} = 0. \tag{27}$$

With the help of (14) we get a condition for this conservation

$$F_0^2 - F_5^2 = 0. \tag{28}$$

This is also the condition of the supersymmetry [3,5] of the spinning particle action.

After quantization the constraint (26) gives the Dirac equation. With the condition (28) its wave function satisfies also equation corresponding to the constraint (25), which is the Klein-Gordon equation.

3. Rotating rod, string and rotator

Before considering a relativistic string in the 4-dimensional space-time, let us imagine a rod in 3 dimensions with "mass density at rest" a and length l rotating in a plane around its center so that the linear velocity of its ends is 1. Let us calculate the energy E and the angular momentum of this rod. Knowing the linear velocity of a point at the distance x from the center to be $v = x/(l/2)$, and using relativistic formulas for the energy and momentum at each x we have

$$E = \int_{-l/2}^{l/2} \frac{adx}{\sqrt{1-v^2}} = \frac{a\pi}{2} l, \tag{29}$$

$$L = \int_{-l/2}^{l/2} \frac{xvadx}{\sqrt{1-v^2}} = \frac{a\pi}{8} l^2 \tag{30}$$

and, since $E = m$ is the mass of the rotating rod,

$$L = \frac{1}{2\pi a} m^2, \tag{31}$$

what is a linearly rising Regge trajectory. Of course, this model is logically inconsistent since the ends of the rod moving with the velocity of light can not be at rest. This inconsistency is eliminated by the notion of relativistic string, described by a 4-vector function $x^\mu(\tau, \sigma)$ of an evolution parameter τ and a parameter σ labelling points on the string, with the Lagrangian

$$\mathcal{L}_{\text{str}} = -a \int_{\sigma_1}^{\sigma_2} ((\dot{x}x')^2 - \dot{x}^2 x'^2)^{1/2} d\sigma \tag{32}$$

(prime means the derivative with respect to σ). It is not difficult to show that for the straight-line string

$$x^\mu(\tau, \sigma) = r^\mu(\tau) + f(\tau, \sigma) q^\mu(\tau) \tag{33}$$

we have exactly the formulas (29)-(31) for the length, energy and angular momentum, the parameter a being interpreted as a "string tension".

We shall not fix the parameters of the string ends $f_i(\tau) = f(\tau, \sigma_i)$ from the beginning, considering them as dynamical variables determined from the Lagrangian by the variational principle. Then the relativistic description of the straight-line string (33) contains physically superfluous variables and the string theory should be invariant under the three sets of τ-dependent transformations:

1) shift of r along q,

$$r \to r + \alpha(\tau)q, \tag{34}$$

2) multiplication of q by an arbitrary function,

$$q \to \beta(\tau)q, \tag{35}$$

3) reparametrization of τ, which for a Lagrangian means the equality

$$\mathcal{L}(\gamma(\tau)\dot{z}) = \gamma(\tau)\mathcal{L}(\dot{z}), \tag{36}$$

where $\dot{z}$ stands for every velocity in the Lagrangian.

We shall call a rotator any system possessing this symmetry. It may be more complicated than the string (32)-(33), may contain spins or other variables, but it always contains a straight-line string (33).

This symmetry implies that the rotator canonical variables obey three constraints (which are in involution with respect to their Poisson brackets since the transformations (34)-(36) form a group) and the rotator canonical Hamiltonian is zero.

The rotator we are interested in has the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{str}} + \sum_{i=1,2} \mathcal{L}_i + \mathcal{L}_{\text{int}}, \tag{37}$$

where $\mathcal{L}_{\text{str}}$ is the Lagrangian of the straight-line string (32)-(33), $\mathcal{L}_i$ is the Lagrangian of a point-like spinning particle (15)-(16), (18)-(19), (28), attached to the i-th end of the string and $\mathcal{L}_{\text{int}}$ is an additional Lagrangian describing an interaction of the string with its end particles if it is necessary. $\mathcal{L}_i$ depends on $\dot{x}_i, \xi_i, \xi_i^5, \dot{\xi}_i, \dot{\xi}^5, \lambda_i$, where $\dot{x}_i$ stands for the velocity of the end of the string perpendicular to the string direction. A possible movement of the end particles along the string is a separate question which is not considered here. Without this movement the Lagrangian (37) contains only first derivatives of r, q and ξ and does not contain $\dot{f}_i$ what makes it possible to easily express f_i through other variables. We shall call this solution as a rigid rotator.

