4 Space Time Measurements of Equivalent Moving Clocks

In this Section space time measurements of an array of synchronised clocks situated in the inertial frame S' will be considered. These clocks may be synchronised by any convenient procedure [4] (see for example Ref.[1]). For an observer in S' all such clocks are 'equivalent' in the sense that each of them records, independently of its position, the proper time τ' of the frame S'. For convenience, the array of clocks is assumed to be placed on the wagons of a train which is at rest in S', as shown in Fig.9a. The clocks are labelled C_m , $m = ... - 2, -1, 0, 1, 2, ...$ and are situated (with the exception of the 'magic clocks' $C_M, \overline{C}_M$, see below) at fixed distances L from each other, along the Ox' axis, which is parallel to the train. Any observer in S' will, after making the necessary corrections for Light Propagation Time Delays (LPTD), note that each Equivalent Clock (EC) indicates the same time, as shown in Fig.9a. It is now asked how the array of EC will appear to an observer at a fixed position in the frame S when the train is moving with velocity βc parallel to the direction Ox in S (Fig.9b). It is assumed that the EC C_0 is placed at $x' = 0$ and that it is synchronised with the Standard Clock C_S, placed at $x = 0$ in S, when $t = t' = 0$. The EC C_m and C_S record exactly equal time intervals when they are situated in the same inertial frame.

The appearence of the moving EC to an observer in S (after correction for LPTD; their actual appearence, including this effect, is considered later) at $t = 0$ is shown in Fig.9b, and in more detail in Fig.10 for both $t = 0$ and $t = \tau$. The period τ is the time between the passage of successive EC past C_S. The big hand of C_S in Fig.10 rotates through 180° during the time τ. Explicit expressions for the apparent times are presented in Table 5. In Fig.9b,10 the apparent positions of the clocks are shown for $\beta = 0.6$. The apparent times are readily calculated using the LT equations (3.1),(3.2). Consider the time indicated by C_1 at $t = 0$. The space-time points are:

$$S' : (L, t') \; ; \quad S : (x, 0)$$

Hence, Eqns.(3.1),(3.2) give:

$$x = \gamma(L + vt') \tag{4.1}$$
$$0 = \gamma(t' + \frac{\beta L}{c}) \tag{4.2}$$

which have the solution [$C_1(t = 0)$]:

$$t' = -\frac{\beta L}{c} \tag{4.3}$$
$$x = \frac{L}{\gamma} \tag{4.4}$$

As shown in Fig 9b, the wagons of the train are apparently shorter due to the LFC effect (Eqn.(4.4)) and also *the wagons at the front end of the train are seen at an earlier time*

[4] If an observer in S' knows the distance D to any of the clocks then the clock is synchronised relative to a local clock at the same position as the observer, when it is observed to lag behind the latter by the time D/c

than those at the rear end. Thus a $t = 0$ snapshot in S corresponds, not to a fixed t' in S' but one which depends on x': $t' = -\beta x'/c$. This is a consequence of the relativity of simultaneity of space-time events in S and S', as first pointed out by Einstein in Ref.[1]. Here it appears in a particularly graphic and striking form. The part of the space-time domain in S' that may be observed from S is considered in detail below. Consider now the time indicated by C_{-1} at $t = \tau$, i.e. when C_{-1} is at the origin of S. The space-time points are:

$$S' : (-L, t') \; ; \quad S : (0, \tau)$$

Hence, Eqns.(3.1),(3.2) give:

$$0 = \gamma(-L + vt') \tag{4.5}$$
$$\tau = \gamma(t' - \frac{\beta L}{c}) \tag{4.6}$$

with the solutions [$C_{-1}(t = \tau)$]:

$$t' = \frac{L}{v} \tag{4.7}$$
$$\tau = \frac{L}{\gamma v} = \frac{t'}{\gamma} \tag{4.8}$$

so that

$$t' = \gamma \tau \tag{4.9}$$

The EC at the origin of S at $t = \tau$ indicates a later time than C_S i.e. it is apparently running *faster* than C_S. This is an example of Time Contraction (TC). As shown below *TC is exhibited by the EC at any fixed position in S.* In fact, if the observer in S can see the EC only when they are near to C_S he (or she) will inevitably conclude that the clocks on the train run fast, not slow as in the classical TD effect (see below). Suppose that the observer is sitting in a waiting room with the clock C_S and notices the time on the train (the same as C_S) by looking at C_0 as it passes the waiting room window. If he (or she) then compares C_{-1} as it passes the window with C_S it will be seen to be running fast relative to the latter. In order to see the TD effect the observer would (as will now be shown), have to note the time shown by, for example C_0, at time $t = \tau$ as recorded by C_S. Indeed, to do this he would first have to correct his observation for the LPTD between himself and C_0. At time $t = \tau$ at C_S he would actually see C_0 as it appeared at an earlier time to a nearby observer in S. Using Eqn.(4.8),Eqn.(4.3) may be written as [$C_1(t = 0)$]:

$$t' = -\beta^2 \gamma \tau = -\frac{(\gamma^2 - 1)\tau}{\gamma} \tag{4.10}$$

This is the formula for the apparent time reported in Table 5. Now consider C_0 at time $t = \tau$. The space-time points are:

$$S' : (0, t') \; ; \quad S : (x, \tau)$$

Hence, Eqns.(3.1),(3.2) give:

$$x = \gamma v t' \tag{4.11}$$
$$\tau = \gamma t' \tag{4.12}$$

with the solutions [$C_0(t=\tau)$]:

$$t' = \tau/\gamma \quad (4.13)$$
$$x = v\tau = L/\gamma \quad (4.14)$$

So *the EC C_0 at time $t=\tau$ indicates an earlier time, and so is apparently running slower than C_S*. This is the classical Time Dilatation (TD) effect. It applies to observations of all *local clocks in S'*,(i.e. those situated at a fixed value of x') as well as any other EC that has the same value of x'.

