EVOLUTION OF THE UNIVERSE AND GRAVITON MASS

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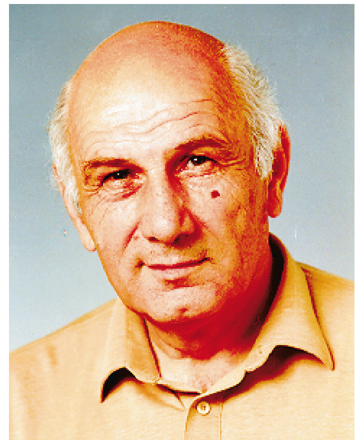

The paper considers the evolution of homogeneous and isotropic Universe in the framework of the Relativistic Theory of Gravitation (RTG). It is stated that in the framework of this theory, the Universe can be only "flat", i.e., its spatial metric is the Euclidean one. Introduction of the graviton mass results in the elimination of cosmological singularity and the development of the Universe in series of alternating cycles from high density $\rho_{\max}$ to minimal $\rho_{\min}$, etc. The Mach principle is naturally realized in the theory: the inertial system is determined by distribution of masses in the Universe. The theory testifies the presence of the big "hidden" mass in the Universe. The Hubble constant H, deceleration parameter q and the evolution period of the Universe have been estimated. The present values $H(\tau_c)$, $q(\tau_c)$ end the time from the onset of evolution τ_c are practically independent of the maximum density of matter $\rho_{\max}$ related to the integral of motion. In the present model the usual problems of singularity, flatness, causality are absent; and therefore there is no need to introduce the stage of inflational expansion.

The Relativistic Theory of Gravitation (RTG) treats the gravitational field as a physical one, i.e. as a field of Faraday-Maxwell type. The conserved density of the energy-momentum tensor of all matter forms is a source of this universal field. As the source is the tensor density, the gravitational field is characterized by the tensor density also. The gravitational field as all other physical fields is considered in Minkowski space. Therefore the inertia forces are separated from the gravitational forces since they are of different nature. The choice of Minkowski space has been dictated by the fundamental physical principles, i.e. the integral conservation laws of the energy-momentum and angular momentum. The origin of the effective Riemannian space comes from this point. It literally has the field origin.

The realization of a tensor gravitational field in the Minkowski space inevitably requires an introduction of the graviton mass and, as a consequence, it requires some unique and completely fixed algebraic structure in the field equations. This structure includes a cosmological term and a term with the Minkowski metric tensor. Both of these terms appear in the field equations with the same constant that is the graviton mass squared. The introduction of a cosmological term was also discussed in the General Relativity Theory. L.D.Landau and E.M.Lifshits wrote in this relation [2]: *"We stress that it would mean a change that had a deep physical meaning: the introduction of a constant term which was in general independent of the field state into the Lagrangian function; this would mean the attributing a principally unavoidable curvature to the space-time and the curvature was neither related to matter nor to the gravitational waves".*

This problem does not arise in the RTG, because the presence of the unique algebraic structure with the same constant in the field equations leads to the elimination of a space-time curvature in the absence of matter and the gravitational field. So, the space-time has a pseudo-Euclidean metric $\gamma_{\mu\nu}$, which has been chosen apriori. According to the RTG the inertial and gravitational masses are equal as a straightforward consequence of the fact that the conserved energy-momentum tensor density of the entire matter is a source of the gravitational field. The notion of the inertial reference frame is preserved in the RTG and therefore the acceleration has an absolute meaning. The RTG field equations are form-invariant under Lorentz transformations. The correspondence between an inertial system and the distribution of matter means that the Mach principle is fulfilled in the RTG. The correspondence principle is also realized in the RTG as matter equations of motion changes for equations of motion of the Special Relativity Theory under switching off the gravitational field. This change takes place in the coordinate system, which was chosen from the very beginning. It is a straightforward consequence of the RTG equations of motion that the gravitational field has polarization properties corresponding to spin values 2 and 0.

The evolution of the isotropic and homogeneous Universe in the RTG was treated in articles [3,4]. It was found there that the Universe could be "flat" only (i.e. the spatial geometry of it should be Euclidean) and its density should be higher than the critical density ρ_c, determined by the Hubble "constant". On this ground it was concluded that the Universe should contain a large "hidden" mass (the so-called "dark matter"). Also it was shown that the presence of a non-zero graviton mass (this is of a principal importance for the RTG) played a constructive role at the same stages of the Universe evolution. Just due to this, the gravitational repulsion forces arise in the process of the Universe contraction, which remove the cosmological singularity. Therefore, this was shown to be a consequence of the RTG that the Universe evolves cyclewise, expanding from some maximal $\rho_{\max}$ density to minimal $\rho_{\min}$, then contracting back to $\rho_{\max}$ and so on.

Article [5] contains a further analysis of the evolution of the homogeneous and isotropic Universe and also a value of the maximal density of matter in the Universe has been found on the base of an additional proposal.

In this paper we come back again to this problem in order to show that the evolution of the homogeneous and isotropic Universe is determined by the contemporary observational data, the maximal matter density $\rho_{\max}$ and the graviton mass m. As we will see below, the modern parameters of the Hubble "constant" and the deceleration coefficient q_c are practically independent of the value of the maximal density $\rho_{\max}$. The maximal density of matter is an integral of motion of the dynamical system and it is determined by the initial conditions. A choice of these initial conditions must be provided in such a way that does not contradict the contemporary observational data. According to the RTG the maximal density of matter in the Universe can be lower, than the Planckian density. Let us write the RTG equations in the form [1]

$$R^\nu_\mu - \frac{1}{2}\delta^\nu_\mu R + \frac{m^2}{2}[\delta^\nu_\mu + g^{\nu\alpha}\gamma_{\alpha\mu} - \frac{1}{2}\delta^\nu_\mu g^{\alpha\beta}\gamma_{\alpha\beta}] = 8\pi T^\nu_\mu \ , \tag{1}$$

$$D_\mu \tilde{g}^{\mu\nu} = 0 \ , \tag{2}$$

where D_μ is a covariant derivative in Minkowski space with the metric $\gamma_{\mu\nu}$, m is the graviton mass. For our convenience we choose the units $G = \hbar = c = 1$.