4. Rigid rotator with n spins

Let a rigid rotator contains n spins. The spin variables of different spins commute with each other (and anticommute with themselves). Knowing the configuration space and the symmetry of the rotator we can write a general form of its Lagrangian

$$\mathcal{L} = -\sqrt{-\dot{n}^2}F(l, u) + \sum_{i=1}^{n} \mathcal{L}_1(\xi_i), \tag{38}$$

where $\mathcal{L}_1$ is given in (16),

$$n = q/\sqrt{-q^2} \tag{39}$$

and F is a real even function of the invariants

$$l = \sqrt{-\dot{r}_\perp^2/\dot{n}^2}, \tag{40}$$

$$\dot{r}_\perp = \dot{r} + (\dot{r}n)n - (\dot{r}\dot{n})\dot{n}/\dot{n}^2, \tag{41}$$

$$u = \{u_i^a\}, \quad u_i^a = v^a\xi_i, \quad a = 0,1,2,3, \quad u_i^5 = \xi_i^5, \tag{42}$$

$$v^0 = \dot{r}_\perp/\sqrt{\dot{r}_\perp^2}, \quad v^1 = \dot{n}/\sqrt{-\dot{n}^2}, \quad v_\mu^2 = \epsilon_{\mu\nu\rho\sigma}v^{0\nu}v^{3\rho}v^{1\sigma}, \quad v^3 = n. \tag{43}$$

Calculating the momenta p and π conjugate to r and q we get the three constraint functions:

$$\varphi_1 = pq, \quad \varphi_2 = \pi q, \tag{44}$$

$$\varphi_3 = \left((q^2 - \frac{(qp)^2}{p^2})\pi_n^2\right)^{1/2} - K(l,u), \tag{45}$$

where

$$K = lF_l - F, \tag{46}$$

an argument as a lower index means the partial derivative with respect to this argument, l is a function of p_n and u, implicitly given by the equation

$$F_l(l,u) = \sqrt{p_n^2}, \tag{47}$$

p_n and π_n are functions of p and π and spin variables (summation over i is implied)

$$p_n = p + l^{-1}(F_{u_i^0}u_i^2 + F_{u_i^2}u_i^0)v^2, \tag{48}$$

$$\pi_n = \left(g^{\mu\nu} - \frac{p^\mu p_n^\nu}{pp_n}\right)\left\{\pi_n - \frac{1}{\sqrt{-q^2}}[(F_{u_i^0}u_i^1 + F_{u_i^1}u_i^0)v_\nu^0 + (F_{u_i^2}u_i^1 - F_{u_i^1}u_i^2)v_\nu^2]\right\} \tag{49}$$

and the velocity variables are expressed through the phase space variables by successive use of the formulas

$$v^0 = \frac{p_n}{\sqrt{p_n^2}}, \quad v^1 = \frac{\pi_n}{\sqrt{-\pi_n^2}}. \tag{50}$$

The Hamiltonian of the rotator is a linear combination of the constraint functions

$$H = \sum_{i=1,2,3} c_i\varphi_i. \tag{51}$$

The non-zero Poisson brackets and the dynamical equations are

$$\{p^\mu, r^\nu\} = \{\pi^\mu, q^\nu\} = g^{\mu\nu}, \tag{52}$$

$$\{\xi_i^\mu, \xi_i^\nu\} = ig^{\mu\nu}, \ \ \{\xi_i^5, \xi_i^5\} = -i, \tag{53}$$

$$\dot{X} = \{X, H\}. \tag{54}$$

After calculating the brackets and for the initial conditions $\varphi_i = 0$.

The canonical quantization of the rotator can be done in the usual way

$$x \to \hat{x}, \ \ \{,\} \to -i[,]_{\mp} \tag{55}$$

$$\varphi_i \psi = 0, \tag{56}$$

where φ_i are the constraint functions (44-45) and other ones which may follow from the Hamiltonian (51) and φ_3 (45) (spin constraints, for instance). The solution of the (anti)commutation relations for two spin variables is

$$\hat{\xi}_1^\mu = \frac{1}{\sqrt{2}}\gamma^5\gamma^\mu \otimes I, \ \ \hat{\xi}_1^5 = \frac{1}{\sqrt{2}}\gamma^5 \otimes I, \tag{57}$$

$$\hat{\xi}_2^\mu = I \otimes \frac{1}{\sqrt{2}}\gamma^5\gamma^\mu, \ \ \hat{\xi}_2^5 = I \otimes \frac{1}{\sqrt{2}}\gamma^5. \tag{58}$$

The generalization for any number of spins is straightforward. The quantization is relativistic since the replacement (55) preserves the Poincaré algebra.