As a last example consider the 'Magic Clock' C_M shown in Fig 9a at time $t=\tau$. With the space-time points:

$$S' \; : \; (-L/(1+\gamma), t') \;\; ; \;\;\; S \; : \; (x, \tau)$$

Eqns.(3.1),(3.2) give:

$$x = \gamma[-L/(1+\gamma) + vt'] \quad (4.15)$$
$$\tau = \gamma[t' - \frac{\beta}{c}L/(1+\gamma)] \quad (4.16)$$

with the solutions [$C_M(t=\tau)$]:

$$t' = \tau \quad (4.17)$$
$$x = \gamma v\tau/(1+\gamma) \quad (4.18)$$

where the relation $L = \gamma v \tau$ from Eqn.(4.14) has been used. *Thus C_M indicates the same time as C_S at $t=\tau$*. Similar 'Magic Clocks' can be defined that show the same time as C_S at any chosen time t in S. Such a clock is, in general, situated at $x' = -ct(\gamma-1)/\beta\gamma$. All of the other apparent times presented in Table 5 and shown in Figs. 9b, 10 are calculated in a similar way to the above examples by choosing appropriate values of x' and t.

It is straightforward to derive a general formula for the apparent time of any EC, C_m after the passage of an arbitary number j of wagons past the clock C_S. Still neglecting LPTD, the result is :

$$t'_{m,j} = -\frac{[m\gamma^2 - (m+j)]\tau}{\gamma} \quad (4.19)$$

A consequence of (4.19) is :

$$t'_{m,j+1} - t'_{m,j} = \tau/\gamma \quad (4.20)$$

This is the general TD result for any local (x' = constant) EC C_m. Eqn.(4.19) may, alternatively, be written in terms of $(n,\ j)$ where the index n labels the position of an EC in S rather than in S'. So the clocks at $x = L/\gamma,\ 2L/\gamma,\ ...$ (see Fig. 9b) have $n = 1, 2, ...$,those with $x = -L/\gamma,\ -2L/\gamma,\ ...$ have $n = -1, -2, ...$ Using the general relation :

$$n = m + j \quad (4.21)$$

Eqn.(4.19) may be written as :

$$t'_{n,j} = -\frac{[(n-j)\gamma^2 - n]\tau}{\gamma} \quad (4.22)$$

so that

$$t'_{n,j+1} - t'_{n,j} = \gamma\tau \quad (4.23)$$

This is the general TC effect for an EC at fixed n (x = constant). Eqn.(4.22) may also be used to calculate the apparent time delay between the EC on successive wagons at a fixed time in S :

$$t'_{n+1,j} - t'_{n,j} = -\gamma\beta^2\tau \quad (4.24)$$

The effects of LPTD on the apparent times indicated by the clocks on the moving train will now be taken into account. Only propagation times parallel to the train are considered and it is assumed that the clocks are orientated in such away that they can be seen by an observer placed beside C_S. Consider the clock C_n at the time Δt before the passage of the jth wagon past C_S (Fig.11). If the clock m is at the position x_m in S at this time then the inverse of the LT Eqn.(3.1) gives:

$$mL = \gamma[x_m - v(j\tau - \Delta t)] \quad (4.25)$$

The corresponding time shown by C_m, $t'^D_{m,j}$ is, using the inverse of the LT Eqn.(3.2):

$$t'^D_{m,j} = \gamma[j\tau - \Delta t - \frac{\beta x_m}{c}] \quad (4.26)$$

There are now two cases to consider:

(i) $x_m > 0$, C_m receding from the observer;

(i) $x_m < 0$, C_m approaching the observer;

If $|x_m| = c\Delta t$ the observer beside C_S in S will see the time $t'^D_{m,j}$ indicated by the clock C_m at time $j\tau$. Since Δt is, by definition, positive then in case (i) above x_m in (4.25) and (4.26) is replaced by $c\Delta t$. Eliminating Δt between the equations, after this replacement, gives for the apparent time:

$$t'^D_{m,j} = \gamma[j(1-\beta) - \beta m]\tau \quad (x_m > 0) \quad (4.27)$$

In case (ii) x_m in Eqns.(4.25),(4.26) is replaced by $-c\Delta t$, giving the solution:

$$t'^D_{m,j} = \gamma[j(1+\beta) + \beta m]\tau \quad (x_m < 0) \quad (4.28)$$

so that

$$t'^D_{m,j+1} - t'^D_{m,j} = \sqrt{\frac{1-\beta}{1+\beta}}\tau \quad (x_m > 0) \quad (4.29)$$

$$t'^D_{m,j+1} - t'^D_{m,j} = \sqrt{\frac{1+\beta}{1-\beta}}\tau \quad (x_m < 0) \quad (4.30)$$

Comparing Eqns(4.29),(4.30) with (4.20) it can be seen that, when the LPTD are taken into account the TD formula for a local clock is replaced by the Relativistic Doppler Effect formulae Eqns(4.29),(4.30). Actually, for $x_m < 0$ the clock appears to run fast, not slow. Just, as pointed out in Refs.[2,3,4], a moving sphere does not appear flattened by the LFC effect, Eqns.(4.29) and (4.30) demonstrate that a moving local clock does not show the TD effect. Weinstein [5] considered length measurements (for example the distance between successive clocks on the train in the present example) under the same conditions as the time measurements decscribed by Eqns.(4.29), (4.30) where a single observer is close to a moving object. If l_0, l denote the lengths of an object viewed in S', S then the relation between l_0 and l is given by the replacements $\tau \rightarrow l_0$, $\Delta t' = l$ in Eqns.(4.29),(4.30). Thus an approaching clock (apparently running fast) appears more distant than a receding clock which is apparently running slow. Neither the LFC nor the TD effects are directly observed when LPTD are taken into account.