The energy-momentum tensor of mater has the form

$$T^\nu_\mu = (\rho + p)u^\nu u_\mu - \delta^\nu_\mu p, \quad u^\nu = \frac{dx^\nu}{ds} \ . \tag{3}$$

Here ρ is the density of matter, p is the pressure, ds is the interval of the effective Riemannian space.

For the homogeneous and isotropic model of the Universe the effective Riemannian space interval ds has a general form

$$ds^2 = U(t)dt^2 - V(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\Theta^2 + \sin^2\Theta d\phi^2)\right] . \tag{4}$$

Here k takes values $1, -1, 0$. $k = 1$ corresponds to the closed Universe, $k = -1$ — to the hyperbolic one and $k = 0$ — to the "flat" one.

The whole consideration will be made in an inertial reference frame with spherical coordinate system r, Θ, ϕ. The interval of the Minkowski space here has the form

$$d\sigma^2 = dt^2 - dx^2 - dy^2 - dz^2 = dt^2 - dr^2 - r^2(d\Theta^2 + \sin^2\Theta d\phi^2) \ . \tag{5}$$

The determinant g, composed from the components $g_{\mu\nu}$, is equal to

$$g = -UV^3(1-kr^2)^{-1}r^4\sin^2\Theta \ . \tag{6}$$

The tensor density

$$\tilde{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu} \tag{7}$$

has according to (4) the following components:

$$\begin{aligned} \tilde{g}^{00} &= \sqrt{\frac{r^4V^3}{U(1-kr^2)}}\sin\Theta \ , \quad \tilde{g}^{11} = -r^2\sqrt{UV(1-kr^2)}\sin\Theta \ , \\ \tilde{g}^{22} &= -\sqrt{\frac{UV}{1-kr^2}}\sin\Theta \ , \quad \tilde{g}^{33} = -\sqrt{\frac{UV}{1-kr^2}}\cdot\frac{1}{\sin\Theta} \ . \end{aligned} \tag{8}$$

The Christoffel symbols for the Minkowski space are

$$\begin{aligned} \gamma^1_{22} &= -r,\ \ \gamma^1_{33} = -r\sin^2\Theta,\ \ \gamma^2_{12} = \gamma^3_{13} = \frac{1}{r}\ , \\ \gamma^2_{33} &= -\sin\Theta\cos\Theta,\ \ \gamma^3_{23} = \mathrm{ctg}\Theta\ . \end{aligned} \tag{9}$$

Equation (2) has the form

$$D_\mu \tilde{g}^{\mu\nu} = \partial_\mu \tilde{g}^{\mu\nu} + \gamma^\nu_{\alpha\beta}\tilde{g}^{\alpha\beta} = 0\ . \tag{10}$$

Substituting (9) into (10), we get

$$\frac{\partial}{\partial t}\left(\sqrt{\frac{V^3}{U}}\right) = 0\ , \tag{11}$$

$$\frac{\partial}{\partial r}(r^2\sqrt{1-kr^2}) = 2r(1-kr^2)^{-1/2}. \tag{12}$$

From equation (11) we derive

$$V = aU^{1/3}. \tag{13}$$

Here a is an integration constant. It is a straightforward calculation to get from equation (12) that

$$k = 0\ , \tag{14}$$

i.e. the spatial metric is Euclidean. It is timely to stress that this conclusion directly follows from relation (2) for the gravitational field and does not depend on the density of matter for the homogeneous and isotopic Universe. Therefore, equation (2) forbids the closed and the hyperbolic Universe models. The homogeneous and isotropic Universe according to the RTG can be just only "flat". In other words, there is no "flatness problem" for the Universe in the RTG.

The effective Riemannian metric (4) with account for (13) and (14) takes the form

$$ds^2 = U(t)dt^2 - aU^{1/3}(t)[dr^2 + r^2(d\Theta^2 + \sin^2\Theta d\phi^2)]\ . \tag{15}$$

The affine connection coefficients of the Riemannian space

$$\Gamma^\rho_{\mu\nu} = \frac{1}{2}g^{\rho\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu})\ , \tag{16}$$

for interval (15) have the form

$$\begin{aligned} \Gamma^0_{00} &= \frac{1}{2U}\frac{dU}{dt},\ \ \Gamma^0_{11} = \frac{a}{6U^{5/3}}\frac{dU}{dt},\ \ \Gamma^0_{22} = \frac{ar^2}{6U^{5/3}}\frac{dU}{dt},\ \ \Gamma^0_{33} = \sin^2\Theta\Gamma^0_{22}\ , \\ \Gamma^1_{01} &= \frac{1}{6U}\frac{dU}{dt}\ ,\ \ \Gamma^1_{22} = -r,\ \ \Gamma^1_{33} = \sin^2\Theta\Gamma^1_{22},\ \ \Gamma^2_{02} = \frac{1}{6U}\frac{dU}{dt}, \\ \Gamma^2_{12} &= r,\ \ \Gamma^2_{33} = -\sin\Theta\cos\Theta,\ \ \Gamma^3_{03} = \frac{1}{6U}\frac{dU}{dt},\ \ \Gamma^3_{13} = r,\ \ \Gamma^3_{23} = \cos\Theta\ . \end{aligned} \tag{17}$$

All the other Christoffel symbols are zero. The variables $R_{\mu\nu}$ and R, which enter equations (1) are:

$$R_{\mu\nu} = \partial_\rho \Gamma^\rho_{\mu\nu} - \partial_\nu \Gamma^\rho_{\mu\rho} + \Gamma^\rho_{\mu\nu}\Gamma^\sigma_{\rho\sigma} - \Gamma^\rho_{\mu\sigma}\Gamma^\sigma_{\rho\nu} \qquad (18)$$

$$R = R_{\alpha\beta} g^{\alpha\beta} = g^{\alpha\beta}(\partial_\rho \Gamma^\rho_{\alpha\beta} - \partial_\beta \Gamma^\rho_{\alpha\rho}) + g^{\alpha\beta}(\Gamma^\rho_{\alpha\beta}\Gamma^\sigma_{\rho\sigma} - \Gamma^\rho_{\alpha\sigma}\Gamma^\sigma_{\rho\beta}) \; . \qquad (19)$$

