LONG-RANGED FORCES AND ENERGY NON-CONSERVATION IN (1+1)-DIMENSIONS

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We consider whether local and causal non-conservation of energy may occur in generally covariant theories with long-ranged fields (analogs of Newton's gravity) whose source is energy–momentum. We find that such a possibility exists in (1+1) dimensions.

1. Introduction

Generally covariant (1+1)-dimensional theories provide convenient framework for considering various suspected properties of quantum gravity in (3+1) dimensions (for reviews see, e.g., refs.[1, 2]). A special feature of (1+1)-dimensional models is that some of them admit the interpretation as string theories in higher dimensional target space.

The simplest model of this sort is literally the theory of closed strings in Minkowski target space of critical dimensions. Indeed, macroscopic string states may be interpreted as (1+1)-dimensional universes [3, 4, 5, 6]. However, one important feature present in (3+1)-dimensional gravity is missing in the simplest stringy model. Namely, in (3+1) dimensions there exists long-ranged gravitational field whose source in energy–momentum (Newton's gravity law), while there is no such field in that stringy model.

A particularly simple (1+1)-dimensional model where the mass (energy–momentum of matter fields) produces long-range effects, is the dilaton gravity with matter that has been widely discussed from the point of view of black hole physics [7] (for careful analysis of the notion of ADM mass in that model see refs.[8, 9]). Here we do not consider black holes, so we simplify the model as much as possible. In particular, we set the (1+1)-dimensional cosmological constant to zero.

An issue that we discuss in this paper within the framework of (1+1)-dimensional dilaton gravity is whether energy conservation is absolute necessity in models exhibiting Newton's gravity law or its analogs. Intuitively, one may suspect that the existence of the long-ranged field associated with energy and momentum may be an obstacle to energy non-conservation in local processes. Indeed, naively any local process in which energy changes would lead to the instantaneous change of the long-ranged field everywhere in space. This "action-at-a-distance" would itself contradict locality and causality. We will see, however, that this is not the case, at least in (1+1) dimensions.

A motivation for our analysis comes from the long-lasting discussion of baby universes/wormholes and their possible effects on the parent universe [10, 11, 12, 13, 14, 15]. It has been argued [5, 16] that the emission of baby universes in some (1+1)-dimensional models, including dilaton gravity, necessarily leads to energy non-conservation in the parent universe. This in fact may be the case in most theories allowing for baby universes/wormholes: these processes may give rise to the loss of quantum coherence in the parent universe, and it has been argued on general grounds [17] that the energy non-conservation is inevitable in modifications of quantum mechanics allowing for the loss of quantum coherence (see, however, refs.[18, 19]).

We will not go into physics of baby universes/wormholes in this paper, and take wore "phenomenological" point of view. Namely, we will assume that classical physics is valid everywhere in space-time except for its small region where unknown processes may violate field equations and lead to non-conservation of energy. The question we address is whether the latter is compatible with locality and causality (no action-at-a-distance).

2. Model and classical solutions

The action for the simplest version of (1+1)-dimensional dilaton gravity with conformal matter can be written in the form similar to ref.[20],

$$S = -\frac{1}{\pi}\int d^2\sigma \,\sqrt{-g}\left(-\frac{\gamma^2}{4}\phi R + g^{\alpha\beta}\partial_\alpha f^i \partial_\beta f^i\right), \tag{1}$$

where ϕ is the dilaton field, f^i are matter fields, and γ is a positive coupling constant analogous to the Planck mass of (3+1)-dimensional gravity. The coupling constant γ may be absorbed into the dilaton field, but we will not do this for book-keeping purposes. The field equations are simplified in the conformal gauge

$$g_{\alpha\beta} = \mathrm{e}^{2\rho}\eta_{\alpha\beta},$$

where η is the Minkowskian metrics in (1+1) dimensions. In this gauge, the fields ρ, ϕ and f^i obey massless free field equations. There are also constraints

$$-\frac{1}{2}\gamma^2\left(\partial_\pm\phi\partial_\pm\rho - \frac{1}{2}\partial_\pm^2\phi\right) + \frac{1}{2}\left(\partial_\pm \mathbf{f}\right)^2 = 0 \tag{2}$$

ensuring that the total energy–momentum tensor vanishes.