It is interesting to note the identity of Eqns.(4.29),(4.30) with the usual Relativistic Doppler Shift formulae which, following Ref.[1] are usually derived by considering the LT properties of Electromagnetic Waves. Here they have been derived purely from considerations of space-time geometry.

Writing Eqns.(4.27),(4.28) in terms of (n, j) gives the equations:

$$t'^{D}_{n,j} = \gamma[j - \beta n]\tau \quad (n > 0) \tag{4.31}$$

$$t'^{D}_{n,j} = \gamma[j + \beta n]\tau \quad (n < 0) \tag{4.32}$$

Both Eqns.(4.31) and (4.32) yield the result:

$$t'^{D}_{n,j+1} - t'^{D}_{n,j} = \gamma\tau \tag{4.33}$$

Thus the TC effect of Eqn(4.23) is unchanged by LPTD corrections (they must clearly be the same at fixed n or x). The apparent time delay between the clocks on successive wagons is, including the effect of LPTD :

$$t'^{D}_{n+1,j} - t'^{D}_{n,j} = -\gamma\beta\tau \quad (n > 0) \tag{4.34}$$

$$t'^{D}_{n+1,j} - t'^{D}_{n,j} = \gamma\beta\tau \quad (n < 0) \tag{4.35}$$

Comparing with Eqn.(4.24) it can be seen that the LPTD increases the absolute size of the delay and, for $n < 0$ (EC approaching the observer) changes the sign of the effect.

The effect of LPTD corrections on the clock C_m may be calculated by taking the difference between $t'^{D}_{m,j}$ given by Eqn.(4.27) or (4.28) and $t'_{m,j}$ given by Eqn.(4.19). The results are:

$$\Delta'_{j,m} \equiv t'^{D}_{m,j} - t'_{m,j} = \frac{[\gamma^2(1-\beta) - 1]}{\gamma}[m + j] \quad (x_m > 0) \tag{4.36}$$

$$\Delta'_{j,m} \equiv t'^{D}_{m,j} - t'_{m,j} = \frac{[\gamma^2(1+\beta) - 1]}{\gamma}[m + j] \quad (x_m < 0) \tag{4.37}$$

The apparent times shown by the EC at $t = 0$, $t = \tau$, taking into account LPTD are presented in Table 6 and shown, for the special case $\beta = 0.6$ in Fig 12. Included also in Table 6 and Fig 12 is the 'Magic Clock' $\overline{C}_M$ situated at $x' = x'_M$ where :

$$x'_M = -\frac{L}{\gamma\beta}\left[1 - \sqrt{\frac{1-\beta}{1+\beta}}\right] \tag{4.38}$$

which indicates the same time as C_S at $t = \tau$. Table 6 and Fig.12 show the perhaps surprising result that the EC situated symmetrically in x relative to C_S at time $t = 0$ and

$t = \tau$ apparently lag C_S by identical times. This is because the time asymmetry between positive and negative x produced by the LT (see Fig.10) is exactly compensated by the LPTD. For example. at $t = \tau$ the LT gives the result for $\beta = 0.6$ that C_0 lags C_S by 0.2τ. However, the longer time delay from C_{-2} as compared to C_0 (see Fig.11) means that after correcting for LPTD, C_{-2} appears also to lag C_S, by just the same amount as C_0, which has a smaller LPTD correction.

Finally in this section the region of the space-time domain of S', that is visible to the observer in S is discussed. In particular the space-time observations which may be made of the wagon holding the EC C_0 during the period $0 < t < \tau$ when it is passing by the Standard Clock C_S will be considered. The situation is shown for the three cases: $\beta = 0.0,\ 0.6,\ 0.943$ ($\gamma = 1.0,\ 1.25,\ 3.0$) in Figs 13a),b),c) respectively. For $\beta = 0$ the history of each part of the wagon may be observed in an unbiased manner over the whole period. When the wagon is moving as in Fig 9b), late times at the front and early times at the rear of the wagon are no longer observable. The observable region of the (x', ct') plane is only that between the lines L1, L2 (Fig 13b)) where:

$$L1\ : ct' = -\beta x' \tag{4.39}$$

$$L2\ : ct' = -\beta x' + \frac{c\tau}{\gamma} \tag{4.40}$$

As β approaches one (Fig 13c) the observable domain occupies only a narrow region around the backward light cone of the origin of S'. So, although the wagons of the train are more concentrated in the field of vision of an observer in S, due to the LFC, the fraction of the total space-time area of S' that may be observed becomes vanishingly small. Note that the boundaries of the observable area in the space-time of S' are easily read off from the apparent time of the clocks C_{-1}, C_0 recorded in Table 5. Eqns.(4.41) and (4.42) are derived from Eqn.(4.19) with $j = 0,\ 1$ respectively on making the replacements: $\tau \rightarrow L/\gamma v$, $m \rightarrow = x'/L$. The situation shown in Fig 13 Corresponds to the observation of the train at a distance such that the angle subtended by the wagon between C_0 and C_{-1} at the observer is small. In this case the effects of LPTD essentially cancel. It is interesting to compare this with the case of an observer close to the train when the LPTD of photons moving almost parallel to the train must be taken into account. The (x', ct') domain seen by such an observer, for the same conditions as in Fig 13, is shown in Fig 14. It is derived in a similar way as for Fig.13, starting from Eqn.(4.28) instead of Eqn.(4.19). When the train is moving the observable range of t' is always greater at the rear end (position of C_{-1}) than at the front (position of C_0) of the wagon. As $\beta \rightarrow 1$ the t' range at C_0 vanishes and that at C_{-1} approaches a constant $-L/c < t' < L/c$, corresponding to the full region between the forward light cone ($x' = ct'$) and the backward light cone ($x' = -ct'$) of the origin of S'.