By substituting expressions (17) into formulas (18) and (19) and by using the previously obtained expressions from equations (1), we find

$$\frac{1}{12U^3}\dot{U}^2 - 8\pi\rho(t) + \frac{m^2}{2}\left(1 + \frac{1}{2U}\right) - \frac{3m^2}{4a}U^{-1/3} = 0 \; , \qquad (20)$$

$$\frac{1}{3U^2}\ddot{U} - \frac{5}{12U^3}\dot{U}^2 + 8\pi p(t) + \frac{m^2}{2}\left(1 - \frac{1}{2U}\right) - \frac{m^2}{4a}U^{-1/3} = 0 \; . \qquad (21)$$

Here $\dot{U} = \frac{dU}{dt}, \ddot{U} = \frac{d^2U}{dt^2}$. If we write equation (1) in the form

$$\frac{m^2}{2}\gamma_{\mu\nu} = 8\pi\left(T_{\mu\nu} - \frac{1}{2}g_{\mu\nu}T\right) - R_{\mu\nu} + \frac{m^2}{2}g_{\mu\nu} \; , \qquad (1a)$$

then it is easy to get convinced that for the homogeneous and isotropic Universe defined by interval (15) the following equations are valid:

$$g_{0i} = 0, \quad R_{0i} = 0, \quad i = 1, 2, 3 \; .$$

From equation (1a) it follows that in an inertial reference frame with interval (5) we have

$$T_{0i} = 0, \quad \text{and therefore } \; u_i = 0 \; .$$

It means that the reference frame in which the matter of the Universe is at rest in an inertial frame. Therefore, the so-called "expansion" of the Universe, which is observed from the red shift effects arises due to the change of the gravitational field in time. It should be especially stressed that the Nature itself prefers this inertial reference frame. So, the considered theory automatically fulfils the Mach principle.

If we make a transformation to the proper time $d\tau$

$$d\tau = \sqrt{U}dt \; ,$$

and introduce the notion

$$R^2 = U^{1/3}(t) \; , \qquad (22)$$

then interval (15) takes the following form

$$ds^2 = d\tau^2 - aR^2(\tau)[dx^2 + dy^2 + dz^2] \; . \qquad (23)$$

Equations (20) and (21) for the function $R(\tau)$ take the form

$$\left(\frac{1}{R}\frac{dR}{d\tau}\right)^2 = \frac{8\pi G}{3}\rho(\tau) - \frac{\omega}{R^6}\left(1 - \frac{3R^4}{a} + 2R^6\right), \qquad (24)$$

$$\frac{1}{R}\frac{d^2R}{d\tau^2} = -\frac{4\pi G}{3}\left(\rho + \frac{3p}{c^2}\right) - 2\omega\left(1 - \frac{1}{R^6}\right), \tag{25}$$

here

$$\omega = \frac{1}{12}\left(\frac{mc^2}{\hbar}\right)^2 . \tag{26}$$

From equation (24) in the domain, where $R >> 1$, it follows that the matter density of the Universe will be equal to

$$\rho(\tau) = \rho_c(\tau) + \frac{1}{16\pi G}\left(\frac{mc^2}{\hbar}\right)^2 , \tag{27}$$

where $\rho_c(\tau)$ is a critical density determined from the Hubble "constant"

$$\rho_c(\tau) = \frac{3H^2}{8\pi G}, \quad H(\tau) = \frac{1}{R}\left(\frac{dR}{d\tau}\right). \tag{28}$$

The solution of the gravitational equations (24) and (25) for the isotropic vectors V^μ on the light cone of Minkowski space $\gamma_{\mu\nu}V^\mu V^\nu = 0$ should fulfil the causality condition

$$g_{\mu\nu}V^\mu V^\nu \leq 0 , \tag{29}$$

which in our case takes the form

$$R^2(R^4 - a) \leq 0 . \tag{30}$$

To fulfil the causality condition in the entire domain of values for $R(\tau)$, it is quite natural to suppose that

$$a = R^4_{\max} . \tag{31}$$

It follows from equation (24) that $\frac{dR}{dt}$ takes a zero value at the sufficiently large value of $R = R_{\max}$, that is, the "expansion" of the Universe stops and then it changes for the "contraction" phase. This end of expansion is due to the last term in equation (24), which plays the role of a cosmological constant. In this stop point the matter density according to (24) has a minimal value equal to

$$\rho_{\min} = \frac{3}{4\pi}\frac{\omega}{G}\left(1 - \frac{1}{R^6_{\max}}\right) \simeq \frac{1}{16\pi G}\left(\frac{mc^2}{\hbar}\right)^2 . \tag{32}$$

It is easy to see from relation (27) that the matter density of the Universe must be higher than the critical density, that is, there should be a hidden matter mass. At the same time, if we choose the graviton mass equal to $10^{-66}g$, then the contribution of the second term to (27) is not higher than 2% of the density ρ_c, which is not forbidden by the contemporary data.

From the covariant conservation law, which is a consequence of equations (1) and (2)

$$\nabla_\mu \tilde{T}^{\mu\nu} = \partial_\mu \tilde{T}^{\mu\nu} + \Gamma^\nu_{\alpha\beta}\tilde{T}^{\alpha\beta} = 0 , \tag{33}$$

where

$$\tilde{T}^{\mu\nu} = \sqrt{-g}T^{\mu\nu},$$

we can get the following equation

$$\frac{1}{R}\frac{dR}{d\tau} = -\frac{1}{3(\rho + \frac{p}{c^2})}\frac{d\rho}{d\tau} \ . \tag{34}$$

For the radiation dominant stage of the Universe evolution we have

$$p = \frac{1}{3}\rho c^2. \tag{35}$$

From equation (34) with account for (35) we get for the radiation density ρ_r:

$$\rho_r = \frac{A}{R^4(\tau)} \ . \tag{36}$$

Here A is an integration constant. We can get from equation (34) at the stage of the Universe evolution, when the non-relativistic matter is dominant and the pressure can be neglected that

$$\rho_m = \frac{B}{R^3(\tau)} \ , \tag{37}$$

here B is an integration constant.