5. Applications: spinless rotator

For a rigid rotator without spins eqs.(38),(45) and (47) simplify

$$\mathcal{L} = -\sqrt{-\dot{n}^2}F(l), \tag{59}$$

$$\varphi_3 = \sqrt{-L_\mu^2} - K(l(p^2)), \tag{60}$$

$$K = lF_l - F, \ \ F_l(l) = \sqrt{p^2}, \tag{61}$$

where L_μ is the orbital spin of the system. In a gauge in which $b_1 = b_2 = 0$, the wave function of the quantum rotator satisfies the equation

$$(\sqrt{\hat{\vec{L}}^2} - K(l(\hat{p}^2)) - a_0)\psi = 0, \tag{62}$$

where a_0 is a constant of order $\hbar$ which may enter with quantization to improve the agreement with experiment for small L where our model is less reliable. For the eigenstates in (62)

$$\sqrt{L(L+1)} = K(l(m^2)) + a_0, \ \ L = 0, 1, 2, \tag{63}$$

with possible exception for $L = 0$ when the model can be unreliable.

For the straight-line string with point-like spinless particles with masses m_1 and m_2 at the ends the Lagrangian (37) with $L_{\rm int} = 0$ corresponds to

$$F(l) = \frac{a}{2} \sum_{i=1,2} \left\{ l^2 \arcsin \frac{l_i}{l} + \left(\frac{m_i l_i}{a} \right)^{1/2} \left(l_i + 2 \frac{m_i}{a} \right) \right\} \tag{64}$$

$$l_i = \sqrt{l^2 + (m_i/2a)^2} - m_i/2a. \tag{65}$$

This function determined the function $K(l(m^2))$ in the Regge trajectory (63) implicitly, through eqs.(61), and can be calculated numerically for any quark masses m_i. For $m_i << m$, or $m_i << al$, m being the meson mass one can use the expansion

$$F(l) = \frac{1}{2} \pi a l^2 \left[1 + \frac{4}{3\pi} \sum_{i=1,2} x_i^{1/2} \left(1 - \frac{3}{20} x_i + 0(x_i^2) \right) \right] \tag{66}$$

$$x_i = m_i/(la). \tag{67}$$

For the opposite case, when $m_i >> al$ (or $m_1 + m_2$ is close to m, both m_i being of the same order)

$$F(l) = l \sum_{i=1,2} m_i \left(1 + \frac{a^2 l^2}{2m_i^2} + O\left(\left(\frac{al}{m_i} \right)^4 \right) \right). \tag{68}$$

For the intermediate case, when $m_1 << al << m_2$

$$\begin{aligned} F(l) &= \frac{1}{4} \pi a l^2 \left[1 + \frac{8}{3\pi} x_1^{3/2} (1 - \frac{3}{20} x_1 + O(x_1^2)) \right] + \\ &+ l m_2 \left(1 + \frac{a^2 l^2}{2m_2^2} + O\left(\left(\frac{al}{m_2} \right)^4 \right) \right). \end{aligned} \tag{69}$$

For the first case

$$\begin{aligned} K(l(m^2)) &= \frac{m^2}{2\pi a} \left(1 - \frac{4}{3\pi} \sum_{i=1,2} y_i^{3/2} \left(1 - \frac{3}{20} y_i \right) + \frac{1}{(3\pi)^2} \left(\sum_{i=1,2} y_i^{3/2} \right)^2 + \right. \\ &+ \left. O(y_i^{7/2}) \right), \quad y_i = \pi m_i/m. \end{aligned} \tag{70}$$

For the second one

$$K(l(m^2)) = \frac{1}{a} \left[\frac{2}{3} (m - m_1 - m_2) \right]^{3/2} \left(\frac{m_1 m_2}{m_1 + m_2} \right)^{1/2} \left(1 + O\left(\frac{m - m_1 - m_2}{m} \right) \right) \tag{71}$$

and for the third, intermediate case

$$K(l(m^2)) = \frac{1}{\pi a} (m - m_2)^2 \left(1 + O\left(\left(\frac{m_1}{m - m_2} \right)^{3/2}, \frac{m - m_2}{m_2} \right) \right). \tag{72}$$