5 Discussion

The different space-time effects (apparent distortions of space or time) in Special Relativity that have been discussed above are summarised in Table 7. These are the well-known LFC and TD effects, Space Dilatation (SD) introduced in Section 3 above, and Time Contraction (TC) introduced in Section 4. Each effect is an observed difference Δq ($q = x, x', t, t'$) of two space or time coordinates ($\Delta q = q_1 - q_2$) and corresponds to a constant projection $\Delta\tilde{q} = 0$ ($\tilde{q} \neq q$) in another of the four variables x, x', t, t' of the LT. As shown in Table 7, the LFC, SD, TC and TD effects correspond, respectively, to the Δt, $\Delta t'$, Δx and $\Delta x'$ projections. After making this projection, the four LT equations give two relations among the remaining three variables. One of these describes the 'space-time distortion' relating $\Delta t'$ and Δt or $\Delta x'$ and Δx while the other gives the equation shown in the last column, (labelled 'Complementary Effect') in Table 7. These equations relate either Δx to Δt (for SD and TD) or $\Delta x'$ to $\Delta t'$ (for LFC and TC). It can be seen from the Complementary Effect relations that the two space-time points defining the effect (of space-time distortion) are space-like separated for LFC and SD and time-like separated for TC and TD.

For example, for the LFC when $t_1 = t_2 = t$, the LT equations for the two space-time points are:

$$x'_1 = \gamma(x_1 - vt) \quad (5.1)$$
$$x'_2 = \gamma(x_2 - vt) \quad (5.2)$$
$$t'_1 = \gamma(t - \frac{\beta x_1}{c}) \quad (5.3)$$
$$t'_2 = \gamma(t - \frac{\beta x_2}{c}) \quad (5.4)$$

Subtracting (5.1) from (5.2) and (5.3) from (5.4) gives:

$$\Delta x' = \gamma \Delta x \quad (5.5)$$
$$\Delta t' = -\frac{\gamma\beta}{c}\Delta x \quad (5.6)$$

Eqn.(5.5) describes the LFC effect, while combining Eqns.(5.5) and (5.6) to eliminate Δx yields the equation for the Complementary Effect. By taking other projections the other entries of Table 7 may be calculated in a similar fashion. It is interesting to note that the TD effect can be derived directly from the LFC effect by using the symmetry of the LT equations. Introducing the notation: $s \equiv ct$, the LT may be written as:

$$x' = \gamma(x_1 - \beta s) \quad (5.7)$$
$$s' = \gamma(s - \beta x) \quad (5.8)$$

These equations are invariant [5] under the following transformations:

$$T1 \quad : \quad x \leftrightarrow s, \quad x' \leftrightarrow s' \quad (5.9)$$
$$T2 \quad : \quad x \leftrightarrow x', \quad s \leftrightarrow s', \quad \beta \rightarrow -\beta \quad (5.10)$$

[5] Actually the transformation $T1$ yields the inverse of the LT (5.7),(5.8). The inverse equations may then be solved to recover (5.7) and (5.8)

Writing out the LFC entries in the first row of Table 7, replacing t, t' by s/c, s'/c ; gives

$$\Delta x \quad \Delta s = 0 \quad \Delta x = \frac{\Delta x'}{\gamma} \quad \Delta x' = -\frac{\Delta s'}{\beta}$$

Applying $T1$ to each entry in this row results in:

$$\Delta s \quad \Delta x = 0 \quad \Delta s = \frac{\Delta s'}{\gamma} \quad \Delta s' = -\frac{\Delta x'}{\beta}$$

Applying $T2$:

$$\Delta s' \quad \Delta x' = 0 \quad \Delta s' = \frac{\Delta s}{\gamma} \quad \Delta s = \frac{\Delta x}{\beta}$$

Replacing Δs, $\Delta s'$ by $c\Delta t$, $c\Delta t'$ yields the last row of Table 7 which describes the TD effect. Similarly TC can be derived from SD (or vice versa) by successively applying the transformations $T1$, $T2$.

A remark on the 'Observed Quantities' in Table 7. For LFC, SD the observed quantity is a length interval in the frame S. The space distortion occurs because this length differs from the result of of a similar measurement made on the same object in its own rest frame. $\Delta x'$ is not directly measured at the time of observation of the LFC or SD. It is otherwise with the time measurements TD, TC. Here the time interval indicated by a moving clock (TD), or different equivalent clocks at the same position in S (TC) *in their own rest frame* is supposed to be directly observed and compared with the time interval Δt registered by an unmoving clock in the observer's rest frame. Thus the effect refers to two simultaneous observations by *the same observer* not to two separate observations by *two different observers* as in the case of the LFC and SD.