Let us suppose that at some moment τ_0 the density of radiation ρ_r becomes equal to the matter density ρ_m

$$\rho_r(\tau_0) = \rho_m(\tau_0) \ . \tag{38}$$

By substituting expressions (36) and (37) into (38), we get

$$A = BR(\tau_0) = BR_0 \ . \tag{39}$$

As the matter is predominant at the late stages, we get from formula (37)

$$B = \rho_{\text{min}} R^3_{\text{max}} \ . \tag{40}$$

Therefore

$$\rho \simeq \rho_r = \frac{\rho_{\text{min}} R_0 \cdot R^3_{\text{max}}}{R^4}, \quad R \le R_0 \ . \tag{41}$$

$$\rho \simeq \rho_m = \rho_{\text{min}} \left(\frac{R_{\text{max}}}{R}\right)^3, \quad R \ge R_0 \ . \tag{42}$$

The modern radiation density (including three species of neutrino, which are supposed massless for definiteness) and the critical matter density according to the observational data (see, for example, [6]) are equal to

$$\rho_r(\tau_c) = 8 \cdot 10^{-34} \frac{\text{g}}{\text{cm}^3}, \quad \rho_m(\tau_c) = 10^{-29} \frac{\text{g}}{\text{cm}^3} \ . \tag{43}$$

We have to relate the "hidden" mass to the matter density as the matter density $\rho_m(\tau_c)$ is close to the critical density $\rho(\tau_c)$ determined by the Hubble "constant" under our choice for the graviton mass according to relation (27). We mean that matter is all the forms of substance excluding the gravitational field.

According to formulae (41), (42) and (43), we have

$$\rho_r(\tau_c) = \frac{\rho_{\text{min}} R_0 \cdot R^3_{\text{max}}}{R^4(\tau_c)} = 8 \cdot 10^{-34} \frac{\text{g}}{\text{cm}^3} \, . \tag{44}$$

$$\rho_m(\tau_c) = \rho_{\text{min}} \left(\frac{R_{\text{max}}}{R(\tau_c)} \right)^3 = 10^{-29} \frac{\text{g}}{\text{cm}^3} \, . \tag{45}$$

From the above we get correspondingly

$$R(\tau_c) = \left(\frac{R_0 \rho_{\text{min}} R^3_{\text{max}}}{\rho_r(\tau_c)} \right)^{1/4} = 1,9 \cdot 10^8 \cdot (R_0 \rho_{\text{min}} R^3_{\text{max}})^{1/4} \, , \tag{46}$$

$$R(\tau_c) = \left(\frac{\rho_{\text{min}}}{\rho_m(\tau_c)} \right)^{1/3} R_{\text{max}} = 4,6 \cdot 10^9 \cdot \rho^{1/3}_{\text{min}} \cdot R_{\text{max}} \, . \tag{47}$$

We get from expressions (46) and (47)

$$R_0 = \frac{\rho_r(\tau_c)}{\rho_m^{4/3}(\tau_c)} R_{\text{max}} \cdot \rho^{1/3}_{\text{min}} = 3,7 \cdot 10^5 \cdot \rho^{1/3}_{\text{min}} \cdot R_{\text{max}} \, . \tag{48}$$

Let us introduce notations:

$$\sigma = \frac{4}{3} R_0 \cdot R^3_{\text{max}} \, . \tag{49}$$

According to (41), (48) and (49), we get

$$\rho(\tau) \simeq \rho_r(\tau) = \frac{3}{4} \cdot \frac{\sigma \rho_{\text{min}}}{R^4(\tau)}, \quad R \leq R_0 \, . \tag{50}$$

Supposing that the radiation is predominant at the early stage of expansion in the hot Universe model and taking into account (31), (32), (50), we get from equation (24)

$$H^2 = \left(\frac{1}{R} \cdot \frac{dR}{d\tau} \right)^2 = \omega \left[\frac{3\sigma}{2R^4} - 2 + \frac{3}{R^4_{\text{max}} \cdot R^2} - \frac{1}{R^6} \right] . \tag{51}$$

Equation (51) allows one to derive the law of the Universe expansion at the early stage. It is easy to see that the right hand side of equation (51) becomes zero at the sufficiently small values of $R = R_{\text{min}}$. The main role is played here by the first term in the brackets of equation (24), which is related to the graviton mass.

By introducing a variable $x = \frac{1}{R^2}$, it is easy to find the approximate values for the roots of equation

$$\frac{3}{2} \sigma x^2 - 2 + \frac{3}{R^4_{\text{max}}} x - x^3 = 0 \, , \tag{52}$$

which are the turning points

$$x_1 = \frac{3}{2}\sigma + 0\left(\frac{1}{\sigma^2}\right), \quad x_{2,3} = \pm\sqrt{\frac{4}{3}}\frac{1}{\sqrt{\sigma}} + 0\left(\frac{1}{\sigma^2}\right). \tag{53}$$

From here we determine the turning point

$$R_{\text{min}} = \sqrt{\frac{2}{3\sigma}}\ . \tag{54}$$

Therefore, the cosmological singularity is removed and the expansion of the Universe starts from a finite value $R = R_{\text{min}}$ due to the presence of the graviton mass. In the framework of equation (50) we get

$$\rho_{\text{max}} = \frac{3}{4}\frac{\sigma\rho_{\text{min}}}{R^4_{\text{min}}} = \frac{27}{16}\sigma^3\rho_{\text{min}}\ . \tag{55}$$