Let us outline some classical solutions in this model. We consider infinite one-dimensional space, $\sigma^1 \in (-\infty, +\infty)$, and study localized distributions of matter. In this case one can further specify the gauge and choose

$$\rho = 0 \tag{3}$$

so that the space-time is flat. Let us make one point here. Unlike the conformal gauge choice, the further specification of the gauge, eq.(3), is possible only if the field equations are satisfied everywhere in space-time. Indeed, the residual gauge transformations in the conformal gauge are

$$\sigma_+ \to \sigma'_+ = \sigma'_+(\sigma_+)\;, \qquad \sigma_- \to \sigma'_- = \sigma'_-(\sigma_-).$$

These can be used to set $\rho = 0$ only if ρ obeys the massless free field equation $\partial_+\partial_-\rho = 0$ (whose general solution is $\rho = \rho_+(\sigma_+) + \rho_-(\sigma_-)$).

Once the gauge (3) is chosen, equation (2) determines the dilaton field ϕ for a given matter distribution. Indeed, the solution to eq.(2) is, up to an arbitrary linear function of coordinates,

$$\phi(\sigma) = \phi_+(\sigma_+) + \phi_-(\sigma_-)$$

with

$$\phi_\pm = -\frac{1}{\gamma^2}\int d\sigma'_\pm \; |\sigma_\pm - \sigma'_\pm| \; (\partial_\pm \mathbf{f})^2(\sigma'_\pm). \tag{4}$$

Hence, the energy–momentum of matter fields produces long-ranged dilaton field which has linear behavior at large $|\sigma^1|$. In particular, the ADM mass can be defined as follows,

$$\mu_{ADM} = -\frac{\gamma^2}{2\pi}\left[\frac{\partial\phi}{\partial\sigma^1}(\sigma^1 \to +\infty) - \frac{\partial\phi}{\partial\sigma^1}(\sigma^1 \to -\infty)\right]. \tag{5}$$

In virtue of eq.(4) it is equal to

$$\mu_{ADM} = \int_{-\infty}^{+\infty} d\sigma^1 \; \varepsilon_M(\sigma),$$

where

$$\varepsilon_M = \frac{1}{2\pi}\left[(\partial_0\mathbf{f})^2 + (\partial_1\mathbf{f})^2\right]$$

is the energy density of matter.

It is instructive to study two narrow pulses of matter moving left and right and colliding at $\sigma^1 = 0$. These pulses may be approximated by the delta-function distribution,

$$(\partial_\pm\mathbf{f})^2 = \frac{\pi}{2}\mu\delta(\sigma_\pm), \tag{6}$$

where the normalization is such that the constant μ coincides with the ADM mass. In this case eq.(4) has particularly simple form

$$\phi_\pm = -\frac{\pi}{2\gamma^2}(\mu|\sigma_\pm| + C\sigma_\pm), \tag{7}$$

where an arbitrary integration constant is chosen to be the same for ϕ_+ and ϕ_- .

The dilaton field produced by two lumps of matter of equal energy and opposite momenta, moving towards each other (or from each other) with the speed of light, is shown in fig.1. Needless to say, the linear dependence of ϕ on σ^1 at large $|\sigma^1|$ is nothing but the Coulomb behavior of long-ranged field in one-dimensional space. In this respect the dilaton field in (1+1) dimensions is analogous to gravitational field of Newton's law in (3+1) dimensions.

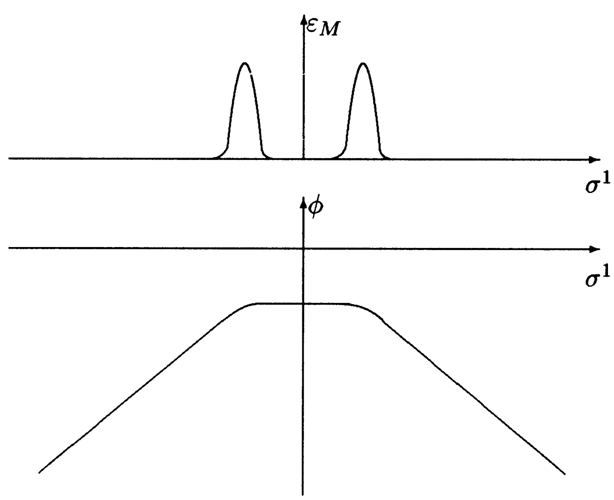

Figure 1:

3. Energy non-conservation

Until now we have considered conventional classical theory. Let us now assume that in a smalll region of space-time the field equations do not hold, and the energy of matter is not conserved.