Einstein's great achievement in his first paper on Special Relativity [1] was, for the first time, to clearly disentangle in Classical Electromagnetism, the purely geometrical and kinematical effects embodied in the Lorentz Transformation from dynamics. In spite of this, papers still appear from time to time in the literature claiming that moving objects 'really' contract [6] or that moving clocks 'really' run slow [7] for dynamical reasons, or even that such dynamical effects are the true basis of Special Relativity and should be taught as such [8]. As it has been shown above that a moving object can apparently shrink or expand, and a moving clock can apparently run fast or slow, depending only on how it is observed, it is clear that they cannot 'really' shrink, or run slow, respectively. If a moving object actually shrinks for dynamical reasons it is hard to see how the same object, viewed in a different way (in fact only illuminated differently in its own rest frame) can be seen to expand. Certainly both effects cannot be dynamically explained. In fact the Lorentz Transformation, as applied to space-time, describes only the *appearence* of space-time events, a purely geometrical property. The apparent distortions are of geometrical origin, the space-time analogues of the apparent distortions of objects in three dimensional space, described by the laws of perspective, when they are linearly projected into a two dimensional sub-space by a camera or the human eye. The Lorentz transformation has nothing to say (in spite of the title of Ref.[1]) on the dynamics of physical objects, and it is the Lorentz Transformation, not Classical Electrodynamics, that is the bedrock of Special Relativity.

In conclusion the essential characteristics of the two 'new' space-time distortions discussed above are summarised :

- Space Dilatation (SD): If a luminous object lying along the Ox' axis in the frame S' has a short luminous lifetime in this frame, it will be observed from a frame S, in uniform motion relative to S' parallel to Ox' at the velocity βc, as a narrow line, perpendicular to the x-axis moving with the velocity c/β in the same direction as the moving object. The total distance swept out along the x-axis by the moving line during the time $\beta l_0/(c\sqrt{1-\beta^2})$, for which the moving line image exists, is $l_0/\sqrt{1-\beta^2}$ where l_0 is the length of the object as observed in S'. Thus the apparent length of the object when viewed with a time resolution much larger than $\beta l_0/(c\sqrt{1-\beta^2})$ is $l_0/\sqrt{1-\beta^2}$. Any effects of LPTD are not taken into account.

- Time Contraction (TC): The equivalent clocks in the moving frame S', viewed at the same position in the stationary frame S, apparently run faster by a factor $1/\sqrt{1-\beta^2}$ relative to a clock at rest in S.

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Point on object	x'	t'	x	t
P	d_2	$-\frac{d_2}{c}\tan\alpha$	$\gamma d_2(1-\beta\tan\alpha)$	$\gamma\frac{d_2}{c}(\beta-\tan\alpha)$
Q	0	0	0	0
R	-d_1	$\frac{d_1}{c}\tan\alpha$	-$\gamma d_1(1-\beta\tan\alpha)$	-$\gamma\frac{d_1}{c}(\beta-\tan\alpha)$

Table 1: Space-time points on the object O_D, illuminated by a short light pulse from the lamp L1 (Fig.2), as observed in the frames S',S.

Point on object	x'	t'	x	t
P	d_2	0	γd_2	$\frac{\gamma v d_2}{c^2}$
Q	0	0	0	0
R	-d_1	0	-γd_1	-$\frac{\gamma v d_1}{c^2}$

Table 2: Space-time points on the object O_D, illuminated by a short light pulse from the lamp L2 (Fig.2), as observed in the frames S',S.

	$\tan\alpha \neq \pm 1$		$\tan\alpha = 1$		$\tan\alpha = -1$	
	$\beta \to 1$	$\beta \to -1$	$\beta \to 1$	$\beta \to -1$	$\beta \to 1$	$\beta \to -1$
$\lvert d_I^{L1}\rvert$	∞	∞	0	∞	∞	0
$\lvert \Delta t^{L1}\rvert$	∞	∞	0	∞	∞	0
β_I^{L1}	$\frac{f}{2l}(1+\frac{d_1}{d_2})$	$-\frac{f}{2l}(1+\frac{d_2}{d_1})$	$-\frac{f}{2l}(1+\frac{d_2}{d_1})$	$-\frac{f}{2l}(1+\frac{d_2}{d_1})$	$\frac{f}{2l}(1+\frac{d_1}{d_2})$	$\frac{f}{2l}(1+\frac{d_1}{d_2})$

Table 3: The size $|d_I^{L1}|$, time duration $|\Delta t^{L1}|$ and velocity parallel to Ox_I β_I^{L1}, of images of the object O_D, illuminated by the lamp L1 for different values of $\tan\alpha$ in the UR limit.

	$\beta \to 1$	$\beta \to -1$
d_I^{L2}	∞	∞
Δt^{L2}	∞	$-\infty$
β_I^{L2}	$\frac{f}{2l}(1+\frac{d_1}{d_2})$	$-\frac{f}{2l}(1+\frac{d_2}{d_1})$

Table 4: The size $|d_I^{L2}|$, time duration $|\Delta t^{L2}|$ and velocity parallel to Ox_I β_I^{L2}, of images of the object O_D, illuminated by the lamp L2 in the UR limit.

C_S	C_{-2}	C_{-1}	C_M	C_0	C_1	C_2
0	$2\frac{(\gamma^2-1)}{\gamma}\tau$	$\frac{(\gamma^2-1)}{\gamma}\tau$	$\frac{(\gamma-1)}{\gamma}\tau$	0	$-\frac{(\gamma^2-1)}{\gamma}\tau$	$-2\frac{(\gamma^2-1)}{\gamma}\tau$
τ	$\frac{(2\gamma^2-1)}{\gamma}\tau$	$\gamma\tau$	τ	$\frac{\tau}{\gamma}$	$-\frac{(\gamma^2-2)}{\gamma}\tau$	$-\frac{(2\gamma^2-3)}{\gamma}\tau$

Table 5: Apparent times of Equivalent Clocks on the moving train in Fig.9, at times $t=0$ and $t=\tau$ of the stationary standard clock C_S. Effects of the LT only.