According to equation (53), expression (51) can be written in the form

$$H^2 = \omega(x_1 - x)(x - x_2)(x - x_3)\ . \tag{56}$$

In the domain, where the scale factor R takes its values

$$R_{\text{min}} \le R \le R_0\ , \tag{57}$$

the expression for H^2 is substantivally simplified

$$H^2 \simeq \omega x^2(x_1 - x) = \frac{3\sigma\omega}{2R^6}(R^2 - R^2_{\text{min}})\ . \tag{58}$$

Equation (51) in this approximation takes the form

$$\frac{1}{R^2}\left(\frac{dR}{d\tau}\right)^2 = \frac{3\sigma\omega}{2R^6}(R^2 - R^2_{\text{min}})\ . \tag{59}$$

We get from the above

$$\tau = \frac{\sqrt{2}}{\sqrt{3\sigma\omega}}\int\limits_{R_{\text{min}}}^{R}\frac{y^2dy}{\sqrt{y^2 - R^2_{\text{min}}}}\ . \tag{60}$$

After the integration we obtain

$$\tau = \frac{R^2_{\text{min}}}{\sqrt{6\sigma\omega}}[z\sqrt{z^2-1} + \ln(z + \sqrt{z^2-1})]\ , \tag{61}$$

where

$$z = \frac{R}{R_{\text{min}}}\ .$$

By using relations (54) and (55), we get

$$\frac{R_{\mathrm{min}}^2}{\sqrt{6\sigma\omega}} = \frac{1}{2\sqrt{2\omega}} \left(\frac{\rho_{\mathrm{min}}}{\rho_{\mathrm{max}}}\right)^{1/2} . \tag{62}$$

Substituting the value of ρ_{min} from equation (32), we obtain from the above

$$\frac{R_{\mathrm{min}}^2}{\sqrt{6\sigma\omega}} = \sqrt{\frac{3}{32\pi G\rho_{\mathrm{max}}}} . \tag{63}$$

Taking into account equation (63) in (61), we get

$$\tau = \sqrt{\frac{3}{32\pi G\rho_{\mathrm{max}}}} \left[z\sqrt{z^2-1} + \ln(z+\sqrt{z^2-1})\right] . \tag{64}$$

In the vicinity of $R \simeq R_{\mathrm{min}}$ we obtain from (64)

$$R(\tau) = R_{\mathrm{min}} \left[1 + \frac{4\pi G}{3}\rho_{\mathrm{max}}\tau^2\right] . \tag{65}$$

Just in this domain a principal difference from the Friedmann model appears as our model removes the cosmological singularity. As $G\rho_{\mathrm{max}}$ is large, $R(\tau)$ rises rapidly from its minimal value.

In this domain $R/R_{\mathrm{min}} >> 1 (R < R_0)$ the dependence of the matter density on the time determined by equation (50) is the following:

$$R(\tau) = R_{\mathrm{min}} \left(\frac{32\pi G}{3}\rho_{\mathrm{max}}\right)^{1/4} \tau^{1/2}, \tag{66}$$

when relations (54), (55) and (66) are taken into account. So, this result coincides with the known expression given by the Friedmann model

$$\rho(\tau) = \frac{3}{32\pi G\tau^2} . \tag{67}$$

Let us now determine the time moment for the transition of the radiation dominated stage of the Universe expansion to the non-relativistic matter dominant stage. According to (66), we have

$$R_0^2 = R_{\mathrm{min}}^2 \left(\frac{32\pi G}{3}\rho_{\mathrm{max}}\right)^{1/2} \tau_0 . \tag{68}$$

We can find from here that

$$\tau_0 = \left(\frac{R_0}{R_{\mathrm{min}}}\right)^2 \left(\frac{3}{32\pi G\rho_{\mathrm{max}}}\right)^{1/2} . \tag{69}$$

Taking into account equation (63), we get

$$\tau_0 = \frac{R_0^2}{\sqrt{6\sigma\omega}} \, . \tag{70}$$

From (49) we can obtain

$$\tau_0 = \frac{1}{2\sqrt{2\omega}} \left(\frac{R_0}{R_{\max}} \right)^{3/2} . \tag{71}$$

Now, with account for (48) and (32), we get

$$\tau_0 = \frac{\rho_r^{3/2}(\tau_c)}{\rho_m^2(\tau_c)} \sqrt{\frac{3}{32\pi G}} = 2.26 \cdot 10^8 \sqrt{\frac{3}{32\pi G}} = 1.5 \cdot 10^{11} \sec \, . \tag{72}$$

Let us now consider the Universe evolution in the case, when the pressure can be neglected. At this stage of the evolution equation (24) can be written in the from

$$\left(\frac{dx}{d\tau} \right)^2 = \frac{\omega x^2}{R_{\max}^6} (x-1)[(2R_{\max}^6 - x^3)(x^2+x+1) - 3x^2] \, , \tag{73}$$

here $x = R_{\max}/R$. From here we get

$$\tau = \tau_0 + \frac{R_{\max}^3}{\sqrt{\omega}} \int\limits_{\frac{R_{\max}}{R}}^{\frac{R_{\max}}{R_0}} \frac{dx}{x\sqrt{(x-1)[(2R_{\max}^6 - x^3)(x^2+x+1) - 3x^2]}} \, . \tag{74}$$

Taking into account that

$$2R_{\max}^6 >> 3 \tag{75}$$

and neglecting the corresponding term, we can write expression (74) in the form

$$\tau = \tau_0 + \frac{R_{\max}^3}{\sqrt{\omega}} \int\limits_{\frac{R_{\max}}{R}}^{\frac{R_{\max}}{R_0}} \frac{dx}{x\sqrt{(x^3-1)(x_1^3 - x^3)}} \, . \tag{76}$$

Here $x_1 = 2^{1/3} R_{\max}^2$.