Note the doubling of the slope in the intermediate case as compared with the first one. To get a feeling of these formulas we can express the strange quark mass m_s through meson masses. From (63)

$$K(l(m_{\rho L}^2)) = K(l(m_{K^*L}^2)), \tag{73}$$

where $m_{\rho L}$ and m_{K^*L} are masses of the mesons lying on the leading ρ- and K^*-trajectories with the same spin L. Neglecting the u- and d-quark masses we have

$$\frac{m_s}{m_{K^*L}} = \frac{1}{\pi} Z_L^{2/3}\left(1 + \frac{1}{10} Z_L^{2/3} + \frac{1}{18\pi} Z_L + O(Z_L^{4/3})\right) < \tag{74}$$

$$Z_L = \frac{3\pi}{4}\left(1 - \frac{m_{\rho L}^2}{m_{K^*L}^2}\right).$$

For $L = 1$ ($m_\rho = 768$ MeV, $m_{K^*} = 892$ MeV) we get $m_s = 218$ MeV. For $L = 2$ ($m_{a_2} = 1318$ MeV, $m_{K_2^*} = 1425$ MeV) we get $m_s = 234$ MeV. For $L = 3$ ($m_{\rho_3} = 1690$ MeV, $m_{K_3^*} = 1780$ MeV) we get $m_s = 223$ MeV. We conclude that $m_s = (225 \pm 5)$ MeV is a good determination of the strange quark mass.

Comparison of a simplified formula (70), obtained in a different way, with experiment has been done in [6].

6. Rotator with one spin

The Lagrangian of a rotator with one spin 1/2 ("squark-quark meson") is

$$\mathcal{L} = -(-\dot{n}^2)^{1/2} F(l, u) + \mathcal{L}_1, \tag{75}$$

where $\mathcal{L}_1$ is given by eq.(16) and

$$\begin{aligned} F(l,u) &= F^0(l) + \frac{i}{2} F_{ab}(l) u^a u^b + \\ &+ (iF_a(l) u^a + \frac{1}{3!} F_{abc} u^a u^b u^c)\lambda, \end{aligned} \tag{76}$$

where $a, b, c = 0, 1, 3, 5$ (see eqs. (39-43)). The constraint function φ_3 is given by the expression

$$\varphi_3 = \sqrt{-J^2} - K^0 + \frac{i}{2} V_{ab} c^a c^b + (iF_a c^a + \frac{1}{3!} V_{abc} c^a c^b c^c)\lambda, \tag{77}$$

where J_μ is the total spin of the rotator,

$$K^0 = lF^0 - F^0, \quad F_l^0(l) = \sqrt{p^2}, \tag{78}$$

$$V_{ab} = F_{ab} - \epsilon_{oab5}, \quad V_{abc} = F_{abc} + F_{l[ab} F_{c]l} (F_{ll}^0)^{-1}, \tag{79}$$

$$c^0 = \frac{p\xi}{\sqrt{p^2}}, \quad c^1 = \frac{\pi_1 \xi}{\sqrt{-\pi_1^2}}, \quad c^3 = n\xi, \quad c^5 = \xi^5, \tag{80}$$

$$\pi_1^\mu = (g^{\mu\nu} - \frac{p^\mu p^\nu}{p^2} - \frac{q_p^\mu q_p^\nu}{q_p^2})\pi_\nu, \quad q_p = q - \frac{(qp)p}{p^2}. \tag{81}$$

The conservation of the spin constraint

$$\varphi_4 = iF_a c^a + \frac{1}{3!} V_{abc} c^a c^b c^c \tag{82}$$

leads to the conditions

$$F^a F_a = 0, \quad F^a V_{ab} = 0, \quad F^a V_{abc} = 0, \tag{83}$$

or

$$V_{ab} = \epsilon_{abcd} F^c X^d, \quad V_{abc} = \epsilon_{abcd} F^d Y, \tag{84}$$

where

$$F^a = g^{ab} F_b, \tag{85}$$

and g^{ab} is diagonal with $-g^{00} = g^{11} = g^{33} = g^{55} = -1$. The consistency of quantization (non-zero solution of the generalized Dirac equation corresponding to (82)) puts further limitations on the interaction and for the leading Regge trajectory we have the same formula as for the spinless rotator with the substitution J for L.

Conclusion

We see that the simple string model can explain main meson states including their dependence on the quark masses and spins. It seems worthwhile to explore this model further and try to consider string oscillations to account for daughter meson states and to investigate electroweak quark interactions inside a rotator to describe electroweak meson formfactors [7].

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