C_S	C_{-2}	C_{-1}	C_M	C_0	C_1	C_2
0	$-2\gamma\beta\tau$	$-\gamma\beta\tau$	$\left[\sqrt{(1-\beta)/(1+\beta)}-1\right]\tau$	0	$-\gamma\beta\tau$	$-2\gamma\beta\tau$
τ	$\gamma(1-\beta)\tau$	$\gamma\tau$	τ	$\gamma(1-\beta)\tau$	$\gamma(1-2\beta)\tau$	$\gamma(1-3\beta)\tau$

Table 6: Definitions as for Table 5, except that the effects of LPTD for an observer close to the train are also included.

Name	Observed Quantity	Projection	Effect	Complementary Effect
Lorentz-Fitzgerald Contraction (LFC)	Δx	$\Delta t = 0$	$\Delta x = \frac{1}{\gamma}\Delta x'$	$\Delta x' = -\frac{c}{\beta}\Delta t'$
Space Dilatation (SD)	Δx	$\Delta t' = 0$	$\Delta x = \gamma\Delta x'$	$\Delta x = \frac{c}{\beta}\Delta t$
Time Contraction (TC)	$\Delta t'$	$\Delta x = 0$	$\Delta t' = \gamma\Delta t$	$\Delta x' = -c\beta\Delta t'$
Time Dilatation (TD)	$\Delta t'$	$\Delta x' = 0$	$\Delta t' = \frac{1}{\gamma}\Delta t$	$\Delta x = c\beta\Delta t$

Table 7: The different apparent distortions of space-time in Special Relativity (see text).

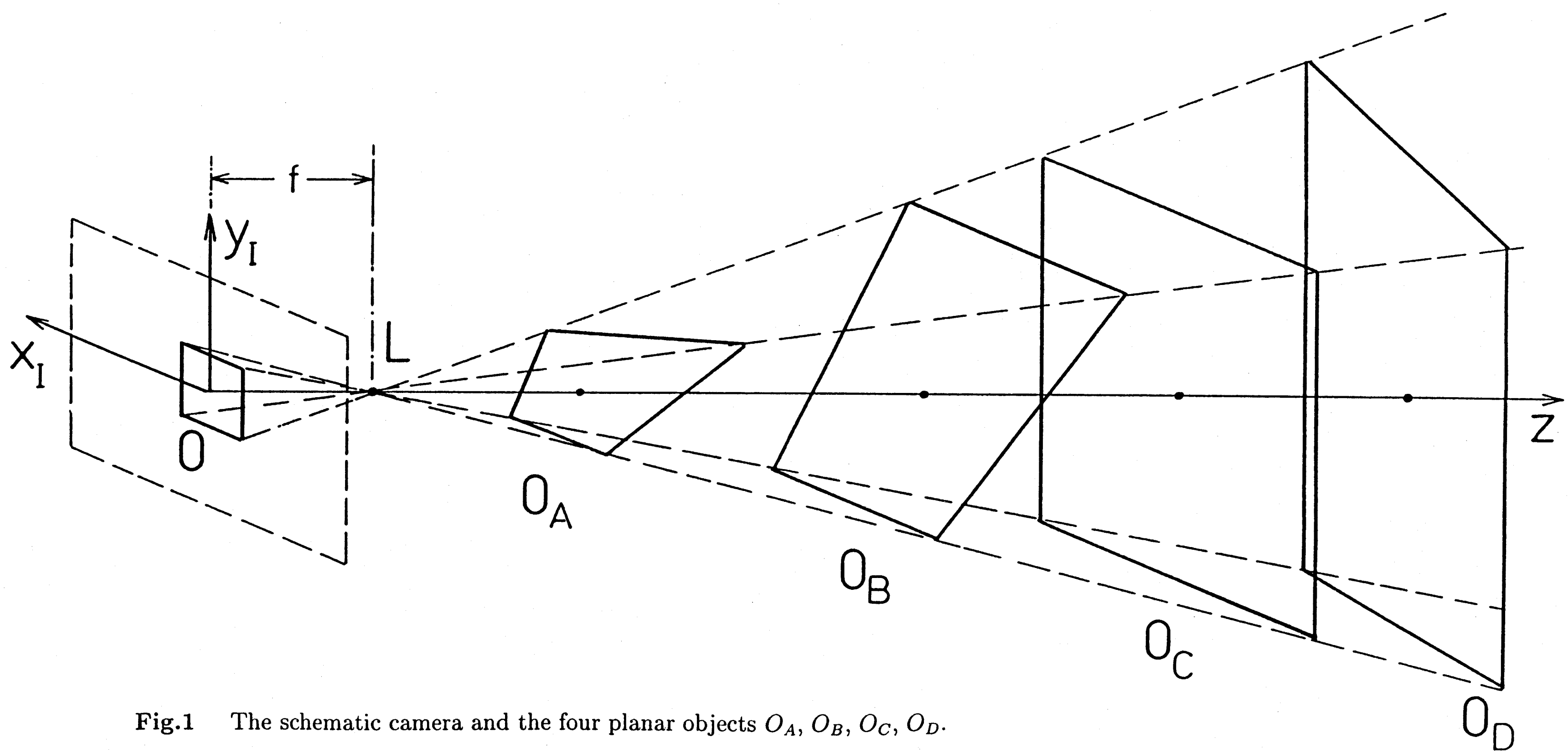

Fig.1 The schematic camera and the four planar objects O_A, O_B, O_C, O_D.