After the integration we get

$$\tau = \tau_0 + \frac{R_{\max}^3}{3\sqrt{x_1^3\omega}} \left[\arcsin \frac{(x_1^3+1)(\frac{R_{\max}}{R_0})^3 - 2x_1^3}{(\frac{R_{\max}}{R_0})^3(x_1^3-1)} - \arcsin \frac{(x_1^3+1)(\frac{R_{\max}}{R})^3 - 2x_1^3}{(\frac{R_{\max}}{R})^3(x_1^3-1)} \right] . \tag{77}$$

Let us estimate the first term in the square brackets

$$\frac{(x_1^3+1)(\frac{R_{\max}}{R_0})^3 - 2x_1^3}{(\frac{R_{\max}}{R_0})^3(x_1^3-1)} \simeq 1 - 2\left(\frac{R_0}{R_{\max}} \right)^3 , \tag{78}$$

$$\arcsin\left[1-2\left(\frac{R_0}{R_{\rm max}}\right)^3\right] \simeq \arccos 2\left(\frac{R_0}{R_{\rm max}}\right)^{3/2} \simeq \frac{\pi}{2} - 2\left(\frac{R_0}{R_{\rm max}}\right)^{3/2} . \tag{79}$$

Taking into account (79), we find

$$\tau = \tau_0 - \frac{2}{3\sqrt{2\omega}}\left(\frac{R_0}{R_{\rm max}}\right)^{3/2} + \frac{1}{3\sqrt{2\omega}}\left[\frac{\pi}{2} - \arcsin\frac{(x_1^3+1)(\frac{R_{\rm max}}{R})^3 - 2x_1^3}{(\frac{R_{\rm max}}{R})^3(x_1^3-1)}\right] . \tag{80}$$

By using equation (71), we obtain

$$3\sqrt{2\omega}(\tau+\beta\tau_0) = \frac{\pi}{2} - \arcsin\frac{(x_1^3+1)(\frac{R_{\rm max}}{R})^3 - 2x_1^3}{(\frac{R_{\rm max}}{R})^3(x_1^3-1)} , \tag{81}$$

where $\beta = \left(\frac{4\sqrt{2}}{3} - 1\right)$. Expression (81) can be written as follows:

$$\cos\lambda(\tau+\beta\tau_0) = \frac{(\alpha+1)(\frac{R_{\rm max}}{R})^3 - 2\alpha}{(\frac{R_{\rm max}}{R})^3(\alpha-1)} , \tag{82}$$

where

$$\lambda = 3\sqrt{2\omega} = \sqrt{\frac{3}{2}}\left(\frac{mc^2}{\hbar}\right), \quad \alpha = 2R_{\rm max}^6 . \tag{83}$$

From expression (82) we get

$$R(\tau) = \left[\frac{\alpha}{2}\right]^{1/6} \cdot \left[\frac{(\alpha+1) - (\alpha-1)\cos\lambda(\tau+\beta\tau_0)}{2\alpha}\right]^{1/3} . \tag{84}$$

Due to equation (42), there is a relation

$$\frac{\rho_m(\tau)}{\rho_{\rm min}} = \left[\frac{R_{\rm max}}{R(\tau)}\right]^3 , \tag{85}$$

By taking into account (84), we obtain

$$\rho_m(\tau) = \frac{2\alpha\rho_{\rm min}}{(\alpha+1) - (\alpha-1)\cos\lambda(\tau+\beta\tau_0)} . \tag{86}$$

Starting from formula (86) under condition $\tau >> \tau_0$, we obtain

$$\rho_m(\tau) = \frac{\rho_{\rm min}}{\sin^2\frac{\lambda(\tau+\beta\tau_0)}{2}} , \tag{87}$$

and analogously from formula (84) we get

$$R(\tau) = R_{\rm max}\sin^{2/3}\frac{\lambda(\tau+\beta\tau_0)}{2} . \tag{88}$$

In the domain $\frac{\lambda(\tau+\beta t_0)}{2} << 1$ we have

$$\rho_m(\tau) = \frac{1}{6\pi G(\tau + \beta\tau_0)^2} \ , \tag{89}$$

$$R(\tau) = R_{\max}\left[\frac{\lambda(\tau+\beta\tau_0)}{2}\right]^{2/3} . \tag{90}$$

Under the condition $\tau >> \beta\tau_0$ formulae (89) and (90) give the time dependence of $\rho_m(\tau)$ and $R(\tau)$, which is analogous to the obtained from the Friedmann model. By using expression (88), we define the Hubble "constant"

$$H = \frac{1}{R}\frac{dR}{d\tau} = \frac{\lambda}{3}\text{ctg}\frac{\lambda(\tau+\beta\tau_0)}{2} \ , \tag{91}$$

and the deceleration parameter of the Universe

$$q = -\ddot{R}\frac{R}{\dot{R}^2} = -1 + \frac{3}{1+\cos\lambda(\tau+\beta\tau_0)} = \frac{1}{2} + \frac{3}{2}\text{tg}^2\frac{\lambda(\tau+\beta\tau_0)}{2} \ . \tag{92}$$

Therefore, the deceleration parameter q for the flat Universe is higher, than 1/2, when the graviton mass is present (it is different from the Friedmann model case, where it is equal to 1/2 exactly). There is a relation between the deceleration parameter q and the Hubble "constant"

$$H = \frac{\lambda}{\sqrt{3}}\cdot\frac{1}{\sqrt{2q-1}} = \frac{\pi}{\sqrt{3}}\cdot\frac{1}{\tau_{max}\sqrt{2q-1}} \ , \tag{93}$$

where

$$\tau_{\max} = \frac{\pi}{\lambda} = \sqrt{\frac{2}{3}}\cdot\frac{\pi\hbar}{mc^2} \ . \tag{94}$$

It follows from formulae (93) and (94) that, if the modern values of the Hubble constant and the parameter q_c could be measured with the required precision, then it will be possible to determine the graviton mass. The critical density defined by the Hubble "constant" is equal to

$$\rho_c = \frac{3H^2}{8\pi G} = \rho_{\min}\text{ctg}^2\frac{\lambda(\tau+\beta\tau_0)}{2} \ . \tag{95}$$