At first sight, there appears to be a conflict between the locality of the non-conservation of matter energy, and the existence, in the gauge (3), of the long-ranged dilaton field whose strength is determined by the matter energy. Let us see that this conflict is only apparent.

Let us again consider the collision of two narrow pulses of matter. Let us assume that the non-conservation of energy occurs when and where the two pulses collide; let us also assume that the total momentum does not change in the center of mass frame of the colliding pulses. If we were able to use the gauge (3) everywhere in space-time, we would have to introduce action-at-a-distance, as the field ϕ after the collision of the two pulses would be given by the expression similar to eq.(7) but with matter distribution characterized by the final ADM mass μ_f which is different from the initial ADM mass μ_i. However, we cannot insist on imposing the gauge (3) everywhere in space-time, as the field ρ does not obey the field equation $\partial_\alpha\partial^\alpha\rho = 0$ in the collision region at the collision time, by our assumption. We can choose the gauge $\rho = 0$ for the initial configuration only.

If we insist on locality (no action-at-a-distance) we have to require that the field ϕ has the same asymptotics as eq.(7) with $\mu = \mu_i$ even *after* the collision. Then it is possible to set this field equal to eq.(4), with $\mathbf{f}$ equal to the *initial* field distribution, everywhere in space-time. In other words, for infinitely narrow pulses we set ϕ equal to

$$\phi_{i,\pm} = \phi_{f,\pm} = -\frac{\pi}{2\gamma^2}(\mu_i|\sigma_\pm| + C\sigma_\pm) \tag{8}$$

both before and after the collision. Hereafter the subscripts i and f refer to the fields before and after the collision.

The requirement (8) of course implies a certain gauge choice for the field configuration after the collision. The field ρ in the final state is no longer zero, and has to be found by solving eq.(2) with the final distribution of matter. More precisely, we have to find ρ_f from the following equations,

$$-\frac{1}{2}\gamma^2\partial_\pm\phi_i\partial_\pm\rho_f + \frac{1}{4}\gamma^2\partial_\pm^2\phi_i + \frac{1}{2}\left(\partial_\pm\mathbf{f}_f\right)^2 = 0\ . \tag{9}$$

Here ϕ_i is given by eq.(8). We obtain in the case of infinitely narrow pulses

$$\rho_f = -\frac{1}{2C}(\mu_i - \mu_f)[\epsilon(\sigma_+) + \epsilon(\sigma_-)], \tag{10}$$

where ϵ is the usual step function. The integration constants here are chosen in such a way that ρ is equal to zero at spatial infinity, so that ρ does not change instantaneously outside of the collision region either.

The initial and final configurations of the fields are shown in figs. 1 and 2, respectively, by solid lines; the final configuration of the conventional proccess with energy conservation is shown by dashed lines in fig. 2 for comparison. In fig. 2 and in what follows we consider for definiteness the case

$$C > 0\ , \qquad \mu_i > \mu_f .$$

The process shown in these figures is perfectly local and causal: the deviations of all fields from their conventional values propagate out of the collision region (where energy is assumed to be violated) with the speed of light. The final dilaton field ϕ is the same as that of the conventional process; in particular, its long-range behavior is not affected by energy non-conservation. On the other hand, the field ρ in the final state is non-trivial and corresponds to longitudinal gravitational waves. It is the presence of these longitudinal waves that ensures the validity of the constraints after the collision, even though the energy of matter is not conserved and the dilaton field does not change asymptotically.