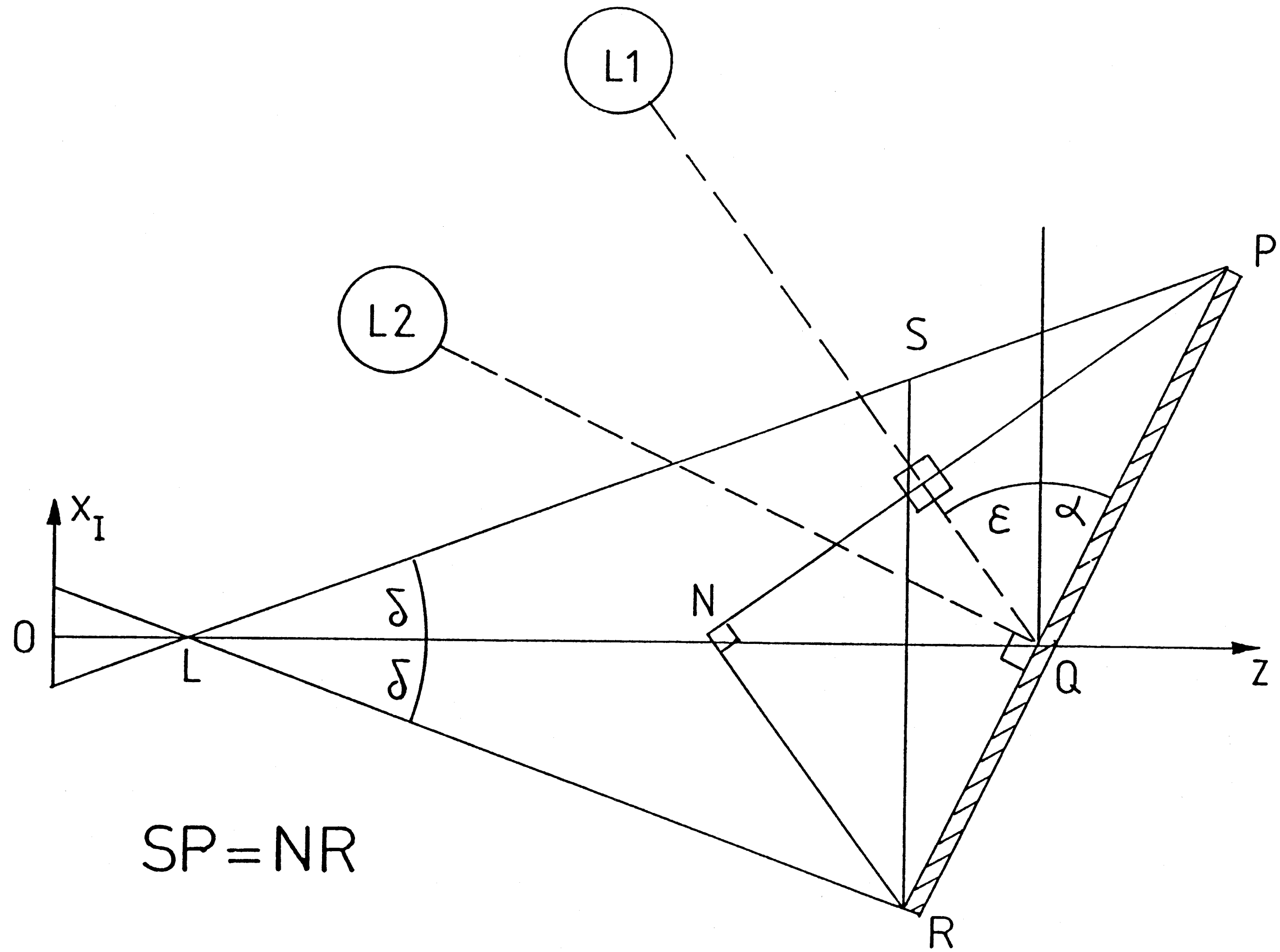

Fig.2 The projection into the x_I-z plane of the object O_D, showing the positions in this plane of the lamps L1 and L2.