Comparing the expression for ρ_c with expression (87), we obtain

$$\rho_m(\tau) = \rho_c(\tau) + \rho_{\min} \ . \tag{96}$$

This relation takes place in the domain $\tau >> \tau_0$. As the critical density in our time is much higher than the observed matter density, it follows from relation (96) that the Universe contains a large "hidden" matter density. We show now that the main parameters of the Universe evolution $R_{\max}$ and $R_{\min}$, which determine the turning points, are expressed through the maximal density of matter $\rho_{\max}$ and the graviton mass. The graviton mass m determines the minimal matter density $\rho_{\min}$. By using formulae (48), (49) and (55), we can easily derive the following relation:

$$R_{\max} = \frac{\rho_m^{1/3}(\tau_c)}{\rho_r^{1/4}(\tau_c)} \cdot \left(\frac{\rho_{\max}}{4\rho_{\min}^2}\right)^{1/12} = 3.6 \cdot 10^{-2} \left(\frac{\rho_{\max}}{\rho_{\min}^2}\right)^{1/12}. \tag{97}$$

The second turning point $R_{\min}$ can be similarly expressed through $\rho_{\max}$ by using relations (62) and (54)

$$R_{\min} = \left(\frac{\rho_{\min}}{2\rho_{\max}}\right)^{1/6}. \tag{98}$$

It is evident from (98) and (97) that the presence of a graviton mass does not only remove the cosmological singularity, but also stops the process of Universe expansion, which transforms into the contraction phase.

Therefore, the evolution of the homogeneous and isotropic Universe is determined by the contemporary observational data (43), i.e. by the maximal matter density and the graviton mass. Let us mention that the homogeneous and isotropic matter distribution is possible in the framework of the considered model only when the graviton mass is not equal to zero. Indeed, according to equations (50) and (55), the constant A in the expression (36) is equal to $A = \rho_{\max}^{1/3}\left(\frac{\rho_{\min}}{2}\right)^{2/3}$. Therefore the constant A in accordance with (32) is proportional to $m^{4/3}$ under the fixed value of $\rho_{\max}$ and turns to zero under $m = 0$. So, in this case the Universe does not contain any matter and its geometry is pseudo-Euclidean. The maximal matter density of the Universe is not fixed in this model. It is related to the integral of motion. This is easy to see. Let us write equation (25) as follows:

$$\frac{d^2R}{d\tau^2} = -4\pi G\left(\rho + \frac{p}{c^2}\right)R + \frac{8\pi G}{3}\rho R - 2\omega\left(R - \frac{1}{R^5}\right). \tag{99}$$

By determining the value of $\left(\rho + \frac{p}{c^2}\right)$ from (34) and substituting it into (99), we obtain

$$\frac{d^2R}{d\tau^2} = \frac{4\pi G}{3}\frac{d}{dR}(\rho R^2) - \omega\frac{d}{dR}\left(R^2 + \frac{1}{2R^4}\right). \tag{100}$$

If we introduce the notion

$$V = -\frac{4\pi G}{3}\rho R^2 + \omega\left(R^2 + \frac{1}{2R^4}\right), \tag{101}$$

it is possible to write equation (100) as a Newtonian equation of motion

$$\frac{d^2R}{d\tau^2} = -\frac{dV}{dR}, \tag{102}$$

where V plays a role of the potential.

Multiplying equation (102) onto $\frac{dR}{d\tau}$ we get

$$\frac{d}{d\tau}\left(\frac{1}{2}\left(\frac{dR}{d\tau}\right)^2 + V\right) = 0 \, . \tag{103}$$

We obtain from the above

$$\frac{1}{2}\left(\frac{dR}{d\tau}\right)^2 + V = E \, . \tag{104}$$

Here E is an integral of motion, the analog of the energy in classical mechanics. By comparing eqs.(104) and (25) and taking into account (31), we get

$$R^4_{\mathrm{max}} = \frac{1}{8E}\left(\frac{mc^2}{\hbar}\right)^2 . \tag{105}$$

By substituting expression (97) into equation (105), we obtain

$$E = 7.4 \cdot 10^4 \left[\frac{(\frac{mc^2}{\hbar})^{10}}{(16\pi G)^2 \rho_{\mathrm{max}}}\right]^{1/3} . \tag{106}$$

So, ρ_{max} is an integral of motion, which is given by the initial conditions of the dynamical system. The above analysis shows, that the model of homogeneous and isotropic Universe according to the RTG evolves cyclewise from some maximal density ρ_{max} to the minimal one ρ_{min} and vise versa.

The Universe can be "flat" only. This structure of the homogeneous and isotropic Universe maintains a definite relation (96) between the matter density and the critical density defined by the Hubble "constant". This theory predicts the presence of a large "hidden" mass of mater in the Universe due to this relation. The Universe is infinite and it exists an infinite time. During this infinite time an intensive exchange of information between its domains has taken place and just this has led to the homogeneity and isotropy of the Universe with some structure for inhomogeneities. For simplicity this inhomogeneity is not taken into account in the model of homogeneous and isotropic Universe. The obtained results are considered as the zero order approximation on the background of which the evolution of inhomogeneities arising from the gravitational instability are usually studied.

The cyclic model of the Universe attracted the attention of A.D.Sakharov, who wrote the following in this relation [7]: "*I wrote on pulsations in the future. But is it possible to imagine such a model of the Universe, which leads to the infinite sequence of pulsations containable both to the future and to the past? Probably there exists at least one variant. Let us consider the spatially flat infinite Universe. Let us suppose that a term with the so-called cosmological constant is present in equations of the General Relativity Theory..... We suggest that the cosmological constant is negative, which is equivalent to the "self-attraction" of the vacuum and leads to the periodical pulsations of the Universe. Then the growth of entropy taking place* *due to the Second Law of Thermodynamics does not lead to any quality differences between the pulsations because the volume, the radius of curvature and entropy of the Universe are infinite.*"

It is worth to mention that the introduction of the negative cosmological term does not lead to the oscillating model for the Universe because the process of contraction does not stop and leads to the infinite density. Indeed according to the RTG the homogeneous and isotropic Universe model can be spatially flat and oscillating only. What was the maximal density of matter ρ_{max} in the Universe? There is an attractive possibility to make a hypothesis that $\rho_{\max}$ is fixed by the world constants. In this case the Planckian density is usually taken as $\rho_{\max}$. But there is a problem of extra production of monopoles arising in the Grand Unification theories. To remove it one usually exploits the mechanism for "burning off" the monopoles in the process of the inflationary expansion caused by the Higgs bosons.