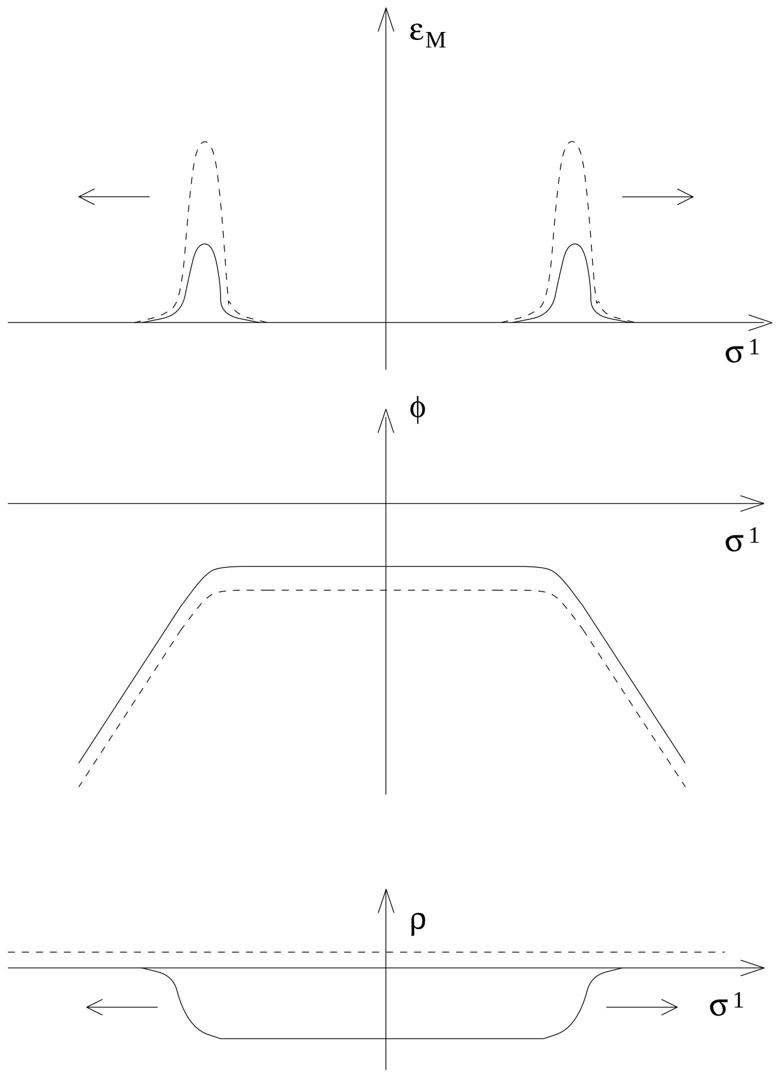

Figure 2:

Of course, the longitudinal ρ-wave of the final state may be gauged away. The corresponding gauge transformation is non-trivial far away from the collision region, i.e., this gauge transformation describes the change of the reference frame everywhere, including spatial infinity. As this gauge transformation provides additional insight, let us perform it explicitly. In the limit of infinitely narrow pulses the gauge transformation to the frame where $\rho'(\sigma')=0$ is determined by the following relations between new and old coordinates σ' and σ,

$$\log \frac{d\sigma'_+}{d\sigma_+} = -\frac{1}{C}(\mu_i - \mu_f)\epsilon(\sigma_+),$$

$$\log \frac{d\sigma'_-}{d\sigma_-} = -\frac{1}{C}(\mu_i - \mu_f)\epsilon(\sigma_-).$$

At *large positive* σ_1 (i.e., to the right of the pulses) we have

$$\sigma'_+ = \mathrm{e}^{\eta}\sigma_+ + \text{const} , \qquad \sigma'_- = \mathrm{e}^{-\eta}\sigma_- + \text{const}$$

with

$$\eta = -\frac{1}{C}(\mu_i - \mu_f).$$

This correspond to a Lorentz boost in the negative direction. In other words, an observer, that is at rest in the old coordinate system, moves towards the pulses with rapidity η in the frame where space-time is flat. This explains why the long-ranged dilaton field measured by this observer remains equal to the initial one even though energy is not conserved.

At *large negative* σ_1 we have

$$\sigma'_+ = \mathrm{e}^{-\eta}\sigma_+ + \text{const} , \qquad \sigma'_- = \mathrm{e}^{\eta}\sigma_- + \text{const}$$

so the original observer on the left of the pulses also moves towards the pulses, as viewed from the new frame. Thus, the inertial observers on the left and on the right of the pulses, that were at rest before the collision, move towards each other after the collision (if energy is not conserved). To establish this fact, however, the two observers have to communicate with each other, so they are able to find out that the total energy of the pulses has changed only after they exchange signals[1]. This explains the causal nature of the whole process.

Thus, we find that energy non-conservation is compatible with locality and causality in (1+1) dimensions even in the presence of long-ranged fields of Newtonian type. This possibility may well be peculiar to (1+1) dimensions, as only in that case spatial infinity is disconnected. Also, Birkhoff's theorem does not hold in (1+1) dimensions, which seems to be of importance too. Still, it remains to be understood whether similar possibility of local and causal energy non-conservation exists in higher dimensions.

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[1] Note that the definition (5) of the ADM mass is given in flat space-time. Note also that this definition involves the dilaton field both to the left and to the right of the pulses.

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