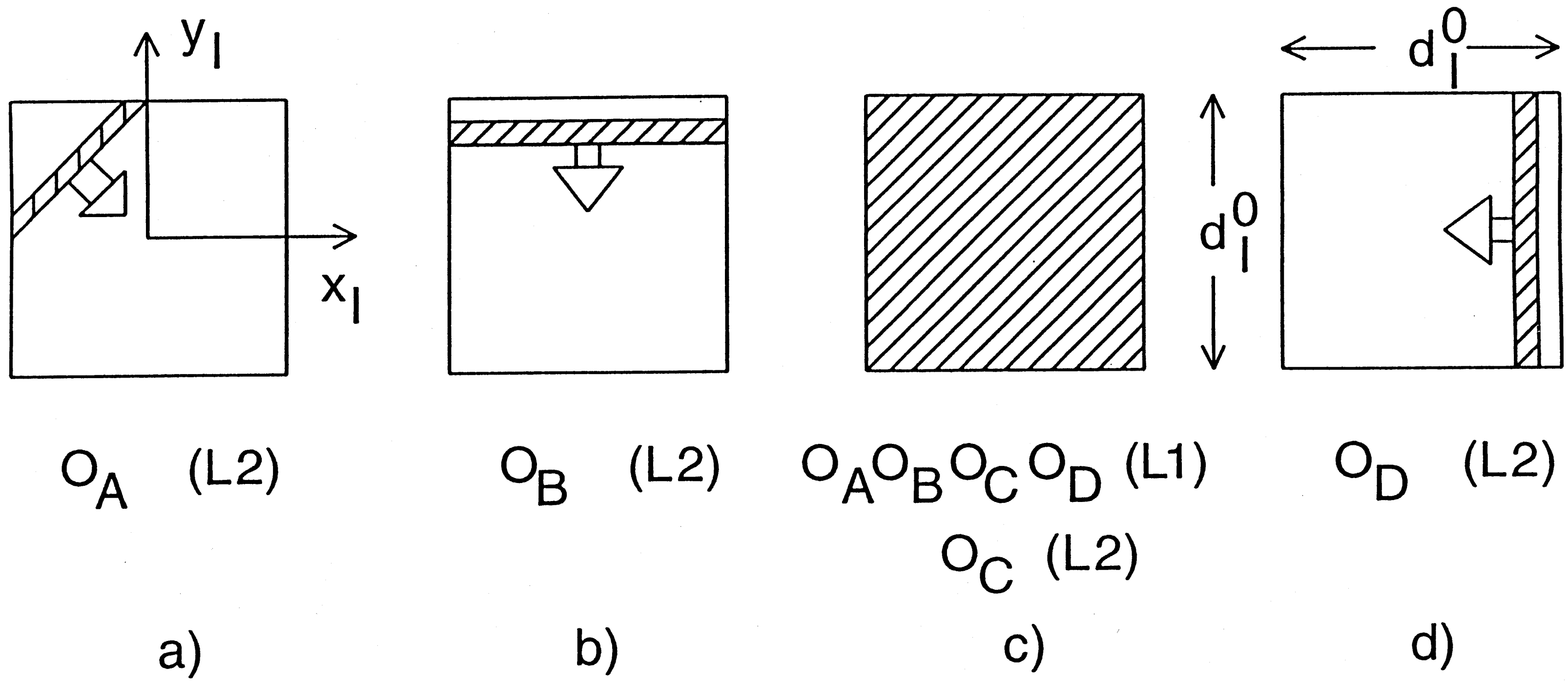

Fig.3 Images of the objects observed in the camera (viewed from the side of the objects) when lamps L1 and L2 are flashed. The arrows show the direction of motion of the instantaneous line image. The dotted squares show the total areas swept out in the image plane by the line image. a),b),c),d) correspond to O_A, O_B, O_C and O_D for L2 whereas L1 gives the image shown in c) for all four objects.

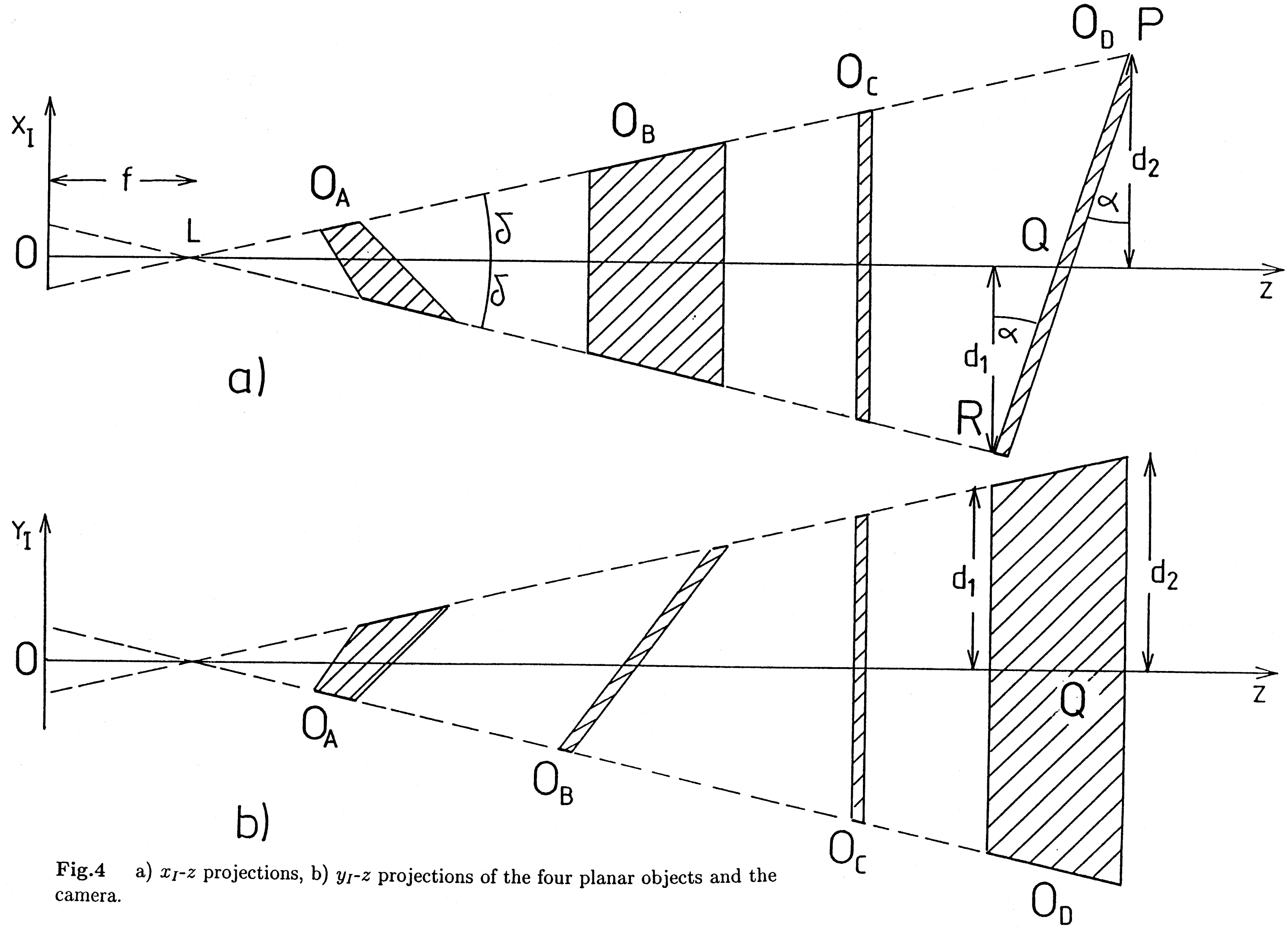

Fig.4 a) x_I-z projections, b) y_I-z projections of the four planar objects and the camera.