The model considered in this article gives another, alternative, opportunity. The value of $\rho_{\max}$ is connected to the integral of motion E according to eq.(106) and can be considerably lower than the Planckian density. In that case the temperature of the early Universe can be not enough for the production of monopoles and the problem of their extra production is trivially removed.

The homogeneous and isotropic model of the Universe is applicable even in the absence of matter for the GRT with the cosmological term. The solution of the GRT equations for that case was found by de Sitter. This solution corresponds to the curved four-dimensional space-time. It means that gravitational field can exists without matter. What is the source of this field? One treats the energy of vacuum as the source of this gravitational field and identifies it with a cosmological constant [8]. In the RTG equation (24) in the absence of matter ($\rho = 0$) according to expression (32) $R_{max} = 1$ takes the following form:

$$\left(\frac{1}{R}\frac{dR}{d\tau}\right)^2 = -\frac{2\omega}{R^6}(R^2-1)^2(R^2+\frac{1}{2})\ . \tag{107}$$

It follows from the above that $R \equiv 1$ and, consequently, the space-time geometry is pseudo-Euclidean in the absence of matter. So, according to the RTG the gravitational field is absent, if there is no matter in the Universe and therefore the vacuum has no energy. Due to that the cosmological constant can not be equivalent to the vacuum energy, which is equal to zero. The cosmological constant is determined by the graviton mass and is equal to

$$\Lambda = \frac{1}{2}\left(\frac{mc}{\hbar}\right)^2 \tag{108}$$

in the RTG. The graviton mass is extremely small.

Let us give here the values of the same quantities, which define the evolution of the homogeneous and isotropic Universe. Take the graviton mass equal to $m = 10^{-66}$g and the modern Hubble constant value as

$$H_c \simeq 74\frac{\text{km}}{\text{secMps}}\ . \tag{109}$$

Then for our moment of time the values of τ_c, q_c are equal to

$$\tau_c \simeq 3 \cdot 10^{17}\text{sec}, \quad q_c = 0.59, \quad \rho_c \simeq 10^{-29}\frac{g}{\text{cm}^3} \ . \tag{110}$$

According of relation (94) half of the cycle evolution period is equal to

$$\tau_{max} = 9\pi \cdot 10^{17} sec \ . \tag{111}$$

It is worth to especially stress that parameters τ_c, H_c, q_c defining the Universe evolution are now practically independent of the maximal matter density ρ_{max}. The maximal temperature (and, consequently, the maximal density also), which could be reached in the Universe may be determined by such phenomena taking part under these extreme conditions, the consequences of which are observable now. The special role is played here by the gravitational field containing the most complete information on the extreme conditions in the Universe. There are no famous problems of singularity, causality and flatness, which take place in the GRT for the homogeneous and isotropic Universe models. We need not any stage of inflation for solving these problems.[1]

In conclusion let us define the particle horizon and the event horizon. In correspondence with integral (23), we have for a light ray

$$\frac{dr}{d\tau} = \frac{1}{\sqrt{a}R(\tau)} \ . \tag{112}$$

The distance, which is passed by the light up to the moment τ equals

$$d_r(\tau) = R(\tau)\int\limits_0^{r(\tau)} dr = R(\tau)\int\limits_0^{\tau} \frac{d\tau'}{R(\tau')} \ . \tag{113}$$

We should substitute for R expression (64) for interval $(0, \tau_0)$ and expression (88) for interval (τ_0, τ). To make an approximate estimation of $d_r(\tau)$ let us take expression

$$R(\tau) = R_{\text{max}} \sin^{2/3}\frac{\pi\tau}{2\tau_{\text{max}}} \ , \tag{114}$$

for the entire interval of integration

$$d_r(\tau) = \left[\sin\frac{\pi\tau}{2\tau_{\text{max}}}\right]^{2/3} \cdot \int\limits_0^{\tau} \frac{d\tau'}{(\sin^{2/3}\frac{\pi\tau'}{2\tau_{\text{max}}})} \ . \tag{115}$$

By introducing a change of variable

$$x = \sin\frac{\pi\tau'}{2\tau_{\text{max}}} \ ,$$

[1] Of course, this does not exclude the possibility of the inflationary expansion of the Universe in case it occurs to have equation of state $p = -\rho$ at some stage of its evolution.

we get

$$d_r(\tau) = \frac{2\tau_{\max}}{\pi}\left[\sin\frac{\pi\tau}{2\tau_{\max}}\right]^{2/3}\cdot\int\limits_0^{\sin\frac{\pi\tau}{2\tau_{\max}}}\frac{dx}{x^{2/3}\sqrt{1-x^2}} =$$

$$= \frac{6\tau_{\max}}{\pi}\sin\frac{\pi\tau}{2\tau_{\max}}F(\frac{1}{2},\frac{1}{6},\frac{7}{6},y)\ ,$$

here $y = sin^2\frac{\pi\tau}{2\tau_{\max}}$, $F(a,b,c,z)$ is the hypergeometric Gauss function.

For the modern time moment τ_c taking into account (110), we find the radius of the observable part of the Universe

$$d_r(\tau_c) \simeq 3c\tau_c = 2,7\cdot 10^{28}\text{cm}\ . \tag{116}$$

The particle horizon for the half-period of evolution $\tau_{\max}$ is equal to

$$d_r(\tau_{\max}) = \frac{c\tau_{\max}}{\sqrt{\pi}}\cdot\frac{\Gamma(1/6)}{\Gamma(2/3)}\ . \tag{117}$$

The event horizon is given by the following expression

$$d_c(\tau) = R(\tau)\int\limits_\tau^\infty\frac{d\tau'}{R(\tau')}\ . \tag{118}$$

As integral (118) turns into infinity, the event horizon is absent in our case. This means that the information on events taking part in any domain of the Universe at the moment τ will reach us.

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